Polaritás növekedés monotonitás 1

$\left(a_n=\dfrac{3n+2}{2n}\ n\in\mathbb{N}\right)$ $\quad a_{n+1}=\dfrac{(3n+1)+2}{2n+1}$

$$a_{n+1}-a_n=\frac{3\cdot(n+1)+2}{2\cdot(n+1)}-\frac{3n+2}{2n}=\frac{(3n+5)\cdot n-(3n+2)(n+1)}{2n(n+1)}=\frac{3n^2+5n-3n^2-5n-2}{2n(n+1)}=$$

$$=\frac{-2\ \}<0}{2n(n+1)\}>0}<0 \text{ szig. mon. csökk.}$$

$\left(a_n=1+\dfrac{(-1)^n}{n}\ n\in\mathbb{N}\right)$

$$a_{n+1}-a_n=\frac{1+(-1)^{n+1}}{n+1}-\left(1+\frac{(-1)^n}{n}\right)=\frac{(-1)^{n+1}}{n+1}-\frac{(-1)^n}{n}=\frac{n\cdot(-1)^{n+1}-(n+1)(-1)^n}{n\cdot(n+1)}=$$

$$=\frac{(-1)^n\cdot(-n-n-1)}{n\cdot(n+1)}=\frac{(-1)^n\cdot(-2n-1)}{n\cdot(n+1)}\Rightarrow \text{nem monoton.}$$

Korlátosság: Határozd meg a korlátot $k=?$

$\left(a_n=\dfrac{2n-5}{n+1}\ n\in\mathbb{N}\right)$ alsó: $k=-\dfrac{3}{2}$ $\quad \dfrac{2n-5}{n+1}\geq-\dfrac{3}{2}\quad \Big|\ 2\cdot(n+1)>0$

$7n\geq 7$

$n\geq 1$ ✓

Felső korlát: $K=2$ $\quad \dfrac{2n-5}{n+1}\leq 2\quad 2n-5\leq 2n+2$

$-5\leq 2$

$\Downarrow$

$K_2=$ felső korlát.

Monotonitás korlátosság

$\left(a_n=\dfrac{n-8}{4n}\ n\in\mathbb{N}\right)$

Monotonitás:

$$a_{n+1}-a_n=\frac{n+1-8}{4n+1}-\frac{n-8}{4n}=\frac{n(n-7)-(n+1)(n-8)}{4n(n+1)}=\frac{n^2-7n-n^2+8n+8}{4n\cdot(n+1)}=\frac{8}{4n(n+1)}\ \overset{0}{1}\Big\}\ \text{Szig. mon. nő}$$

korlátosság $a_1=\dfrac{1-8}{4\cdot1}=-\dfrac{7}{4}$ $\quad$ felső $K=10$ $\quad \dfrac{n-8}{4n}\leq 10\quad n-8\leq 40n$

$-8\leq 39n$ ✓

korlátos

$a_n=\dfrac{2n-1}{n}=2-\dfrac{1}{n}$ $\quad$ sup $a_n=2$ $\forall n\leq 2$

inf $a_n=1$ $\quad$ max nincs $\dfrac{1}{n}>0=2-\dfrac{1}{n}<2$

min $a_n=1$

$c_n=\dfrac{1-(-1)^n}{2}\geq 0$ $\quad 1-(-1)^n\geq 0$

$1\geq(-1)^n$

$(-1)^{2n-1}=-1$

$(-1)^{2n}=1$

$K=1$

$\dfrac{1-(-1)^n}{2}\leq 1\quad 1-(-1)^n\leq 2$

$-1\leq(-1)^n$

$\left(1\ \dfrac{-1^n}{n}\ n\in\mathbb{N}\right)$ $\quad K=\dfrac{1}{2}$ $\ a_n\leq\dfrac{1}{2}$ $\ \forall n\in\mathbb{N}$

$\dfrac{(-1)^n}{n}\leq\dfrac{1}{2}$

$(-1)^n\leq\dfrac{n}{2}$

$\hookrightarrow$ 1 ha páros
-1 ha páratlan

$k=1$ $\quad \dfrac{(-1)^n}{n}\geq-1$ $\quad$ sup $(a_n=$ max $a_n=\dfrac{1}{2}$

$(-1)^n\geq -n$ $\quad$ min a_n inf $a_n=-1$

$d_n=\dfrac{2n+4}{3n}\ (n\in\mathbb{N})$

Monotonitás:

$$d_{n+1}-d_n=\frac{2(n+1)+4}{3(n+1)}-\frac{2n+4}{3n}=\frac{2n+6}{3n+1}-\frac{2n+4}{3n}=\frac{(2n+6)n-(2n+4)(n+1)}{3n(n+1)}=$$

$$=\frac{\cancel{2n^2}+\cancel{6n}-\cancel{2n^2}-2n-\cancel{4n}-4}{3n(n+1)}=\frac{-4}{3n(n+1)}=<\ \text{szig. mon. csökk.}$$

korlátosság $\quad d_1=2$ felső korlát

$k=\dfrac{2}{3}$ $\quad \dfrac{2}{3}\leq\dfrac{2n+4}{3n}$ $\overset{|\cdot 3n>0}{}$ $\ 2n\leq 2n+4\quad 0\leq 4\Rightarrow k=\dfrac{2}{3}$ alsó korlát

$e_n = \frac{2n}{n^2+1} \quad (n \in \mathbb{N})$

Monotonitás:

$$e_{n+1} - e_n = \frac{2(n+1)}{(n+1)^2+1} - \frac{2n}{n+1} = \frac{(2n+2)\cdot(n^2+1) - 2n\cdot(n^2+2n+2)}{(n^2+2n+2)(n^2+1)} =$$

$$= \frac{2n^3+2n+2n^2+2n^3-4n^2-4n}{(n^2+2n+2)(n^2+1)} = \frac{-2n^2-2n+2}{(n+1)^2+1\,[n^2+1]} < 0 \qquad n_{1,2} = \frac{2 \pm \sqrt{20}}{-4} \begin{matrix} \frac{-1+\sqrt5}{2} \\ \frac{1-\sqrt5}{2} \end{matrix}$$

$f_n = \frac{(-1)^n}{n} \quad n \in \mathbb{N})$

$$f_{n+1} - f_n = \frac{(-1)^{n+1}}{n+1} - \frac{(-1)^n}{n} = \frac{n(-1)^{n+1} - (n+1)(-1)^n}{n\cdot(n+1)} = \frac{\overbrace{(-1)^n}^{\pm 1} \cdot \overbrace{(-n-n-1)}^{-2n-1 < 0}}{\underbrace{n\cdot(n+1)}_{>0}} = \text{nem monoton}$$

Küszöbindex

$$\lim_{n\to\infty} \frac{4n+1}{3n-2} = \frac{4}{3}$$

$$\left|\frac{4n+1}{3n-2} - \frac{4}{3}\right| < \varepsilon \qquad \left|\frac{12n+3-4(3n-2)}{3\cdot(3n-2)}\right| < \varepsilon \qquad \left|\frac{11}{3(3n-2)}\right| < \varepsilon$$

$$\left|\frac{12n+3-12n+8}{3(3n-2)}\right| < \varepsilon \qquad \frac{11}{3(3n-2)} < \varepsilon \qquad /\cdot(3n-2)>0 \ /\cdot\frac{1}{\varepsilon}$$

$$\frac{11}{3\varepsilon} < 3n-2 \qquad \frac{\frac{11}{3\varepsilon}+2}{3} < n$$

$$N = \left[\frac{\frac{11}{3\varepsilon}+2}{3}\right] + 1$$

Határozatlan eset

$$\lim_{n\to\infty} (7n^3-2n+1) \overset{\text{"}\infty-\infty\text{"}}{=} \lim_{n\to\infty} n^3\cdot(7 - \underbrace{\tfrac{2}{n^2}}_{0} + \underbrace{\tfrac{1}{n^3}}_{0}) = +\infty$$

$$\lim_{n\to\infty} (-4n^7+2n^6-5n^4+3) \overset{\infty-\infty}{=} \lim_{n\to\infty} n^7\cdot(-4 + \underbrace{\tfrac{2}{n}}_{0} - 5\cdot\underbrace{\tfrac{1}{n^3}}_{0} + \underbrace{\tfrac{3}{n^7}}_{0}) = -\infty$$

$$\lim_{n\to\infty} \frac{2n^2+3n-2}{4n^3-5n+7} = \lim_{n\to\infty} \frac{n^2\cdot(2+\overbrace{\tfrac{3}{n}}^{\to 0} - \overbrace{\tfrac{2}{n^2}}^{\to 0})}{n^2(4 - \underbrace{\tfrac{5}{n^2}}_{0} + \underbrace{\tfrac{7}{n^3}}_{0})} \overset{\frac{2}{\infty}}{=} \underline{\underline{0}}$$

$$\lim_{n\to\infty} \frac{7n^5-2n+1}{5n^5+3} \overset{\frac{\infty}{\infty}}{=} \lim_{n\to\infty} \frac{n^5(7-\frac{2}{n^4}+\frac{1}{n^5})}{n^5(5+\frac{3}{n^5})} = \frac{7}{5}$$

$$\underline{\lim_{n\to\infty}} \frac{-4n^5+2n^3+1}{2n^4+3n-1} \overset{\frac{\infty}{\infty}}{=} \lim_{n\to\infty} \frac{n^5(-4+\frac{2}{n^2}+\frac{1}{n^5})}{n^4(2+\frac{3}{n^3}-\frac{1}{n^4})} \overset{\frac{-\infty}{2}}{=} -\infty$$

$$\underline{\lim_{n\to\infty}} = \frac{5n^6+7n-1}{-4n^2+2} = \frac{n^6(5+\frac{7}{n^5}-\frac{1}{n^6})}{n^2(4+\frac{2}{n^2})} \overset{\frac{+\infty}{-4}}{=} -\infty$$

$$\lim_{n\to\infty} \frac{1+2+\dots+n}{n^2} = \lim_{n\to\infty} \frac{n\cdot(n+1)}{2n^2} = \lim_{n\to\infty} \frac{n^2+n}{2n^2} = \lim_{n\to\infty} \frac{\cancel{n^2}(1+\frac{1}{n})}{\cancel{n^2}\cdot 2} = \underline{\underline{\frac{1}{2}}}$$

Határozatlansági eset

$$\lim_{n\to\infty}\left(\frac{1^2+2^2+\dots+n^2}{n+2}-\frac{n^2}{3}\right)=\lim_{n\to\infty}\left(\frac{n\cdot(n+1)(2n+1)}{6(n+2)}-\frac{n^2}{3}\right)=\lim_{n\to\infty}\left(\frac{2n^3+3n^2+n-(2n^3+4n^2)}{6\cdot(n+2)}\right)$$

$$=\lim_{n\to\infty}\left(\frac{-n^2+n}{6(n+2)}\right)=-\infty$$

$1^2+2^2+\dots+n^2=\frac{n\cdot(n+1)(2n+1)}{6}$ teljes indukcióval $n=1$ $1^2=\frac{1\cdot 2\cdot 3}{6}$ ✓

Konjugálttal való bővítés:

$$\lim_{n\to\infty}\left(\sqrt{n+2}-\sqrt{n+1}\right)\overset{\infty-\infty}{=}\lim_{n\to\infty}\frac{(\sqrt{n+2}-\sqrt{n+1})(\sqrt{n+2}+\sqrt{n+1})}{\sqrt{n+2}+\sqrt{n+1}}=\lim_{n\to\infty}\frac{n+2-(n+1)}{\sqrt{n+2}+\sqrt{n+1}}=$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt{n+2}+\sqrt{n+1}}=\cancel{\frac{1}{\infty}}=0$$

$$\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n+3}\right)=\lim_{n\to\infty}\frac{2n+1-(2n+3)}{\sqrt{2n+1}+\sqrt{2n+3}}=\frac{-2}{\infty+\infty}=0$$

$$\lim_{n\to\infty}\left(\sqrt{n^2+2n+3}-n\right)\overset{\infty-\infty}{=}\lim_{n\to\infty}\frac{n^2+2n+3-n^2}{\sqrt{n^2+2n+3}+\sqrt{n^2}}=\lim_{n\to\infty}\frac{2n+3}{\sqrt{n^2+2n+3}+\sqrt{n^2}}=$$

$$=\lim_{n\to\infty}\frac{n\cdot\left(2+\frac{3}{n}\right)}{n\cdot\sqrt{1+\frac{2}{n}+\frac{3}{n^2}}+1}=\frac{2}{1+1}=\underline{1}$$

$$\lim_{n\to\infty}\frac{\sqrt{n+7}-\sqrt{n-3}}{\sqrt{n+1}}=\lim_{n\to\infty}\frac{\sqrt{n+7}-\sqrt{n-3}\,(\sqrt{n+7}+\sqrt{n-3})}{\sqrt{n+1}\cdot(\sqrt{n+7}+\sqrt{n-3})}=\lim_{n\to\infty}\frac{n+7-(n-3)}{\sqrt{n+1}\cdot\sqrt{n+7}+\sqrt{n-3}}=$$

$$=\lim_{n\to\infty}\frac{10}{\sqrt{n+1}\,(\sqrt{n+7}+\sqrt{n-3})}=0$$

$$\lim_{n\to\infty}\frac{1}{\sqrt{n}\cdot(\sqrt{n^2+1}-n)}=\lim_{n\to\infty}\frac{\sqrt{n^2+1}+\sqrt{n^2}}{\sqrt{n}\cdot\sqrt{n^2+1}-\sqrt{n^2}\quad\sqrt{n^2+1}+\sqrt{n^2}}=\lim_{n\to\infty}\frac{\sqrt{n^2+1}+n}{\sqrt{n}\cdot(n^2+1-n^2)}=$$

$$=\lim_{n\to\infty}\frac{\cancel{n}\cdot\sqrt{1+\frac{1}{n^2}}+1}{\sqrt{n}}=+\infty$$

Következmény

$\lim_{n\to\infty}\left(2+\frac{1}{n}\right)^3=2^3=\underline{8}$ | $\lim_{n\to\infty}\frac{3n^2-n+1}{n^2+3}=2^3=\underline{8}$ | $\lim_{n\to\infty}\sqrt{\frac{2n^2+1}{8n^2+3}}=\sqrt{\frac{2}{8}}=\sqrt{\frac{1}{4}}=\frac{1}{2}$

$\lim_{n\to\infty}\sqrt{\frac{2n+1}{n^5-3n+1}}=\sqrt{0}=0$ | $\lim_{n\to\infty}\left(\frac{2n}{n+1}\right)^{11}=2^{11}$ | $\lim_{n\to\infty}\sqrt[5]{\frac{32n^2+1}{n^2-7}}\ \sqrt[5]{32}=\underline{2}$

Nevezetes sorozatok

$$\lim_{n\to\infty}\frac{n^5-3n^2+1}{100n^2+2n-7}\overset{\frac{\infty}{\infty}}{=}\lim_{n\to\infty}\frac{n^5\cdot\left(1-\frac{3}{n^3}+\frac{1}{n^5}\right)}{n^2\cdot\left(100+\frac{2}{n}-\frac{7}{n^2}\right)}=+\infty$$

$$\lim_{n\to\infty}\left(\sqrt{n^2+6n+1}-\sqrt{n^2+5n+3}\right)\overset{\infty-\infty}{=}\lim_{n\to\infty}\frac{(\sqrt{n^2+6n+1}-\sqrt{n^2+5n+3})\cdot\sqrt{n^2+6n+1}+\sqrt{n^2+5n+3}}{\sqrt{n^2+6n+1}+\sqrt{n^2+5n+3}}=$$

$$\lim_{n\to\infty}\frac{n^2+6n-(n^2+5n+3)}{\sqrt{n^2+6n+1}+\sqrt{n^2+5n+3}}=\lim_{n\to\infty}\frac{n-2}{\sqrt{n^2+6n+1}+\sqrt{n^2+5n+3}}=\lim_{n\to\infty}\frac{n\cdot\left(1-\frac{2}{n}\right)}{n\cdot\left(\sqrt{1+\frac{6}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{5}{n}+\frac{3}{n^2}}\right)}=\frac{1}{2}$$

4

$\left(b_n = \dfrac{\sin n \cdot \frac{\pi}{3}}{n^2} \quad n \in \mathbb{N}\right)$

$-1 \le \sin\left(n \cdot \frac{\pi}{3}\right) \le 1 \quad | \frac{1}{n^2} > 0$

$\underbrace{-\frac{1}{n^2}}_{a_n} \le \underbrace{\frac{\sin\left(n \cdot \frac{\pi}{3}\right)}{n^2}}_{b_n} \le \underbrace{\frac{1}{n^2}}_{c_n}$

$\to 0 \leftarrow$ rezultă $\lim_{n\to\infty} b_n = 0$

$a_n = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)}, \ n \in \mathbb{N}$

$a_1 = \frac{1}{1\cdot 3}$

$a_2 = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5}$

$a_3 = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7}$

Intuitiva

$\frac{1}{2} \frac{1}{(2n-1)(2n+1)} = \frac{1}{2} \frac{(2n+1)-(2n-1)}{(2n-1)(2n+1)} = \frac{1}{2}\left[\frac{2n+1}{(2n-1)(2n+1)} - \frac{2n-1}{(2n-1)(2n+1)}\right] = \cancel{\frac{1}{2} \frac{1}{(2n-1)(2n+1)}}$

$= \frac{1}{2} \cdot \frac{1}{(2n-1)} - \frac{1}{2} \cdot \frac{1}{(2n+1)}$

mătcai rocentor

$\lim_{n\to\infty} (2^n - 3^n - 5^n + 1) \overset{\infty-\infty}{=} \lim_{n\to\infty} 5^n\left(\left(\frac{2}{5}\right)^n - \left(\frac{3}{5}\right)^n - 1 + \left(\frac{1}{5}\right)^n\right) = -\infty$

$\lim_{n\to\infty} \frac{2\cdot 3^n + 4\cdot 5^n}{2^n - 5^n} = \lim_{n\to\infty} \frac{5^n \cdot \left(2\cdot\left(\frac{3}{5}\right)^n + 4\right)}{5^n \cdot \left(\left(\frac{2}{5}\right)^n - 1\right)} = -4$

$\lim_{n\to\infty} \frac{2\cdot 4^n - 3\cdot 5^n + 4\cdot 7^n}{3^{n+1} + 2\cdot 6^n - 8\cdot 7^n} = \lim_{n\to\infty} \frac{7^n\left(2\cdot\frac{4^n}{7} - 3\left(\frac{5}{7}\right)^n + 4\right)}{7^n\left(3\left(\frac{3}{7}\right)^n + 2\left(\frac{6}{7}\right)^n - 8\right)} = \frac{4}{-8} = -\frac{1}{2}$

$\lim_{n\to\infty} \frac{2\cdot 3^{n+2} - 7^{n+1}}{6\cdot 7^n + 9^n} = \lim_{n\to\infty} \frac{7^n\left(18\cdot\left(\frac{3}{7}\right)^n - 7\right)}{9^n\left(6\cdot\frac{7}{9}\right) + 1)} = 0(-7) = 0$

$\lim_{n\to\infty} \frac{4\cdot 5^{n+1} + 3\cdot 11^n}{2\cdot 6^n - 4\cdot 7^n} \quad \lim_{n\to\infty} \frac{11(20\cdot\left(\frac{5}{11}\right) + 3\cdot 1}{7(2\cdot\frac{6}{11} - 4} = \frac{+\infty\cdot 3}{-4} = -\infty$

$\lim_{n\to\infty} \sqrt[6n]{3n+1} \quad 1 \le \sqrt[6n]{3n+1} < \sqrt[n]{3n+1} < \sqrt[n]{3n+n} = \sqrt[n]{4n} = \sqrt[n]{4n} = \sqrt[n]{4} \cdot \sqrt[n]{n}$

$a_n \to 1$, b_n, $\sqrt[n]{4} \to 1$, $\sqrt[n]{n} \to 1$

rezultă $\lim_{n\to\infty} \sqrt[6n]{3n+1} = 1$

$\lim_{n\to\infty} \underbrace{\sqrt[3n]{2n+5}}_{b_n} \le \sqrt[6n]{2n+n} = \sqrt[3n]{2} \cdot \sqrt[3n]{n} = \left(\sqrt[n]{2}\right)^{\frac{1}{3}} \left(\sqrt[n]{n}\right)^{\frac{1}{3}} \to 1 \quad \Rightarrow \lim_{n\to\infty} b_n = 1$

$1 \le a_n$

$\lim_{n\to\infty} \sqrt[7n]{3n\cdot 2} \le \sqrt[7n]{3n\cdot n} = \sqrt[7n]{3} \cdot \sqrt[7n]{n} = \left(\sqrt[n]{3}\right)^{\frac{1}{7}} \cdot \left(\sqrt[n]{n}\right)^{\frac{1}{7}} \to 1 \Rightarrow \lim_{n\to\infty} b_n = 1$

$\lim_{n\to\infty} \left(1 + \frac{1}{2n}\right)^{7n+1} = \lim_{n\to\infty} \left[\overbrace{\left(1 + \frac{1}{2n}\right)^{2n}}^{\to e}\right]^{\frac{7n+1}{2n}} = e^{\frac{7}{2}}$

$\lim_{n\to\infty} \left(\frac{2n+1}{2n-2}\right)^{5n+3} \quad \frac{2n+1}{2n-2} = \frac{(2n-2)+3}{2n-2} = 1 + \frac{1}{\frac{2n-2}{3}} = \lim_{n\to\infty} \left[\left(1 + \frac{1}{\frac{2n-2}{3}}\right)^{\frac{2n-2}{3}}\right]^{\frac{15}{2}} \cdot \left[1 + \frac{1}{\frac{2n-2}{3}}\right]^{8} = e^{\frac{15}{2}} \cdot 1^8 = e^{\frac{15}{2}}$

$\lim_{n\to\infty} \left(\frac{7n+1}{7n+2}\right)^{3n} = \lim_{n\to\infty}$ $\frac{7n+1}{7n+2} = \frac{7n+2-1}{7n+2} = 1 - \frac{1}{7n+2} = 1 + \frac{1}{-7n-2}$ $\lim_{n\to\infty} \left[\left(1+\frac{1}{-7n-2}\right)^{-7n-2}\right]^{\frac{3n}{-7n-2}} = e^{-\frac{3}{7}}$

$\lim_{n\to\infty} \left(\frac{2n+3}{5n+1}\right)^{3n} = 0$ $\lim_{n\to\infty} \frac{2n+3}{5n+1} = \frac{2}{5}$ $\frac{2n+3}{5n+1} \leq \frac{5}{6}$ $\forall n > 1$ $\underset{\downarrow 0}{0} \leq \left(\frac{2n+3}{5n+1}\right)^{3n} \leq \underset{\downarrow 0}{\left[\left(\frac{5}{6}\right)^{n}\right]^{3}}$

$\lim_{n\to\infty} \left(\frac{2n-3}{5n+1}\right)^{3n}$ $\frac{2n-3}{5n-1} \to \frac{2}{5}$ $0 \leq \frac{2n-3}{5n-1} < \frac{2n}{5n-1} \leq \frac{2n}{5n-n} = \frac{1}{2}$

$\left(a_n = \sqrt[7n]{3n^3}\ n \in \mathbb{N}\right)$ $\underset{\downarrow 1}{1} < \sqrt[7n]{3n^3} < \sqrt[n]{3n^3} = \underset{\downarrow 1}{\sqrt[n]{3}} \cdot \underset{\downarrow 1}{\left(\sqrt[n]{n}\right)^3}$ $\lim_{n\to\infty} a_n = 1$

$\left(a_n = \sqrt[3n]{7n-4}\ n \in \mathbb{N}\right)$ $1 < \sqrt[3n]{7n-4} < \sqrt[3n]{7n+0} = \sqrt[3n]{7n} < \sqrt[n]{7n} = \underset{\downarrow 1}{\sqrt[n]{7}} \cdot \underset{\downarrow 1}{\sqrt[n]{n}}$ $\lim_{n\to\infty} a_n = 1$

$\left(a_n = \left(1+\frac{1}{n}\right)^{\frac{1}{n}}\ n \in \mathbb{N}\right)$ $\underset{\downarrow 1}{1} < \left(1+\frac{1}{n}\right)^{\frac{1}{n}} < (1+1)^{\frac{1}{n}} = \underset{\downarrow 1}{\sqrt[n]{2}}$ $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{\frac{1}{n}} = 1$

—o—

$\lim_{n\to\infty} \frac{1}{\sqrt{n+1}} \cdot \frac{3n}{\sqrt{n-1} - \sqrt{2n+3}}$ $\lim_{n\to\infty} \frac{1}{\sqrt{n+1}} \cdot \frac{3n}{\sqrt{2n-1} - \sqrt{2n+3}} = \lim_{n\to\infty} \frac{3n \cdot (\sqrt{2n-1} + \sqrt{n+3})}{(\sqrt{\ } - \sqrt{\ })(\sqrt{\ } + \sqrt{\ })} =$

$\lim_{n\to\infty} \frac{3n \cdot (\sqrt{2n-1} + \sqrt{2n+3})}{\sqrt{n+1}\,(2n-1-(2n+3))} = \lim_{n\to\infty} \frac{3n\,(\sqrt{2n-1}}{-4 \cdot \sqrt{n+1}} + \sqrt{2n+3}) \overset{\frac{\infty}{\infty}}{=} \lim_{n\to\infty} \frac{3\sqrt{n} \cdot \sqrt{n} \cdot (\sqrt{2n+1}\sqrt{2n+3}}{\sqrt{n} \cdot (-4(1+\frac{1}{4})} \overset{\frac{+\infty}{-4}}{=} -\infty$

$\lim_{n\to\infty} \frac{4 \cdot 2^{n+1} - 3 \cdot 5^n}{2 \cdot 3^n - 5^{n+2}} = \frac{4 \cdot 2^{n+1} - 3 \cdot 5^n}{2 \cdot 3^n - 5^n + 2} = \lim_{n\to\infty} \frac{5^n(8 \cdot (\frac{2}{5}) - 3}{5^n(2 \cdot (\frac{3}{5})^2 - 25} = \frac{3}{25}$

$\lim_{n\to\infty} \left(\frac{3n-2}{3n+5}\right)^{4n-2} =$ $\frac{3n-2}{3n+5} = \frac{3n+5-7}{3n+5} = 1 - \frac{7}{3n+5} = 1 + \frac{1}{\frac{3n+5}{-7}}$ $\lim_{n\to\infty} \left[\left(1+\frac{1}{\frac{3n+5}{-7}}\right)^{\frac{3n+5}{-7}}\right]^{\frac{-7(4n-2)}{3n+5}} = e^{-\frac{28}{3}}$