1, $\lim\limits_{n\to\infty}(\sqrt{2}-1)^{2n}\cdot(\sqrt{2}+1)^{n} \overset{0\cdot\infty}{=} \lim\limits_{n\to\infty}(\sqrt{2}-1)^{n}\cdot(\sqrt{2}+1)^{n}$

$[(\sqrt{2}-1)(\sqrt{2}+1)]^n=(2-1)^n$ $\quad \lim\limits_{n\to\infty}(\underbrace{\sqrt{2}}_{\approx 1,4}-1)^n\cdot 1=\underline{0}$

2, $\lim\limits_{n\to\infty}(\sqrt{2}-1)^n(\sqrt{2}+1)^{2n} \overset{0\cdot\infty}{=} +\infty$

3, Legyen $-1<q<1$ számra

$\lim\limits_{n\to\infty}(1+q+q^2+\dots+q^n = \lim\limits_{n\to\infty}\frac{1-q^{n+1}}{1-q}=\frac{1}{1-q}$

$a_n=1+q+q^2+\dots q^n$

$a_n\cdot q=q+q^2+q^3+\dots+q^{n+1}$ $\Big\}\cdot q$

$a_n(1-q)=1-q^{n+1} \Rightarrow a_n=\frac{1-q^{n+1}}{1-q}$

4, $\lim\limits_{n\to\infty}(1+\frac{1}{2}+\frac{1}{4}+\dots\frac{1}{2^n})=\lim\limits_{n\to\infty}\frac{1-\overbrace{(\frac{1}{2})^{n+1}}^{0}}{\underbrace{1-\frac{1}{2}}_{\frac{1}{2}}}=2$

5, $\lim\limits_{n\to\infty}(\frac{3}{10}+\frac{3}{100}+\frac{3}{1000}+\dots+\frac{3}{10^n}=\lim\limits_{n\to\infty}\frac{3}{10}(1+\frac{1}{10}+\frac{1}{10^{n-1}})=\lim\limits_{n\to\infty}\frac{3}{10}\cdot\frac{1-(\frac{1}{10})^n}{1-\frac{1}{10}}=\frac{3}{10}\cdot\frac{10}{9}=\frac{1}{3}$

FÜGGVÉNYEK HATÁRÉRTÉKE

1/

a, $\lim\limits_{x\to 1}\frac{2x+3}{4x-1}=\lim\limits_{n\to\infty}\frac{2\cdot(1+\frac{1}{n})+3}{4(1+\frac{1}{n})-1}=\frac{5}{3}$ $\quad(x_n=1+\frac{1}{n}\ n\in\mathbb{N})$ $\lim\limits_{n\to\infty}x_n=1$

b, $\lim\limits_{x\to+\infty}\frac{2x+3}{4x-1}=\lim\limits_{n\to\infty}\frac{2n+3}{4n-1}=\frac{1}{2}$ $\quad(x_n=n\ n\in\mathbb{N})$ $\lim\limits_{n\to\infty}n=+\infty$

c $\lim\limits_{x\to-\infty}\frac{2x+3}{4x-1}=\lim\limits_{n\to\infty}\frac{2(-n)+3}{4(-n)-1}=\frac{-2}{-4}=\frac{1}{2}$ $\quad(x_n=-n\ n\in\mathbb{N})$ $\lim\limits_{n\to\infty}x_n=-\infty$

2, $f(x)\ \frac{3x^2-2x^2+1}{x-1}$ $\quad$ a, $x_0=2$ $\quad$ b, $x_0=+\infty$ $\quad$ c, $x_0=-\infty$

a, $\lim\limits_{x\to 2}\frac{3x^3-2x^2+1}{x-1}=\frac{3\cdot 2^2-2\cdot 2^2+1}{2-1}=\frac{24-8+1}{1}=17$

b, $\lim\limits_{x\to+\infty}\frac{3x^3-2x^2+1}{x-1}=\lim\limits_{n\to\infty}\frac{3n^3-2n^2+1}{n-1}=+\infty$ $\quad x_n=n\in\mathbb{N}$

c, $\lim\limits_{x\to-\infty}\frac{3x^3-2x^2+1}{x-1}=\lim\limits_{n\to\infty}\frac{3\cdot(-n)^3-2(-n)^2+1}{-n-1}=\lim\limits_{n\to\infty}\frac{(-n)^3[3-2\frac{1}{n}+\frac{1}{(-n)^3}]}{(-n)(1-\frac{1}{-n})}=+\infty$

3, a, $\lim_{x\to 2}(\sqrt{x+5}-\sqrt{x+1})=\sqrt{7}-\sqrt{3}$

b, $\lim_{x\to+\infty}(\sqrt{x+5}-\sqrt{x+1})=\lim_{n\to\infty}(\sqrt{n+5}-\sqrt{n+1})=\frac{(\sqrt{n+5}-\sqrt{n+1})(n+5\cdot\sqrt{n+1})}{\sqrt{n+5}+\sqrt{n+1}}=$

$=\lim_{n\to\infty}\frac{n+5-(n+1)}{\sqrt{n+5}+\sqrt{n+1}}=\lim_{n\to\infty}\frac{4}{\sqrt{n+5}+\sqrt{n+1}}=0$ $\quad x_n=n\in\mathbb{N}$

4, $\lim_{x\to+\infty}(\sqrt{x^2+3x-1}-x)=\lim_{n\to\infty}(\sqrt{n^2+3n-1}-\sqrt{n^2})=\frac{n^2+3n-1-n^2}{\sqrt{n^2+3n-1}+\sqrt{n^2}}=$

$(x_n=n,\ n\in\mathbb{N})\ \lim_{n\to\infty}x_n=+\infty$

$=\lim_{n\to\infty}\frac{n^2+3n-1-n^2}{\sqrt{n^2+3n-1}+\sqrt{n^2}}=\lim_{n\to\infty}\frac{n\cdot(3-\frac{1}{n})}{n\cdot(\sqrt{1+\frac{3}{n}-\frac{1}{n^2}}+1)}=\frac{3}{2}$

5, $\lim_{x\to 1}\left(\frac{2x+3}{2x-5}\right)^{4x}=\left(\frac{5}{-3}\right)^4$

$\lim_{x\to+\infty}\left(\frac{2x+3}{2x-5}\right)^{4x}=\lim_{n\to\infty}\left(\frac{2n+3}{2n+5}\right)=\ldots$

$x_n=n\ n\in\mathbb{N}$

$\frac{0}{0}$ eset

$\lim_{x\to 5}\frac{x^2-5}{x^2-7x+10}\overset{"\frac{0}{0}"}{=}\lim_{x\to 5}\frac{(x-5)(x+1)}{(x-5)(x-2)}\ \lim_{x\to 5}\frac{x+1}{x-2}=\frac{6}{3}=2$

$x^2\cdot 4x+5=0$ $\qquad x^2-4x-5=(x-5)\cdot(x+1)$

$x_{12}\begin{smallmatrix}\nearrow 5\\ \searrow 1\end{smallmatrix}$ $\qquad x^2-7x+10=0$

$x_{12}=\begin{smallmatrix}5\\2\end{smallmatrix}$

2, $\lim_{x\to 5}\frac{x^2-3x-10}{x^2-2x-15}=\lim_{x\to 5}\frac{\cancel{(x-5)}(x+2)}{\cancel{(x-5)}(x+3)}=\frac{7}{8}$ $\quad x^2-3x-10=0\ x_{12}\begin{smallmatrix}\nearrow 5\\ \searrow -2\end{smallmatrix}$ $\quad x^2-2x-15=0\ x_{12}=\begin{smallmatrix}5\\-3\end{smallmatrix}$

$\frac{0}{0}$ eset $\begin{cases}\frac{\sin x}{x}\\ \frac{\operatorname{tg}x}{x}\end{cases}$ esetén $\lim_{x\to 0}\frac{\sin x}{x}=1\quad\lim_{x\to\infty}\frac{\operatorname{tg}x}{x}=1$

1, $\lim_{x\to 0}\frac{\sin x3}{4x}=\lim_{x\to 0}\frac{3}{4}\frac{\sin 3x}{3x}=\frac{3}{4}$

2, $\lim_{x\to 0}\frac{\sin 7x}{\operatorname{tg}5x}\ \lim_{x\to 0}\frac{\sin 7x}{7x}\cdot\frac{5x}{\operatorname{tg}5x}=\frac{7}{5}\cdot 1\cdot 1=\frac{7}{5}$

$\frac{1}{0}$ eset $\lim_{x\to 2}\frac{1}{x-2}=\lim_{x\to 2}\frac{1}{x-2}=\lim_{n\to\infty}\frac{1}{2\times\frac{1}{n}-2}=\lim_{n\to\infty}<+\infty$ $\quad x_n=2\times\frac{1}{n}\ n\in\mathbb{N}$

baloldal $\lim_{\substack{x\to 2\\ x<2}}\frac{1}{x-2}=\lim_{n\to\infty}\left(\frac{1}{(2-\frac{1}{n})-\cancel{2}}\right)=\lim_{n\to\infty}(-n)=-\infty$ $\quad x<2\ (x_n=2-\frac{1}{n}\ n\in\mathbb{N}$

Elemi fgvek deriváltja

$(x^{\alpha})' = \alpha \cdot x^{\alpha - 1}$ $(\alpha \in \mathbb{R})$

$(x^1)' = 1 \cdot x^0 = 1 \cdot 1 = 1$

$(1)' = (x^0)' = 0 \cdot x^{-1} = 0$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(e^x)' = e^x$

$(a^x)' = a^x \ln a$

$(\ln x)' = \frac{1}{x}$

$(\cancel{\log}\ a^x)' = \frac{1}{x \cdot \ln a}$

deriválási szabályok

$(f+g)' = f' + g'$

$(\lambda f)' = \lambda \cdot f'$

$(f \cdot g)' = f' \cdot g + f \cdot g'$

$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$

$(f \circ g)' = (f' \circ g) \cdot g'$

$\left[f''(g(x))\right]' = f'(g(x)) \cdot g'(x)$

1, $f(x) = 3x^5 - 2x^2 + 5$ $\quad f: \mathbb{R} \to \mathbb{R}$

$f'(x) = 15x^4 - 4x \, (+5 \cdot 1 = 1 = 0)$

2, $f(x) = 4x + \frac{1}{x^2} - 3 = 4x + x^{-2} - 3$

$f'(x) = 4 \cdot 1 - 2 \cdot x^{-3}$ 0

3, $f(x) = 7 - \frac{4}{x} + \sqrt{x} - \frac{3}{\sqrt{x^2}} = 7 - 4 \cdot x^{-1} + x^{\frac{1}{2}} - 3 \cdot x^{-\frac{2}{3}}$

$f'(x) = 0 - 4(-1)x^{-2} + \frac{1}{2} \cdot x^{-\frac{1}{2}} - 3 \cdot \left(-\frac{2}{3}\right) x^{-\frac{5}{3}}$

4, $f(x) = 3 \cdot 5^x + 4 \cdot \sin x - 2 \ln x + 7$

$f'(x) = 3 \cdot 5^x \cdot \ln 5 + 4 \cos x - 2 - \frac{1}{x}$

5, $f(x) = 3x\sqrt{x} - \frac{3}{x} + 4 \frac{x\sqrt[3]{x}}{\sqrt{x^3}} = 3 \cdot x^{\frac{3}{2}} - 3 \cdot x^{-7} + 4 - x^{-\frac{1}{6}}$

$f(x) = 3 \cdot \frac{3}{2} \cdot x^{\frac{1}{2}} - 3 \cdot (-7)x^{-8} + 4 - \left(-\frac{1}{6}\right) x^{-\frac{7}{6}}$

6, $f(x) = x^2 \cdot \sin x$

$f'(x) = 2x \sin x + x^2 \cdot \cos x$

7, $f(x) = (x^2 + 2\pi \cdot x + 3) \cdot 4^x + 2$

$f'(x) = (2x + 2\pi) 4^x + (x^2 + 2\pi x + 3) \cdot 4^x \cdot \ln 4$

8 $h(x) = \frac{x+1}{2x-1}$

$h'(x) = \frac{1 \cdot (2x-1) - (x+1) \cdot 2}{(2x-1)^2}$

$h(x)=\frac{\sqrt{x}\cdot\cos x}{4x-1}=\frac{f}{g}$ $\quad f'=\frac{1}{2}\cdot x^{-\frac{1}{2}}\cdot\cos x+\sqrt{x}\cdot(-\sin x)$

$h'(x)=\frac{f'g-f\cdot g'}{g^2}=\frac{\left(\frac{1}{2\sqrt{x}}\cos x-\sqrt{x}\sin x\right)\cdot(4x-1)-\sqrt{x}\cos x\cdot 4}{(4x-1)^2}$

5, $h(x)=\frac{2^x\cdot 3x^2-1}{\pi+\cos x}=\frac{f}{g}$ $\quad f'\ 2^x\ln 2\,(3x^2+1)+2^x-6x$

$h'(x)=\frac{f'\cdot g-f\cdot g'}{g^2}=\frac{(2^x\ln 2\cdot(3x^2+1)+2^x\cdot 6x)(\pi+\cos x)-2^x(3x^2+1)\cdot(-\sin x)}{(\pi+\cos x)^2}$

6, $h(x)=\frac{\sin x+\cos x}{\sin x-\cos x}=\frac{f}{g}$

$h'(x)\ \frac{(\cos x-\sin x)(\sin x-\cos x)-(\sin x+\cos x)(\cos x+\sin x)}{(\sin x-\cos x)^2}$

7 $h(x)=\frac{1+\sqrt{x}}{1-\sqrt{x}}$ $\quad \sqrt{x}=x^{\frac{1}{2}}$

$(x^{\frac{1}{2}})'=\frac{1}{2}x^{-\frac{1}{2}}$

8, $h(x)=\sin(2x+3)$ $\quad f(x)=\sin x\to f'(x)=\cos x$

$g(x)=2x+3\to g'(x)=2$

$h=(f\circ g)'=f'(g(x))\cdot g'(x)=\cos(2x+3)\cdot 2$

9 $h\,x=\cos(x^2)$

$h'(x)=-\sin(x^2)\cdot 2x$

$(x^2)'=2x$

10, $h(x)=\cos^2 x=(\cos x)^2$

$h'(x)=2\cdot\cos x\cdot(-\sin x)$

$f(x)=\cancel{x}^2\to f'=2x$

$g(x)=\cos\to g'=-\sin x$

$h'(g(x))\cdot g'(x)$

11, $h(x)=(4x^5+7)^{10}$ $\to f(x)=x^{10}$ $\quad f'=10x^9$

$\to g(x)=4x^5+7$ $\quad g'=20x^4$

$h'(x)=\underbrace{10(4x^5+7)^9}_{f'(g(x))}\cdot\underbrace{20x^4}_{g'(x)}$

12 $h(x)=\log_2(2x+1)\to f(x)=\log_2 x$ $\quad f'=\frac{1}{x\cdot\ln 2}$

$\to g(x)=2x+1$ $\quad g'=2$

$h'(x)\frac{1}{(2x-1)\ln 2}\cdot 2$

13, $h(x)=\ln\left(\frac{x^2+1}{2x-1}\right)$ $\to f(x)=\ln x$ $\quad f'(x)=\frac{1}{x}$

$\to g(x)=\frac{x^2+1}{2x-1}$ $\quad g'=\frac{2x\cdot(2x-1)-(x^2+1)\cdot 2}{2x-1^2}$

$h'(x)\ \frac{1}{\frac{x^2+1}{2x-1}}\cdot\frac{2x(2x-1)-(x^2+1)\cdot 2}{2x-1}$