1) Vizsgáljuk az $S(5,-2,3)$ gépi számhalmazt

a) adjuk meg M-ben a nevezetes számokat $(\varepsilon_0, \varepsilon_1, M_\infty)$

b) adjuk meg M elemszámát.

$$\pm[\underbrace{\text{mantissza}}_{5}\,|\,K] \qquad -2 \le k \le 3, \quad k \in \mathbb{Z}$$

$$m \cdot 2^k \longleftarrow \quad m = \sum_{i=1}^{5} m_i 2^{-i} \qquad m_1 = 1, \quad m_i \in \{0, 1\} \quad i = 2, 3 ..$$

a)

legkisebb ábrázolható + szám

$$\varepsilon_0 = +[10000|-2] = \frac{1}{2} \cdot 2^{-2} = \frac{1}{8}$$

egyhez 1-el nagyobb szomszéd szám

$$\hookrightarrow \varepsilon_1 = [10001|1] - [10000|1] = [00001|1] = 2^{-5} \cdot 2^1 = 2^{-4} = \frac{1}{16}$$

legnagyobb + szám

$$M_\infty = +[11111|\underset{\text{legnagyobb kitevő}}{3}] = (1 - 2^{-5}) \cdot 2^3 = 2^3 - 2^{-2} = 8 - \frac{1}{4} = 7{,}75$$

kitevők lehetősége -2-től +3-ig 6 lehetőség.

b) $2 \cdot 1 \cdot 2^4 \cdot \acute{6} = 2^6 \cdot 3 = 192$

2, keressük meg a 10,85-nek megfeleltetett gépi számot az $M(5,-3,4)$-ben $\left(Fl(10,85)=?\right)$

$$\begin{array}{c|c} 10 & 0 \\ 5 & 1 \\ 2 & 0 \\ 1 & 1 \end{array} \qquad 10 = 1010$$

$10,85 = 1010.1101$

$$\begin{array}{c|c} & 85 \\ 1 & 70 \\ 1 & 40 \\ 0 & 80 \\ 1 & 60 \end{array} \quad \cdot 2$$

1011.0

kerekítve 1 felfelé 0 lefelé kerekítünk

$+[10110|4]$ 4-et értünk alatta

HF $Fl\left(\frac{1}{10}\right)$, $Fl(\sqrt{2})$, $Fl\left(-\frac{1}{3}\right)$ } közelítőleg lehet felírni

$Fl\ \frac{1}{10}\ M(5,-3,4)$

$0,1 = 0.\dot{0}\dot{0}011$

$0,1 = 0.000\underbrace{11001}_{5}$

$+[11010|-3]$

$$\begin{array}{c|c} 0,1 & 1 \\ 0 & 2 \\ 0 & 4 \\ 0 & 8 \\ 1 & 6 \\ 1 & 2 \end{array}$$

3, Adjunk példát $M(5,-3,4)$-re

a, $a \oplus b = a$

b, $(a \oplus b) \oplus c \neq a \oplus (b \oplus c)$

c, kivonási jegyvesztés

a, a-az a számban található legnagyobb, b a legkisebb

$a = [10011|4]$

$b = [10000|-1] \longrightarrow$

$$\begin{array}{l} +[10011|4] \quad - a \\ +[00000|4] \quad - b \\ \hline [10011|4] = a \oplus b \end{array}$$

b, $a = +[10011|4]$

$b = [10000|-1]$

$c = 10000|-1]$

$\overbrace{a \oplus b}^{a} \oplus b = a$

$a \oplus (b \oplus b) = a$

$\cancel{[10000|-1]}$

$\oplus [10000$

$\cancel{[100001|-1]}$ a

$\cancel{[100001|-1]}$ b

10000|-1

10000|-1

$1[00000|-1] \xrightarrow{norm} [10000|0]$

a $[10011|4]$

a $[10000|0]$

$$\begin{array}{l} [10011|4] \\ [00001|4] \\ \hline [10100|4] \neq a \end{array}$$

$$fl(x) \begin{cases} M_\infty & x \geq M_\infty \\ 0 & |x| \leq \varepsilon_0 \\ \text{x-hez legközelebbi gépi szám} & \\ -M_\infty & x \leq -M_\infty \end{cases}$$

4/ $a=[1\,0\,0\,1\,1|4]$ $b=[1\,0\,0\,0\,1|4]$

$$a \ominus b \quad \begin{array}{r} [1\,0\,0\,1\,1|4] \\ \ominus\, [1\,0\,0\,0\,1|4] \\ \hline [0\,0\,0\,1\,0|4] \end{array} \xrightarrow{\text{norm}} [\underbrace{1}_{}\,\underbrace{0\,0\,0\,0}_{\text{kiegészítő 0-k}}|1]$$

Pontos bitek

Normalizáláskor 4-et törölni

4, Számítsuk ki a $\sqrt{2007}-\sqrt{2006}$ mennyiséget ha ismert hogy a $\sqrt{2007}\approx 44{,}80$ és $\sqrt{2006}\approx 44{,}79$

különbség 0,01

A közös abszolút hibakorlát 0,01

A közös relatív hibakorlát $\frac{0{,}01}{44} \approx 2{,}3\cdot 10^{-4}$

a, Adjuk meg a számított különbség abszolút és relatív hibakorlátját.

b, $(\sqrt{2007}-\sqrt{2006})\frac{\sqrt{2007}+\sqrt{2006}}{\sqrt{2007}+\sqrt{2006}} = \frac{2007-2006}{\sqrt{2007}+\sqrt{2006}} = \frac{1}{\sqrt{2007}+\sqrt{2006}} \approx \frac{1}{89{,}59} \approx 0{,}0111$

abszolút hiba — gyors hibanövekedés

a, $\Delta_{kül} = \Delta_{44,80} + \Delta_{44,79} = 0{,}02$

$\delta_{kül} = \left|\frac{a}{a-b}\right|\delta_a + \left|\frac{b}{a-b}\right|\delta_b \le 2\cdot\frac{45}{0{,}01} \quad 2{,}3\cdot 10^{-4} \approx 2{,}07$ ROSSZ!!!

$\left|\frac{a}{a-b}\right| \le \frac{45}{0{,}01}$

$\Delta_{nevező} = \Delta_{44,80} + \Delta_{44,79} = 0{,}02$

$\delta_{nevező} = \frac{0{,}02}{89{,}59} \approx 2{,}2\cdot 10^{-4}$

$\delta_{\frac{1}{90}} = \delta_1 + \delta_{nev} = \delta_{nev} \approx 2{,}2\cdot 10^{-4}$, $\delta_1 = 0$

számláló nevező

E.A.

2) azonos mátrix LER-ek mo.-ra

$A\underline{x}_1=\underline{b}_1 \quad A\underline{x}_2=\underline{b}_2 \quad A\underline{x}_3=\underline{b}_3$

$$A\,|\,\underline{b}_1\,|\,\underline{b}_2\,|\,\underline{b}_3 \xrightarrow[\text{visszaf.}]{GET} I\,|\,\underline{x}_1\,|\,\underline{x}_2\,|\,\underline{x}_2$$

3) A^{-1} meghat.-ra $A\cdot A^{-1}=I=[\underline{e}_1\ \underline{e}_2\ \ \underline{e}_n] \Rightarrow \begin{matrix} A\underline{x}_1=\underline{e}_1 \\ A\underline{x}_n=\underline{e}_n \end{matrix}$

$\underline{x}_1\ \underline{x}_2\ \ \underline{x}_n$

$$A\,|\,I \xrightarrow[\text{visszaf.}]{GE} I\,|\,A^{-1}$$

Pl

$$\left[\begin{array}{ccc|ccc} 2&2&3&1&0&0\\ 4&3&7&0&1&0\\ 6&7&7&0&0&1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 2&2&3&1&0&0\\ 0&-1&1&-2&1&0\\ 0&1&-2&-3&0&1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 2&2&3&1&0&0\\ 0&-1&1&-2&1&0\\ 0&0&-1&-5&1&1 \end{array}\right]$$

2. sor − 2·1. sor
3. sor − 3·1. sor

3. sor + 2. sor

3. sor / −1

~~0 0 1~~

$$\left[\begin{array}{ccc|ccc} 2&2&0&&&\\ 0&-1&0&&&\\ 0&0&1&5&-1&-1 \end{array}\right]$$

2. sor + 3. sor
1. sor + 3×3. sor

6. GYAKORLAT

5. A 3^π közelítésére 3^3-t használjuk. Adjuk meg az abszolút és relatív hibát.

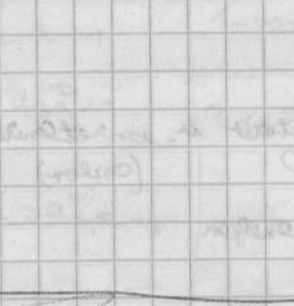
$f(x)=3^x$

$3\approx\pi$

$\Delta_3=0{,}5 \qquad (\Delta_3=0{,}15)$

$\delta=\frac{0{,}5}{3}\approx 0{,}166 \qquad \left(\delta=\frac{0{,}15}{3}=0{,}05\right)$

$\Delta_{3^3}=f'(\xi)\Delta_3 \le \ln 3\cdot 3^{3{,}5}\underbrace{\Delta_3}_{0{,}5}\approx 25{,}6885$



$f(x)=3^x$

$f'(x)=\ln 3\cdot 3^x$

$f'(\xi)\le f'(3{,}5)$

$\left(\Delta_{3^3}=f'(\xi)\Delta_3 \le \ln 3\cdot 3^{3{,}15}\cdot\underbrace{\Delta_3}_{0{,}15}\approx 5{,}2465\right)$

$$\delta_{3^3} = \frac{\Delta_3^3}{\underset{(3^3)}{27}} = \frac{25{,}6885}{27} \approx 0{,}9514$$

$$\left(\delta_{3^3} = \frac{5{,}2465}{27} = \approx 0{,}194\right)$$

A $c(f,x)$ kondíció szám

$$\frac{|x f'(x)|}{|f(x)|} = \frac{|x| \ln 3 \cdot 3^x}{3^x} = |x| \ln 3$$

$|x_0| = 3 \quad c(f,3) \approx 3{,}3$

	$\delta_{f(a)}$	δ_a	$\frac{\delta_{f(a)}}{\delta_a}$
0,5	0,9514	0,167	5,69
0,15	0,194	0,05	3,88

$\delta_{f(a)} \quad c(f,a) \cdot \delta_a$

Gauss eliminációval és főelem kiválasztása

1, 2 1 3
4 4 7

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 4 & 7 \\ 2 & 5 & 9 \end{bmatrix} \underline{x} = \begin{matrix} 1 \\ 1 \\ 3 \end{matrix}$$

jobb oldal / főelemsor

$2x_1 + 1x_2 + 3x_3 = 1$

$4x_1 + 4x_2 + 7x_3 = 1$

$2x_1 + 5x_2 + 9x_3 = 3$

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 4 & 4 & 7 & 1 \\ 2 & 5 & 9 & 3 \end{array}\right] \quad \left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 0 & 2 & 1 & -1 \\ 0 & 4 & 6 & 2 \end{array}\right]$$

az első sor változatlan
$(-2x)$ minden elem kivonása
$(-1x)$ a főelemmel lévő n x-szert

Gauss elimináció vége

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 4 & 4 \end{array}\right] \quad (-2x)$$

1, rész

$4x_3 = 4 \Rightarrow x_3 = 1$

$2x_2 + x_3 = -1 \Rightarrow 2x_2 + 1 = -1 \Rightarrow x_2 = -1$

$2x_1 + x_2 + 3x_3 = 1 \Rightarrow 2x_1 = -1 \Rightarrow x_1 = \frac{1}{2}$

$$\underline{x} = \begin{bmatrix} -\frac{1}{2} \\ -1 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

2, rész

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

utolsó sort osztva 4-el

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & -2 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|c} 2 & 1 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|c} 2 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Részleges főelem kiválasztás   www.TTK.PTE.HU/MATEK/NUMANAL

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ \boxed{4} & 4 & 7 & 1 \\ 2 & 5 & 9 & 3 \end{array}\right] \quad \left[\begin{array}{ccc|c} 4 & 4 & 7 & 1 \\ 2 & 1 & 3 & 1 \\ 2 & 5 & 9 & 3 \end{array}\right] \begin{array}{c} \\ (-\frac{1}{2}x) \\ (-\frac{1}{2}x) \end{array} \quad \left[\begin{array}{ccc|c} 4 & 4 & 7 & 1 \\ 0 & -1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & \boxed{3} & \frac{11}{2} & \frac{5}{2} \end{array}\right] \quad \left[\begin{array}{ccc|c} 4 & 4 & 7 & 1 \\ 0 & 3 & \frac{11}{2} & \frac{5}{2} \\ 0 & -1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right] (+\frac{1}{3}x)$$

$$\left[\begin{array}{ccc|c} 4 & 4 & 7 & 1 \\ 0 & 3 & \frac{11}{2} & \frac{5}{2} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} \end{array}\right]$$

$\frac{4}{3}x_3 = \frac{4}{3} \Rightarrow x_3 = 1$

$3x_2 + \frac{11}{2}\cdot 1 = \frac{5}{2}$

$3x_2 = -\frac{6}{2} = -3$

$x_2 = -1$

$4x_1 - 4 + 7 = 1$

$x_1 = -\frac{1}{2}$

2, $x_1 - x_2 + 2x_3 + x_4 = 3 \qquad ,3$

$2x_1 + 3x_2 + x_3 - x_4 = 5 \qquad ,5$

$3x_1 + 2x_2 + 3x_3 \qquad = 8 \qquad ,8$

$x_1 + 4x_2 - x_3 - 2x_4 = 2 \qquad ,0$

$b_1 \qquad b_2$

első sor

2. sor

$$\begin{array}{r} \\ +(-2x) \\ +(-3x) \\ +(-1x) \end{array}\left[\begin{array}{cccc|cc} 1 & -1 & 2 & 1 & 3 & 3 \\ 2 & 3 & 1 & -1 & 5 & 5 \\ 3 & 2 & 3 & 0 & 8 & 8 \\ 1 & 4 & -1 & -2 & 2 & 0 \end{array}\right] \Rightarrow \begin{array}{r} \\ \\ (-1x) \\ (-1x) \end{array}\left[\begin{array}{cccc|cc} 1 & -1 & 2 & 1 & 3 & 3 \\ 0 & 5 & -3 & -3 & -1 & -1 \\ 0 & 5 & -3 & -3 & -1 & -1 \\ 0 & 5 & -3 & -3 & -1 & -3 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|cc} 1 & -1 & 2 & 1 & 3 & 3 \\ 0 & 5 & -3 & -3 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 \end{array}\right]$$

2. oszlopvektor $\to 0 = -2$ ↯

$5x_2 - 3x_3 - 3x_4 = -1$

$x_2 = \frac{3}{5}x_3 + \frac{3}{5}x_4 - \frac{1}{5}$

$x_1 - x_2 + 2x_3 + x_4 = 3$

$x_1 - \frac{3}{5}x_3 - \frac{3}{2}x_3 + 2x +$

3, $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ Határozzuk meg az inverzét GE-vel

$A \cdot A^{-1} = I \qquad AX = I$

$$\begin{array}{r} \\ (-\frac{1}{2}x) \\ \\ \end{array}\left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & \frac{1}{2} & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right] (\tfrac{2}{3}x) \Rightarrow \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{4}{3} & +\frac{2}{6} & +\frac{2}{3} & 1 \end{array}\right] \Rightarrow /\tfrac{4}{3} \quad \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & 0 & \frac{3}{4} & \frac{3}{2} & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & \frac{3}{2} & 1 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{array}\right]$$

$$\underbrace{\begin{matrix} \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{matrix}}_{A^{-1}}$$

Ezzel nem változtat az oszlop cserélése esetén sem

(4) Írjuk fel L_i mátrixokat az 1. feladat lépéseiben.

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}}_{L_i}$$

amivel a sorokat szorozzuk

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 4 & 7 \\ 2 & 5 & 9 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 4 & 6 \end{bmatrix}}_{A_1} \to$$ háromszög mátrixot

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}}_{L_2} * A_1 = A_2 = U$$

$$L_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$L_2 L_1 A = U$$

$$A = \underbrace{L_1^{-1} L_2^{-1}}_{L} U$$

$$L_2^{-1} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$L = L_1^{-1} L_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$L \cdot U = A$$

$$\begin{array}{r} \\ -2\times \\ -1\times \end{array} \left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 4 & 4 & 7 & 1 \\ 2 & 5 & 9 & 3 \end{array}\right] \quad \left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 2 & 2 & 1 & -1 \\ 1 & 4 & 6 & 2 \end{array}\right] \quad \left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 2 & 2 & 1 & -1 \\ 1 & 2 & 4 & 4 \end{array}\right]$$

$$\begin{matrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{matrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

ELMÉLET

1, Készítsük el az $A=\begin{bmatrix}2&1&3\\4&4&7\\2&3&9\end{bmatrix}$ mátrix LU felbontását a mátrixszorzással

$$\begin{matrix}L & U & = A\end{matrix}$$

$$\begin{bmatrix}1&0&0\\l_1&1&0\\l_2&l_3&1\end{bmatrix}\begin{bmatrix}2&1&3\\0&u_1&u_2\\0&0&u_3\end{bmatrix}=\begin{bmatrix}2&1&3\\4&4&7\\2&5&9\end{bmatrix}$$

$l_1\cdot 2+1\cdot 0+0\cdot 0=4\Rightarrow l_1=2$

$l_2\cdot 2+l_3\cdot 0+1\cdot 0=2\Rightarrow l_2=1$

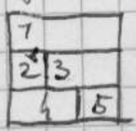
$l_1\cdot 1+1\cdot u_1 = 4\Rightarrow 2+u_1=4\Rightarrow u_1=2$

$3\cdot l_1+1\cdot u_2=7\Rightarrow 6+u_2=7\Rightarrow u_2=1$

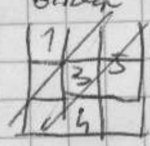
$1\,l_2+u_1\cdot l_3=5\Rightarrow 1+2\,l_3=5\Rightarrow l_3=2$

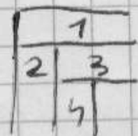
$3\cdot l_2+u_2\cdot l_3+u_3=9\Rightarrow 3+2+u_3=9\Rightarrow u_3=4$

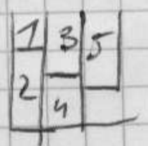
sor



oszlop







$$L=\begin{bmatrix}1&0&0\\2&1&0\\1&2&1\end{bmatrix}\qquad U=\begin{bmatrix}2&1&3\\0&2&1\\0&0&4\end{bmatrix}$$

2 $\quad A=\begin{bmatrix}2&-1&0\\-1&2&-1\\0&-1&2\end{bmatrix}$ LU felb.
LDLT felb.
(LDU = LDLT)

$$\begin{bmatrix}1&0&0\\l_1&1&0\\l_2&l_3&1\end{bmatrix}\begin{bmatrix}2&-1&0\\0&u_1&u_2\\0&0&u_3\end{bmatrix}=\begin{bmatrix}2&-1&0\\-1&2&-1\\0&-1&2\end{bmatrix}$$

$l_1\cdot 2+1\cdot 0+0\cdot 0=-1\Rightarrow l_1=-\frac{1}{2}$

$l_2\cdot 2\ l_3\cdot 0+1\cdot 0=0\Rightarrow l_2=0$

$l_1\cdot -1+1\cdot u_1=2\Rightarrow \frac{1}{2}+u_1=2\Rightarrow u_1=\frac{3}{2}$

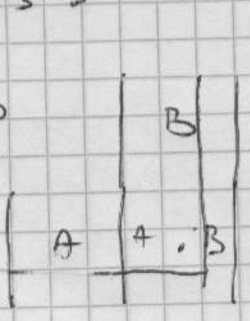
$0\cdot l_1+1\cdot u_2=-1\Rightarrow u_2=-1$

$l_2\cdot(-1)\ l_3\cdot u_1=-1\Rightarrow \frac{3}{2}\,l_3=-1=l_3=\frac{2}{3}$

$l_2\cdot 0+l_3\cdot l_2+1\cdot u_3=2\Rightarrow u_3=\frac{4}{3}$

$$L=\begin{bmatrix}1&0&0\\\frac{1}{2}&1&0\\0&\frac{2}{3}&1\end{bmatrix}\qquad U=\begin{matrix}2&-1&0\\0&\frac{3}{2}&-1\\0&0&\frac{4}{3}\end{matrix}$$

A · B



~~D=~~

$D = \text{diag } u_{11}, u_{22}, u_{nn}$

$D \begin{bmatrix} 2 & 0 & 0 \\ 0 & \frac{3}{2} & 0 \\ 0 & 0 & \frac{2}{3} \end{bmatrix}$ ~~$L \cdot D \cdot U$~~

$L \cdot D\tilde{U} = L \cdot U$

$D\tilde{U} = U$

$U = D^{-1} U$

$D^{-1} \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{3}{4} \end{bmatrix}$

$D^{-1} U \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 1 \end{bmatrix}$

U 2-sora $\times \frac{2}{3}$

utolsó sor $\times \frac{3}{4}$

3 Igazoljuk, hogy az $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 1 \\ 3 & -2 & 7 \end{bmatrix}$-nak ∞ sok LU felbontása van

$\begin{matrix} 1 & -1 \\ 2 & -2 \end{matrix} = -2 - (-2)$

$$\begin{array}{c|c} & \begin{bmatrix} 1 & -1 & 2 \\ 0 & u_1 & u_2 \\ 0 & 0 & u_3 \end{bmatrix} \\ \hline \begin{bmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{bmatrix} & \begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 1 \\ 3 & -2 & 7 \end{bmatrix} \end{array}$$

$1 l_1 + 1 \cdot 0 \cdot 0 = 2 \Rightarrow l_1 = 2$

$1 l_2 = 3 \Rightarrow l_2 = 3$

$-l_1 + u_1 = -2 \Rightarrow -2 + u_1 = -2 + u_1 = 2$ $\boxed{u_1 = 0}$!

$2 \cdot l_1 + u_2 + 0 = 1 \Rightarrow u_2 = -3$

$-l_2 + \underset{0}{u_1} \cdot l_3 = -2$

$-3 = -2$ ↯ $\Rightarrow$ nem létezik LU felbontás

főátlóba 0 gauss-elimináció leáll

$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 1 \\ 3 & -2 & 7 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ -1 \\ 6 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & -3 \\ 3 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 1 \\ 2 & 0 & -3 \end{bmatrix}$

U

L

$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{bmatrix}$

$P \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$P \cdot A = LU = P \cdot \underline{b} = \tilde{b}$

$A\underline{x} = \underline{b}$ $\quad Ly = b$

$(LU)\underline{x} = \underline{b}$ $\quad U\underline{x} = \underline{y}$

$L(\underbrace{U\underline{x}}_{y}) = \underline{b}$

2 lépés $\tilde{v}_2 = \frac{b_1 - \sigma_2 \cdot e_1}{\|b_1 - \sigma_2 e_1\|_2} \in \mathbb{R}^{n-1}$ $\sigma_2 = \text{sign}(b_{11}) \cdot \|b_1\|_2$

ezzel
$$\underset{(n-1)(n-1)}{H(\tilde{v}_2)} \cdot B = \left[\begin{array}{c|c} \sigma_2 & \\ \hline 0 & C \end{array}\right] \text{ stb}$$

Az eddigi transzformációk összegzése:

$$v_2 = \begin{bmatrix} 0 \\ \tilde{v}_2 \end{bmatrix} \in \mathbb{R}^n; \quad H(v_2) = \left[\begin{array}{c|c} 1 & 0 \cdots 0 \\ \hline 0 & \\ \vdots & H(\tilde{v}_2) \\ 0 & \end{array}\right]$$

$$H(v_2) H(v_1) \cdot A = \begin{bmatrix} \sigma_1 & * & * & \cdots & * \\ 0 & \sigma_2 & * & \cdots & * \\ \vdots & 0 & & & \\ 0 & \vdots & & C & \end{bmatrix}$$

1, LER mo.-a Hausen trf.-val $Ax = b$ $A|b$

$$H_1 A x = H_1 b \quad H_1(A|b)$$
$$\underbrace{(H_{n-1} \dots H_1 A)}_{R} x := \underbrace{H_{n-1} \dots H_1 b}_{d} \text{ rendsz.}$$

2, QR felb előáll Hausk. trf.-val:

$$(H_{n-1} \dots H_1) A = R =$$
$$A = H_1^{-1} \dots H_{n-1}^{-1} R = \underbrace{(H_1 \dots H_{n-1})}_{Q} R \qquad \text{Nem gazdaságos}$$

GYAK

~~$A = 2 \; -1 \; 0$~~

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$ ~~Állapítsuk~~ Állítsuk elő az LL^T Cholesky felbontás

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & -\frac{2}{3} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & -1 & 0 \\ 0 & \frac{3}{2} & -1 \\ 0 & 0 & \frac{4}{3} \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & \frac{3}{2} & 0 \\ 0 & 0 & \frac{4}{3} \end{bmatrix} \left(= \text{diag}(2, \tfrac{3}{2}, \tfrac{4}{3})\right) \qquad \tilde{L}^T = (\sqrt{D})^{-1} U$$

$$\sqrt{D} = \text{diagonális}\left(\sqrt{2}, \sqrt{3/2}, \sqrt{4/3}\right)$$
$$\left(\text{diag}(d_{ii})_{i=1}^{n}\right)$$

$$\tilde{L} = L\sqrt{D} \qquad A = L \cdot \underbrace{\left(\sqrt{D}(\sqrt{D})^{-1}\right)}_{I} U$$

$$L\sqrt{D} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & -\frac{2}{3} & 1 \end{bmatrix} \cdot \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & \sqrt{3/2} & 0 \\ 0 & 0 & \sqrt{4/3} \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & \sqrt{\frac{2}{3}} & \frac{2}{\sqrt{3}} \end{bmatrix}$$

$\frac{2}{\sqrt{3}}$

$-\frac{2}{3}\sqrt{\frac{3}{2}}$

$L D L^T$

$\sqrt{D} = diag\ (\sqrt{2}, \sqrt{\frac{3}{2}}, \frac{2}{\sqrt{3}})$

$A = L(\overbrace{\sqrt{D}\,\sqrt{D}}^{D})L^T = \underbrace{(L\sqrt{D})}_{\tilde{L}}\underbrace{(\sqrt{D}L^T)}_{\tilde{L}^T}$

$\begin{bmatrix} e_1 & e_2 & e_4 \\ 0 & e_3 & e_5 \\ 0 & 0 & e_6 \end{bmatrix}$

$\begin{bmatrix} l_1 & 0 & 0 \\ l_2 & l_3 & 0 \\ l_4 & l_5 & l_6 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

$l_1^2 = 2 \to l_1 = \pm\sqrt{2}$

$l_1 l_2 = -1$

$\sqrt{2}\, l_2 = -1 \Rightarrow l_2 = \frac{1}{\sqrt{2}}$

$l_1 l_4 = 0 \Rightarrow l_4 = 0$

$l_2^2 + l_3^2 = 2$

$\frac{1}{2} + l_3^2 = 2$

$l_3 = \pm\sqrt{\frac{3}{2}}$

$L \cdot L^T$

$(-L)(-L^T) = L L^T$

$l_2 l_4 + l_3 \cdot l_5 = -1$

$\sqrt{\frac{3}{2}} \cdot l_5 = -1$

$l_5 = -\sqrt{\frac{2}{3}}$

$l_4^2 + l_5^2 + l_6^2 = 2$

$\frac{2}{3} + l_6^2 = 2$

$l_6 = \frac{4}{3}$

$l_6 = \frac{2}{\sqrt{3}}$

Lenza mero

$LLT \to MF$

$\tilde{L} \begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 2 & 1 & 1 \end{bmatrix}$

1, rémitsür el ar

$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$ vátha QR felbontás Gram Schmidt ortogonalizáció

$\tilde{a}_1 \quad \tilde{a}_2 \quad a_3$

$Q = [q_1, q_2, q_3] \qquad \langle q_i, q_j \rangle \begin{cases} 0 \text{ ha } i \neq j \\ 1 \text{ ha } i = j \end{cases}$

δ_{ij}

1)

$r_{11} = \|a_1\|_2 = \sqrt{2}$ $\quad q_1 = \frac{1}{r_{11}} \cdot \underline{a}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ $\quad q_2 = \frac{1}{r_{22}} \cdot \underbrace{(\underline{a}_2 - r_{12} \cdot q_1)}_{\underline{a}_2}$

$R = \begin{bmatrix} \sqrt{2} & 0 & 2\sqrt{2} \\ 0 & 2 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}$ $\quad Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$ $\quad \Rightarrow \langle q_2, q_1 \rangle = 0 = \frac{1}{r_{22}} \left(\langle \underline{a}_2, q_1 \rangle - r_{12} \overbrace{\langle q_1, q_1 \rangle}^{1} \right) \leftarrow$

$A = Q \cdot R$ $\quad r_{23} = \left\langle \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\rangle = 0$ $\quad r_{12} = \langle q_2, q_1 \rangle =$

$r_{22} = \|\underline{a}_2 - \overbrace{r_{12}}^{0} q_1\| = 2$ $\quad = \left\langle \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\rangle = \frac{1}{\sqrt{2}} \underbrace{\left\langle \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\rangle}_{= 0}$

$q_2 = \frac{1}{2} \cdot \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$r_{13} = \left\langle \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\rangle = \frac{1}{\sqrt{2}} (1 \cdot 1 + 0 \cdot 0 + 3 \cdot 1) = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

3) $q_3 = \frac{1}{r_{33}} (a_3 - r_{13} q_1 - r_{13} q_1 - r_{23} q_2) = \frac{1}{r_{33}} \left(\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} - 2\sqrt{2} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - 0 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right) = \frac{1}{r_{33}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \sqrt{2} \end{bmatrix}$

$r_{13} = \langle a_3 q_1 \rangle \quad r_{23} = \langle a_3 \; q_2 \rangle$

mátrix $\tilde{Q}\tilde{R} = A$ ahol $\tilde{Q}$ oszlopai ortogonálisak (nem feltétlenül normáltak) $\tilde{r}_{ii} = 1$

$\tilde{R} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad \tilde{Q} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $\quad A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$

$\begin{matrix} \downarrow & \downarrow & \downarrow \\ \sqrt{2} & 2 & \sqrt{2} \end{matrix}$

1) $\tilde{q}_1 = a_1$ és $\tilde{r}_{11} = 1$

$(q_1 = \frac{1}{r_{11}} \cdot a_1)$

2) $\tilde{q}_2 = a_2 - \tilde{r}_{12} \tilde{q}_1$

$\langle \tilde{q}_2, \tilde{q}_1 \rangle = 0 = \langle a_2, \tilde{q}_1 \rangle - r_{12} \overbrace{\langle \tilde{q}_1 \, q_1 \rangle}^{\neq 1}$

$\tilde{r}_{12} = \frac{\langle a_2 \; \tilde{q}_1 \rangle}{\langle \tilde{q}_1, q_1 \rangle} = \frac{0}{2}$

$\tilde{q}_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} - 0 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

3) $\tilde{q}_3 = \underline{a}_3 - \tilde{r}_{13} q_1 - \tilde{r}_{23} \cdot \tilde{q}_2$ $\quad r_{13} = \frac{\langle \tilde{a}_3 \, \tilde{q}_1 \rangle}{\langle \tilde{q}_1, \tilde{q}_1 \rangle} = \frac{4}{2} = 2$

$\tilde{q}_3 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} - 2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $\quad r_{23} = \frac{\langle a_3 \; \tilde{q}_2 \rangle}{\langle \tilde{q}_2, \tilde{q}_2 \rangle} = 0$

$A = \tilde{Q}\tilde{R} = \tilde{Q} \underbrace{(D^{-1} D)}_{I} \tilde{R} = \underbrace{(\tilde{Q} D^{-1})}_{Q} \cdot \underbrace{(D \cdot \tilde{R})}_{R}$

$D = \mathrm{diag}(\sqrt{2}, 2, \sqrt{2})$