MÁTRIX MŰVELETEK

	A	B	C
$+\ -$	$m\times n$	$m\times n$	$C=A+B$ $m\times n$
$\cdot$	$m\times n$	$n\times r$	$C=A\cdot B$ $m\times r$
determináns	$n\times n$	—	$\det. A\in\mathbb{R}$
inverz	$n\times n$ $\det A\neq 0$	—	A^{-1} $n\times n$
λ-szoros $\lambda\in\mathbb{R}$	$m\times n$	—	$C=\lambda A$ $m\times n$

1) $A=\begin{bmatrix}-1&-1&-4&-2\\3&4&4&3\end{bmatrix}$ $\quad B=\begin{bmatrix}-3&4\\-1&0\\5&1\\5&2\end{bmatrix}$ $\quad \underset{2\times 2}{C}=A\cdot B$

A sora B oszlopa

$$\begin{array}{cccc|cc} & & & & -3 & 4\\ & & & & -1 & 0\\ & & & & 5 & 1\\ & & & & 5 & 2\\ \hline -1 & -1 & -4 & -2 & -26 & -4\\ 3 & 4 & 4 & 3 & -8 & 22\end{array}$$

$\underset{2\times 1}{\underline{v_1}}=\begin{bmatrix}2\\4\end{bmatrix}$ $\quad \underset{2\times 1}{\underline{v_2}}=\begin{bmatrix}-3\\-3\end{bmatrix}$

$\langle v_1,v_2\rangle = v_1^T\cdot v_2=\begin{array}{c|c} & \begin{bmatrix}-3\\-3\end{bmatrix}\\ \begin{bmatrix}2,4\end{bmatrix} & -18\end{array} = -18$ a skaláris szorzat

$v_2\, v_1^T=\begin{array}{c|cc} & \begin{bmatrix}2&4\end{bmatrix} & \\ \begin{bmatrix}-3\\-3\end{bmatrix} & \begin{matrix}-6&-12\\-6&-12\end{matrix}\end{array}$ $\qquad A\cdot\underline{v}=\underline{v}$

$I=\begin{bmatrix}1&\cdots&0\\ \vdots&\ddots&\vdots\\0&\cdots&1\end{bmatrix}$

egységmátrix

$IA=A \qquad I\cdot\underline{v}=\underline{v}$

Diagonális mátrix

$D=\begin{bmatrix}d_{11}&&0\\&\ddots&\\0&&d_{nn}\end{bmatrix}$

$$\begin{array}{ccc|ccc} & & & 1&2&3\\ & & & 4&5&6\\ & & & 7&8&9\\ \hline 2&0&0 & 2&4&6\\ 0&\frac{1}{2}&0 & \frac{1}{2}&\frac{5}{2}&\frac{6}{2}\\ 0&0&3 & 21&24&27\end{array}$$

diag $(2\ \tfrac{1}{2}\ 3)$

$D\times A$

$A\cdot D=\begin{bmatrix}2&1&9\\8&\frac{5}{2}&18\\14&4&27\end{bmatrix}$

$$\begin{array}{ccc|ccc} & & & & A & \\ & & & 1&2&3\\ & & & 4&5&6\\ & & & 7&8&9\\ \hline 0&1&0 & 4&5&6\\ 1&0&0 & 1&2&3\\ 0&0&1 & 7&8&9\end{array}$$

$A\cdot P=\begin{bmatrix}2&1&3\\5&4&6\\8&7&9\end{bmatrix}$

Determináns csak négyzetes mátrixokra!

$$A=\begin{bmatrix}-4 & -4\\ -3 & 3\end{bmatrix}$$

$$\det A = \underbrace{-4\cdot 3}_{-12} - \underbrace{[(-3)\cdot(-4)]}_{12} = -24$$

$$\begin{vmatrix} \overset{+}{a_{11}} & \overset{-}{a_{12}} & \overset{+ - \cdots}{\cdots} & a_{nn} \\ - & \begin{matrix} + & - & + \\ & n-1\times n-1 & \\ & A_{11} & \end{matrix} & & \\ + & & & \end{vmatrix} = a_{11}\cdot A_{11} + a_{12}\cdot A_{12} + \dots$$

$$A=\begin{bmatrix}3 & 2 & 5\\ 3 & 1 & 1\\ 1 & -5 & 1\end{bmatrix} \quad \det A = 3\cdot\underbrace{(-1)^{1+1}}_{+1}\cdot\underbrace{\begin{vmatrix}1 & 1\\ -5 & 1\end{vmatrix}}_{-1-(-5)} + 2\cdot\underbrace{(-1)^{1+2}}_{-1}\cdot\underbrace{\begin{vmatrix}3 & 1\\ 1 & 1\end{vmatrix}}_{3-1} + 5\cdot\underbrace{(-1)^{1+3}}_{+1}\underbrace{\underbrace{\begin{vmatrix}3 & 1\\ 1 & -5\end{vmatrix}}_{-15-1}}_{-16} = -66$$

Felső háromszög mátrix

$$\begin{bmatrix} U_{11} & U_{12} & \cdots & \\ 0 & U_{22} & \cdots & \\ 0 & 0 & U_{32} & \cdots \\ \vdots & & & \\ 0 & 0 & 0 \cdots & 0\, U_{nn} \end{bmatrix} = U_{nn} \qquad \det U = U_{11}\cdot U_{22}\cdot \dots \cdot U_{nn}$$

$$\begin{vmatrix}3 & 2 & 5\\ 3 & 1 & 1\\ 1 & -5 & 1\end{vmatrix} = \begin{vmatrix}3 & 2 & 5\\ 0 & -1 & -4\\ 0 & -\frac{17}{3} & -\frac{2}{3}\end{vmatrix} = \begin{bmatrix}3 & 2 & 5\\ 0 & -1 & -4\\ 0 & 0 & 22\end{bmatrix} = -66$$

$$-\frac{15}{3}-\frac{2}{3}=\frac{17}{3}$$

$$-\frac{17}{3}\cdot 4 \qquad \frac{68}{3}-\frac{2}{3}=\frac{66}{3}=22$$

$$-\begin{vmatrix}1 & -5 & 1\\ 3 & 1 & 1\\ 3 & 2 & 5\end{vmatrix}$$

Inverz $A\cdot A^{-1} = A^{-1}\cdot A = I$

Speciális mátrix

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A=\begin{bmatrix} -4 & -5 \\ 4 & 0 \end{bmatrix}$$

$$A^{-1}=\frac{1}{\det A}\cdot\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad A^{-1}=\frac{1}{20}\cdot\begin{bmatrix} 0 & 5 \\ -4 & -4 \end{bmatrix}=\begin{bmatrix} 0 & 1/4 \\ -\frac{1}{5} & -\frac{1}{5} \end{bmatrix}$$

$\downarrow$

$a\cdot d - b\cdot c$

$$A\cdot A^{-1}=I$$
$$A^{-1}\cdot A=I$$

Diagonális

$$A=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A^{-1}=\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \underline{a}_1 \cdot \underline{a}_2 \cdot \underline{a}_3 \cdots \underline{a}_n \end{bmatrix}\cdot x=\underline{b}$$

$$A\cdot\underline{x}=\underline{b} \qquad \underline{x_1\cdot a_1}+x_2\cdot\underline{a_2}\ldots x_n\cdot\underline{a_n}=\underline{b_n}$$

Saját érték probléma

$$A\cdot\underline{v}=\lambda\cdot v \qquad v\neq 0$$

λ A saját értéke

$\underline{v}$ λ-hoz tartozó saját vektor

$$(A(c\cdot\underline{v})=\lambda(c\cdot v)$$
$$(A-\lambda I)v=0$$

$\det(A-\lambda\cdot I)=0$

$K_A(\lambda)$

~~karakterisztikus~~

karakterisztikus polinom

$$A=\begin{bmatrix} -2 & -6 \\ 3 & 7 \end{bmatrix} \qquad K_A\lambda\begin{vmatrix} -2-\lambda & -6 \\ 3 & 7-\lambda \end{vmatrix}=(-2-\lambda)(7-\lambda)+18=\lambda^2-5\lambda-14+18=\lambda^2-5\lambda+4=0$$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad \frac{5\pm\sqrt{25-16}}{2}=\frac{5\pm3}{2}=\begin{matrix} 4 \\ 1 \end{matrix}$$

$$\lambda_1=1 \qquad \begin{bmatrix} -2 & -6 \\ 3 & 7 \end{bmatrix}\cdot\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=1\cdot\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$-2\cdot v_1-6\cdot v_2=v_1$$
$$3\cdot v_1+7\cdot v_2=v_2$$
$$v_1=-2v_2$$

$$\underline{v}=\begin{bmatrix} -2v_2 \\ v_2 \end{bmatrix}=\begin{bmatrix} -2 \\ 1 \end{bmatrix}v_2$$

$$\begin{vmatrix} e_1-\lambda & 0 & 0 \\ e_2 & e_3-\lambda & 0 \\ e_4 & e_5 & e_6-\lambda \end{vmatrix}=(e_1-\lambda)(e_3-\lambda)\cdot(e_6-\lambda) \qquad e_1=\lambda \quad e_3=\lambda \quad e_6=\lambda$$