Integrálási szabályok

$$\int(f+g)=\int f+\int g$$
$$\int \lambda f=\lambda\int f$$

1

1, $\int\left(3x^5-4x+\underbrace{\frac{2}{x^3}}_{2\cdot x^{-3}}+\frac{7}{x}-\underbrace{4}_{4\cdot x^0}\right)dx=3\cdot\frac{x^6}{6}-4\cdot\frac{x^2}{2}+2\cdot\frac{x^{-2}}{-2}+7\cdot\ln|x|-4x+c$

2, $\int\left(3\cdot\underbrace{\sqrt{x}}_{x^{\frac{1}{2}}}-4\underbrace{x\sqrt[3]{x}}_{1+\frac{1}{3}=x^{\frac{4}{3}}}+7\cdot\underbrace{x\sqrt{x^3}}_{x^{\frac{5}{2}}\ (1+\frac{3}{2})}+\underbrace{5}_{5\cdot x^0}\right)dx=3\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}-4\cdot\frac{x^{\frac{7}{3}}}{\frac{7}{3}}+7\cdot\frac{x^{\frac{7}{2}}}{\frac{7}{2}}+5x+C$

~~3, $\int(7\cos x-4\cdot 2^x+\frac{7}{x\sqrt[3]{x^2}}$~~

3, $\int\left(7\cos x-4\cdot 2^x+\underbrace{\frac{7}{x\sqrt[3]{x^2}}}_{7\cdot x^{-\frac{5}{3}}}\right)dx=7\cdot\sin x-4\cdot\frac{2^x}{\ln 2}+7\cdot\frac{x^{-\frac{2}{3}}}{-\frac{2}{3}}+c$

4, $\int\sqrt{\sqrt{\sqrt{x}}}\,dx=\int\left(\left(x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}dx=\int x^{\frac{1}{8}}dx=\frac{x^{\frac{9}{8}}}{\frac{9}{8}}+c$

Parciális integrálás $\int f'(x)\cdot g(x)=f(x)\cdot g(x)-\int f(x)\cdot g'(x)$

1, eset $\int \underbrace{P_n(x)}_{g}\cdot\underbrace{\begin{Bmatrix}e^x\\ \sin x\\ \cos x\end{Bmatrix}}_{f'}dx$

1, $\int\underbrace{(4x-3)}_{g}\cdot\underbrace{\cos x}_{f'}\,dx=(4x-3)\cdot\sin x-\int 4\cdot\sin x\,dx=(4x-3)\cdot\sin x-4\cdot(-\cos x)+c$

$g=4x-3\xrightarrow{\text{deriv.}}g'=4$

$f'=\cos x\xrightarrow{\text{integ.}}f=\sin x$

2, $\int\underbrace{(x^2+7x-1)}_{g}\cdot\underbrace{e^x}_{f'}dx=e^x(x^2+7x-1)-\int\underbrace{(2x+7)}_{g}\underbrace{e^x}_{f'}dx=e^x(x^2+7x-1)-\left[(2x+7)\,e^x-\int 2e^x dx\right]=$

$g'=2$ $\quad f=e^x$

$=e^x(x^2+7x-1)-(2x+7)\cdot e^x+2e^x+C$

$g=x^2+7x-1\xrightarrow{\text{deriv.}}g'=2x+7$

$f'=e^x\xrightarrow{\text{int}}f=e^x$

2. Set

$$\int \underbrace{P_n(x)}_{f'} \cdot \underbrace{\begin{Bmatrix} \ln x \\ \log_a x \end{Bmatrix}}_{g} dx$$

2

1, $\int \underbrace{2x^7+3x^2-1}_{f'} \cdot \underbrace{\ln x}_{g}\, dx = \left(\frac{1}{4}x^8+x^3-x\right)\cdot \ln x - \int \underbrace{\left(\frac{1}{4}x^8+x^3-x\right)\cdot\frac{1}{x}}_{\frac{1}{4}x^7+x^2-1} dx =$

$= \underline{\left(\frac{1}{4}x^8+x^3-x\right)\ln x - \left(\frac{1}{4}\cdot\frac{x^8}{8}+\frac{x^3}{3}-x\right)+c}$

$f' = 2x^7+3x^2-1 \xrightarrow{int} f = 2\frac{x^8}{8}+3\cdot\frac{x^3}{3}-x$

$g = \ln x \xrightarrow{der} g' = \frac{1}{x}$

2, $\int \ln x\, dx = \int \underbrace{1}_{f'}\cdot\underbrace{\ln x}_{g}\, dx = x\cdot\ln x - \int \underbrace{x\cdot\frac{1}{x}}_{1} dx = \underline{x\ln x - x + c}$

$f = x \quad g' = \frac{1}{x}$

Hf $\int (4x^2+7)\ln x\, dx$

$\int (3x+1)\cdot \sin x\, dx$

$\int (2x^2+4)\cdot \cos x\, dx$

$\int (7x^5+2x-1)\ln x\, dx$

Helyettesítéses integrálás

1, $\int f(g(x))\, g'(x)\,dx = F(g(x)) + C$ ahol F a f fv. primitív ~~fel~~ fgv-e

$\int \cos(4x+7)\,dx = \int (\cos t)\,\frac{1}{4}\,dt = \frac{1}{4}\cdot \sin t = \frac{1}{4}\sin(4x+7)+C$

$4x+7=t$

$4dx = dt \Rightarrow dx = \frac{1}{4}\,dt$

2, $\int (3x+2)^8\,dx = \int t^8\,\frac{1}{3}\,dt = \frac{1}{3}\cdot\frac{t^9}{9} = \frac{(3x+2)^9}{27}+C$

$3x+2=t$

$3dx = dt$

$dx = \frac{1}{3}\,dt$

3, $\int 2x^2\cdot(x^3+4)^{-3}\,dx = \int 2\cdot t^{-3}\,\frac{1}{6}\,dt = \frac{1}{3}\,\frac{t^{-2}}{-2} = \frac{1}{3}\cdot\frac{(2x^3+4)^{-2}}{-2}+C$

$2x^3-4=t$

$6x^2\,dx = dt$

$x^2\,dx = \frac{1}{6}\,dt$

4, $\int \frac{2x^2+1}{2x^3+3x}\,dx \int \frac{1}{t}\,\frac{1}{3}\,dt = \frac{1}{3}\cdot\ln|t| = \frac{1}{3}\cdot\ln|2x^3+3x|+C$

$2x^3+3x=t$

$(6x^2+3)\,dx = dt$

$3\cdot(2x^2+1)\,dx = dt$

$(2x^2+1)\,dx = \frac{1}{3}\,dt$

5 $\int \frac{\ln^7 x}{x}\,dx = \int t^7\,dt = \frac{t^8}{8} = \frac{(\ln x)^8}{8}+C$

$\ln x = t$

$\frac{1}{x}\,dx = dt$

6, $\int \frac{1}{x\cdot\sqrt[3]{\ln x}}\,dx = \int \frac{1}{\sqrt[3]{t}}\,dt = \frac{t^{\frac{2}{3}}}{\frac{2}{3}} = \frac{(\ln x)^{\frac{2}{3}}}{\frac{2}{3}}+C$

$\ln x = t$

$\frac{1}{x}\,dx = dt$

$\sqrt[3]{t} \quad t^{-\frac{1}{3}}$

7, $\int \frac{\sin x}{1+\cos x}\,dx = \int \frac{1}{t}(-1)\,dt = -\ln|t| = -\ln|1+\cos x| + C$

$1+\cos x = t$

$-\sin x\,dx = dt$

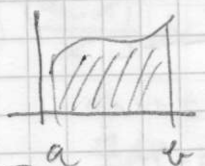
$\sin x\,dx = -dt$

Határozott integrál.

Newton Leibniz szabály

$$\int_a^b f(x)\,dx = \underbrace{F(b) - F(a)}_{[F(x)]_a^b}$$

jelölés



1, $\int_0^{\frac{\pi}{2}} 2\cos x\,dx = [2\cdot\sin x]_0^{\frac{\pi}{2}} = \underbrace{2\cdot\overbrace{\sin\frac{\pi}{2}}^{=1}}_{F(b)} - \underbrace{2\cdot\overbrace{\sin 0}^{=0}}_{F(a)} = 2$

2, $\int_0^1 (3\cdot e^x + 4)\,dx = [3\cdot e^x + 4x]_0^1 = 3\cdot e^1 + 4\cdot 1 - 3\cdot \overbrace{e^0}^{1} - 4\cdot 0 = \underline{\underline{3e+1}}$

3, $\int_1^2 \underbrace{(3x^2+2)}_{f'}\underbrace{\ln x}_{g}\,dx = [(x^3+2x)\cdot\ln x]_1^2 - \int_1^2 \underbrace{(x^3+2x)\cdot\frac{1}{x}}_{x^2+2}\,dx = [(x^3+2x)\cdot\ln x]_1^2 - \left[\frac{x^3}{3}+2x\right]_1^2 =$

$f' = 3x^2+2 \xrightarrow{int} f = \cancel{3}\cdot\frac{x^3}{\cancel{3}} + 2x$

$g = \ln x \xrightarrow{dx} g' = \frac{1}{x}$

$= \overbrace{(2^3+2\cdot 2)\cdot\ln 2}^{F(2)} - \overbrace{(1^3+2\cdot 1)\ln 1}^{F(1)} - \left[\left(\frac{2^3}{3}+2\cdot 2\right) - \left(\frac{1^3}{3}+2\cdot 1\right)\right] = \dots$ ($\ln 1 = 0$)

4, $\int_0^1 (2x+3)^{11}\,dx = \int_3^5 t^{11}\,\frac{1}{2}\,dt = \left[\frac{1}{2}\cdot\frac{t^{12}}{12}\right]_3^5 = \overbrace{\frac{5^{12}}{24}}^{G(2)} - \overbrace{\frac{3^{12}}{24}}^{G(1)}$

$2x+3 = t$

$2\,dx = dt$

$dx = \frac{1}{2}\,dt$

$x = 0 \rightarrow t = 2\cdot 0 + 3 = 3$

$x = 1 \rightarrow t = 2\cdot 1 + 3 = 5$

MF

5

1, $\int \cos(7x-1)\,dx = \frac{1}{7}\int \cos t\,dt = \frac{1}{7}\sin t = \sin\frac{(7x-1)}{7} + C$

$7x-1=t$

$7dx=dt$

$dx=\frac{1}{7}dt$

2, $\int (3x^2+2)^7 \cdot x\,dx = \int t^7 \frac{1}{6}\,dt = \frac{1}{6}\cdot\frac{t^8}{8} = \frac{1}{6}\cdot\frac{(3x^2+2)^8}{8} + C$

$3x^2+2=t$

$6x\,dx=dt$

$x\,dx=\frac{1}{6}dt$

3, $\int \frac{(\ln x)^5}{x}\,dx = \int t^5\,dt = \frac{t^6}{6} = \frac{(\ln x)^6}{6} + C$

$\ln x = t$

$\frac{1}{x}dx = dt$

4, $\int \frac{8x^3+6x}{2x^4+3x^2+5}\,dx = \int \frac{dt}{t} = \ln|t| = \ln|2x^4+3x^2+5| + C$

$\frac{1}{t}dt$

$2x^4+3x^2+5=t$

$(8x+6x)\,dx = dt$

5, $\int 2x\cdot\sin(1x^2)\,dx = \int \sin t\,dt = -\cos t = -\cos(1+x^2) + C$

$1+x^2=t$

$2x\,dx=dt$

1, $\underbrace{(3x-1)}_{f}\cdot\underbrace{\cos x}_{g'}\,dx = (3x-1)\cdot\sin x - \int 3\cdot\sin x\,dx = (3x-1)\cdot\sin x + 3\cdot\cos x + C$

$f'=3$ $\quad g=\sin x$

2, $\int \underbrace{(4x^2+7x-1)}_{g'}\cdot\underbrace{\ln x}_{f}\,dx = (\frac{4}{3}x^3+\frac{7}{2}x^2-x)\cdot\ln x - \int \underbrace{(\frac{4}{3}x^3+\frac{7}{2}x^2-x)\frac{1}{x}}_{\frac{4}{3}\cdot x^2+\frac{7}{2}x-1} =$

$g = 4\cdot\frac{x^3}{3}+7\frac{x^2}{2}-x \quad f'=\frac{1}{x}$

$= \quad (\frac{4}{3}\frac{x^3}{3}+\frac{7}{2}\cdot\frac{x}{2}-x) + C$