KIEGÉSZÍTÉS

1, Alapintegrálok gyakorlása

1, $\int \frac{1}{\sqrt{4-x^2}}\,dx = \arcsin \frac{x}{2} + c$

2, $\int \frac{1}{\sqrt{x^2+9}}\,dx = \ln\left(x+\sqrt{x^2+9}\right) + c$

3, $\int \frac{1}{\sqrt{1-9x^2}}\,dx = \int \frac{1}{3\sqrt{\underbrace{\frac{1}{9}}_{\left(\frac{1}{3}\right)^2}-x^2}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{\left(\frac{1}{3}\right)^2-x^2}}\,dx = \frac{1}{3}\cdot \arcsin \underbrace{\frac{x}{\frac{1}{3}}}_{3x} + c$

(osztás szorzás)

4, $\int \frac{1}{\sqrt{9x^2+4}} = \int \frac{1}{3}\cdot\frac{1}{\sqrt{x^2+\frac{4}{9}}}\,dx = \frac{1}{3}\cdot \ln\left(x+\sqrt{x^2+\frac{4}{9}}\right) + c$

$\boxed{\int \frac{1}{\sqrt{x^2+a^2}}\,dx}$

5, $\int \frac{1}{25+4x^2}\,dx = \frac{1}{4}\int \frac{1}{\frac{25}{4}+x^2}\,dx = \frac{1}{4}\cdot\frac{2}{5}\cdot \mathrm{arctg}\,\frac{x}{\frac{5}{2}} + c$

$\boxed{\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\cdot \mathrm{arctg}\,\frac{x}{a} + c}$

$\boxed{\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\cdot \ln\left|\frac{x-a}{x+a}\right| + c}$

6, $\int \frac{1}{25-4x^2}\,dx = \frac{1}{4}\int \frac{1}{\frac{25}{4}-x^2}\,dx = -\frac{1}{4}\int \frac{1}{x^2-\left(\frac{5}{2}\right)^2}\,dx = \left(-\frac{1}{4}\right)\cdot\frac{1}{2\cdot\frac{5}{2}}\cdot \ln$

$\Rightarrow \left|\frac{x-\frac{5}{2}}{x+\frac{5}{2}}\right| + c$

2 Parciális integrálás

$\int f\cdot g' = f\cdot g - \int f'\cdot g$

I. típus $\int \underbrace{P_n(x)}_{f}\cdot \underbrace{\begin{Bmatrix}\sin x\\ \cos x\\ e^x\end{Bmatrix}}_{g'}\,dx$

$\int \underbrace{(7x-3)}_{f}\cdot \underbrace{\sin x}_{g'}\,dx = \underbrace{(7x-3)}_{f}\cdot \underbrace{(-\cos x)}_{g} - \int \underbrace{7}_{f'}\underbrace{(-\cos x)}_{g}\,dx = (7x-3)(-\cos x) + 7\cdot\sin x + c$

$f = 7x-3 \xrightarrow{der} f' = 7$

$g' = \sin x \xrightarrow{int} g = -\cos x$

II tipas

$$\int \underbrace{P_n(x)}_{g'} \cdot \underbrace{\begin{Bmatrix} \ln x \\ \ln a^x \end{Bmatrix}}_{f} dx$$

$$\int \underbrace{(7x^5+3x-1)}_{g'} \cdot \underbrace{\ln x}_{f} \, dx = \left(\frac{7}{6}x^6+\frac{3}{2}x^2-x\right)\cdot \ln x - \int \underbrace{\left(\frac{7}{6}x^6+\frac{3}{2}x^2-x\right)\frac{7}{x}}_{\frac{7}{6}x^5+\frac{3}{2}x-1} dx =$$

$f=\ln x \xrightarrow{dx} f'=\frac{1}{x}$

$g'=7x^5+3x-1 \xrightarrow{int} g=7\cdot\frac{x^6}{6}+3\frac{x^2}{2}-x$

$$=\left(\frac{7}{6}x^6+\frac{3}{2}x^2-x\right)\cdot \ln x-\left(\frac{7}{6}\cdot\frac{x^6}{6}+\frac{3}{2}\cdot\frac{x^2}{2}-x\right)+C$$

III tipas

$$\int \begin{Bmatrix} \sin x \\ \cos x \end{Bmatrix} e^x \, dx$$

1) $\int \underbrace{\sin x}_{f} \cdot \underbrace{e^x}_{g'} dx = \sin x \cdot e^x - \int \underbrace{\cos x}_{f} \cdot \underbrace{e^x}_{g'} dx = \sin x \cdot e^x - \left[\cos x \cdot e^x - \int \sin x \, e^x \, dx\right] = ⊛$

$f=\sin x \xrightarrow{dx} f'=\cos x$ $\quad$ $f=\cos x \rightarrow f'=-\sin x$

$g'=e^x \xrightarrow{int} g=e^x$ $\quad$ $g'=e^x \rightarrow g=e^x$

$⊛ = \sin x\, e^x - \cos \cdot e^x - \int \sin x \cdot e^x dx$

$I=(\sin x-\cos x)\cdot e^x - I \quad /+I$

$2I=(\sin x-\cos x)\cdot e^x$

$I=\frac{1}{2}\cdot(\sin x-\cos x)\cdot e^x + C$

2) $\int \underbrace{\cos(5x)}_{f} \cdot \underbrace{e^{3x}}_{g'} dx = (\cos 5x)\cdot\frac{1}{3}e^{3x} - \int \underbrace{(-5\sin 5x)}_{f'} \cdot \underbrace{\frac{1}{3}}_{g} \cdot e^{3x} dx = ⊛$

$f=\cos(5x) \xrightarrow{dx} f'=-\sin(5x)\cdot 5$

$g'=e^{3x} \xrightarrow{int} g=\int e^{3x}dx = \int e^t \frac{1}{3}dt = \frac{1}{3}\cdot e^t = \frac{1}{3}\cdot e^{3x}+C$

$3x=t$

$3dx=dt$

$dx=\frac{1}{3}dt$

$⊛\ \frac{1}{3}\cos 5x\, e^{3x} + \frac{5}{3}\cdot\int \underbrace{\sin 5x}_{f} \cdot \underbrace{e^{3x}}_{g'} dx = ⊛^2$

$f=\sin 5x \xrightarrow{dx} f'=\cos(5x)\cdot 5$

$g'=e^{3x} \xrightarrow{int} g=\frac{1}{3}\cdot e$

$⊛^2$

$=\frac{1}{3}\cos 5x\, e^{3x} + \frac{5}{3}\cdot\frac{1}{3}\cdot\sin 5x\cdot e^{3x} - \frac{5}{3}\cdot\frac{5}{3}\int \cos 5x \cdot e^{3x} dx$

$I = e^{3x} \cdot \left(\frac{1}{3}\cdot \cos 5x + \frac{5}{9}\cdot \sin 5x\right) - \frac{25}{9}\cdot I \qquad / + \frac{25}{9} I$

$\frac{34}{9} I = e^{3x} \cdot (\quad - \text{ll} - \quad)$

$I = \frac{9}{34} e^{3x} \cdot \left(\frac{1}{3}\cdot \cos 5x + \frac{5}{9}\cdot \sin 5x\right.$

(3) Helyettesítéses integrál

1, $\int \cos(7x^2-1)\, dx = \int \cos t \frac{1}{14} dt = \frac{1}{14}\cdot \sin t = \frac{1}{14} \sin(7x^2-1) + c$

$7x^2-1 = t$

$14x\, dx = dt$

$x\, dx = \frac{1}{14} dt$

2, $\int \frac{1\cdot 1}{x\cdot(1+\ln^2 x)} dx = \frac{1}{1+t^2} dt = \text{arc}\,\text{tg}\, t \qquad t = \text{arc tg}(\ln x) + c$

$\ln x = t$

$\frac{1}{x} dx = dt$

3, $\int \frac{4x+2}{2x^2+2x-7} dx = \int \frac{1}{t} dt = \ln|t| = \ln|2x^2+2x-7| + c$

$2x^2+2x-7 = t$

$(4x+2)dx = dt$

4, $\int (10x^4+6)\cdot(x^5+3x)^7 dx = \int 2\cdot t^7 dt = 2\cdot \frac{t^8}{8} = 2\cdot \frac{(x^5+3^x)^8}{8} + c$

$5^x + 3x = t$

$(5x^4+3)dx = dt$

5, $\int \frac{e^{2x}}{1+e^x} dx = \int \frac{t+1-1}{1+t} dt = \int \left(1 - \frac{1}{1+t}\right) dt = \int \left(1-\frac{1}{u}\right) du = u - \ln|u| =$

$e^x = t \qquad e^x dt = dt$

$= 1+t - \ln|1+t| =$

$1+t = u$

$dt = du$

$= 1+e^x - \ln|1+e^x| + c$

$\sin x \frac{1}{\cos x}$

6, $\int \frac{\sin x}{\cos x} dx = -\int -\sin x \cdot \frac{1}{\cos x} dx = -\int \frac{1}{t} dt = -\ln|t| = -\ln|\cos x| + c$

$\cos x = t$

$-\sin x\, dx = dt$

Parciális tört fgv-ek integrálása

$\frac{P(x)}{Q(x)}$ mindig felbontható elemi törtek összegére
(Parciális)

$\to$ Polinomok

$\to \frac{A}{(x-a)^n}$

$\to \frac{Ax+B}{(ax^2+bx+c)^n} \Rightarrow n=1$

x^2-ek egyes
x-ek egyes
x^0-ok egyese

1, eset Ha a számláló foka < nevező foka

$$\frac{7x+4}{(x+2)^2\cdot(x-3)} = \frac{A}{(x+2)^2}+\frac{B}{x+2}+\frac{C}{x-3} = \frac{A(x-3)+B\cdot(x+2)(x-3)+C(x+2)^2}{(x+2)^2\cdot(x-3)}$$

$0\cdot x^2 \quad 7x+4 = A\cdot(x-3)+B\cdot(x+2)(x-3)+C\cdot(x+2)^2$

$(x^2-x-6) \qquad (x^2+4x+4)$

$$\left.\begin{array}{ll} x^2 & 0=B+C \\ x & 7=A-B+4C \\ x^0 & 4=-3A-6B+4C \end{array}\right\} \to C=-B$$

$$\left.\begin{array}{l} 7=A-5B \\ 4=-3A-10B \end{array}\right\}$$

$10=-5A$

$\underline{A=2}$

$7=2-5B$

$5=-5B \Rightarrow \underline{B=-1} \Rightarrow \underline{C=1}$

$$\int\frac{7x+4}{(x+2)^2\cdot(x-3)}dx = \int\frac{2}{(x+2)^2}dx+\int\frac{-1}{x+2}dx+\int\frac{1}{x-3}dx = 2\cdot\frac{(x+2)^{-1}}{-1}+$$

$$+(-1)\ln|x+2|+\ln|x-3|+C$$

$$\int\frac{2}{(x+2)^2}dx = \int\frac{2}{t^2}dt = \int 2\cdot t^{-2}dt = 2\cdot\frac{t^{-1}}{-1} = 2\cdot\frac{(x+2)^{-1}}{-1}$$

$x+2=t$

$dx=dt$

$$\int\frac{1}{x+2}dx = \int\frac{1}{t}dt = \ln|t| = \underline{\ln|x+2|+C}$$

$$\int\frac{1}{x-3}dx = \int\frac{1}{t}dt = \ln|t| = \underline{\ln|x-3|+C}$$

2. set ka a zámedlé fatsráme $\geq$ uvoří fatsráme

2, $$\frac{2x^4+6x^3+9x^2+1}{(x+1)^2} = 2x^2+3x+3+\frac{-8x-2}{(x+1)^2}$$

$$\begin{array}{r|l} 2x^4+6x^3+9x^2+1 & x^2+2x+1 \\ \hline -2x^4-4x^3-2x^2 & 2x^2 \\ 2x^3+7x^2+1 & 2x \\ -2x^3-4x^2-2x & \\ 3x^2-2x+1 & 3 \\ -3x^2-6x-3 & \\ -8x-2 & \end{array}$$

$$\frac{-8x-2}{(x+1)^2} = \frac{A}{(x+1)^2} + \frac{B}{x+1} = \frac{A+B\cdot(x+1)}{(x+1)^2}$$

$$-8x-2 = A+B(x+1)$$

$$\left.\begin{aligned} -8 &= B \\ -2 &= A+B \end{aligned}\right\}$$

$$\boxed{B=-8}$$

$$A=-2-B=6 \quad \boxed{A=6}$$

$$\int\frac{2x^4+6x^3+9x^2+1}{(x+1)^2}\,dx = \int\left(2x^2+2x+3+\frac{6}{(x+1)^2}+\frac{-8}{x+1}\right)dx =$$

$$= 2\cdot\frac{x^3}{3}+2\cdot\frac{x^2}{2}+3x+6\cdot\frac{(x+1)^{-1}}{-1}-8\cdot\ln|x+1|+C$$

3, $$\int\frac{1}{x^2+4x+8}\,dx = \int\frac{1}{(x+2)^2+4}\,dx = \int\frac{1}{t^2+2^2}\,dt = \frac{1}{2}\operatorname{arctg}\frac{t}{2} = \frac{1}{2}\operatorname{arctg}\frac{x+2}{2}+C$$

$$x^2+4x+8 = (x^2+4x+4)+4 \qquad \begin{aligned}(x+2)&=t\\ dx&=dt\end{aligned}$$

$$a^2+2ab+b^2=(a\pm b)^2 \Rightarrow$$

4, $$\int\frac{1}{x^2+6x+20}\,dx = \int\frac{1}{(x+3)^2+11}\,dx = \int\frac{1}{t^2+\sqrt{11}^2}\,dt = \frac{1}{\sqrt{11}}\operatorname{arctg}\frac{t}{\sqrt{11}} =$$

$$= \frac{1}{\sqrt{11}}\operatorname{arctg}\frac{x+3}{\sqrt{11}}+C$$

$$x^2+6x+20 = (x^2+2\cdot3\cdot x+9)+11 = (x+3)^2+\sqrt{11}^2 \qquad \begin{aligned}x+3&=t\\ dx&=dt\end{aligned}$$

$2\cdot 3x$

5, $\int \frac{5}{x\cdot(x^2+4)}\,dx = \int \frac{\frac{5}{4}}{x}dx + \int \frac{-\frac{5}{4}\cdot x}{x^2+4}dx = \frac{5}{4}\cdot \ln|x| - \frac{5}{8}\int \frac{2x}{x^2+4}\,dx = ⊛$

$x^2+4=t$

$$\frac{5}{x\cdot(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4} = \frac{A\cdot(x^2+4)+(B\cancel{x}+C)x}{x(x^2+4)}$$

ha $\Delta < 0 \Rightarrow x^2+4 \neq (x-A)(x+B)$

$\Delta \geq 0 \Rightarrow ax^2+bx+c = a(x-x_1)(x-x_2)$

$5 = A(x^2+4)+(Bx+C)\cdot x$

$$\left.\begin{aligned} x^2&: 0 = A+B \\ x^1&: 0 = C \\ x^0&: 5 = 4A \end{aligned}\right\} \to A = \frac{5}{4} \qquad B = -\frac{5}{4} \qquad C=0$$

$$⊛\ \frac{5}{4}\cdot\ln|x| - \frac{5}{8}\underbrace{\int \frac{1\,dt}{t}}_{\ln|t|} = \frac{5}{4}\cdot\ln|x| - \frac{5}{8}\cdot\ln|x^2+4|+C$$

6, $\int \frac{2x^2}{x^4-1}\,dx =$

MT

$x^4-1=(x^2-1)(x^2+1)=(x-1)(x+1)(x^2+1)$

$$\frac{2x^2}{(x-1)(x+1)(x^2+1)} = \frac{A}{x-1}+\frac{B}{x+1}+\frac{Cx+D}{x^2+1} = \frac{A\cdot(x+1)(x^2+1)+B(x-1)(x^2+1)+(Cx+D)(x^2-1)}{(x-1)(x+1)(x^2+1)}$$

$2x^2 = A(x^3+x+x^2+1)+B(x^3+x-x^2-1)+Cx^3-Cx+Dx^2-D$

$x^3: 0\quad A+B+C$

$x^2: 2\quad A-B+D$

$x^1: 0\quad A+B-C$

$x^0: 0\quad A-B-D$

$B=-\frac{1}{2}$

$A=\frac{1}{2}$

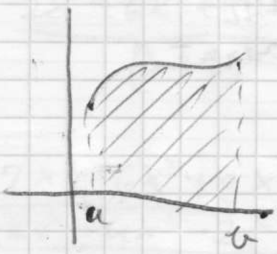
$C=0$

$D=1$

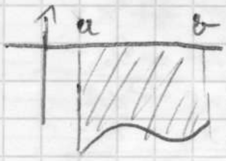
$$\int\frac{2x^2}{x^4-1}\,dx = \int \frac{\frac{1}{2}}{x-1}dx + \int\frac{-\frac{1}{2}}{x+1}dx + \int \frac{1}{x^2+1}dx = \frac{1}{2}\cdot\ln|x-1| - \frac{1}{2}\cdot\ln|x+1| +$$

$$\operatorname{arctg}\frac{x}{1}+C$$

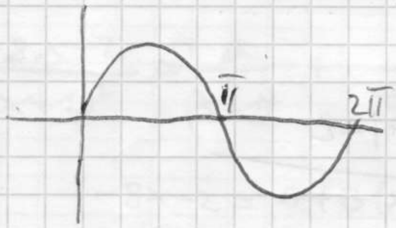
Integrálszámítás alkalmazásai



$\int_a^b f(x)\,dx = T$ ha $f(x) \geq 0$ $(x \in M)$



$\int_a^b f(x)\,dx = -T$ ha $f(x) \leq 0$ $(x \in M)$



$\int_a^{2\pi} \sin x\,dx = 0$

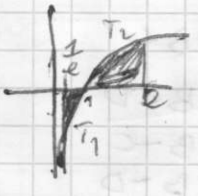
 → Szabgrafikon

1, $\underline{T_1+T_2}$

$T = \int_0^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} \underbrace{|\sin x|}_{-\sin x}\,dx = \left[-\cos x\right]_0^{\pi} + \left[\cos x\right]_{\pi}^{2\pi} = -\cos\pi + \cos 0 + \cos 2\pi - \cos\pi =$

$= 1+1+1+1 = 4$

2, $f: \left[\frac{1}{e}, e\right] \to \mathbb{R}$ $f(x) = \ln x$



$T_1 + T_2 = \int_{\frac{1}{e}}^{e} |\ln x|\,dx = \underbrace{-\int_{\frac{1}{e}}^{1} \ln x\,dx}_{I_1} + \underbrace{\int_1^e \ln x\,dx}_{I_2} =$

$I_1 = \int_{\frac{1}{e}}^{1} \underbrace{1}_{g'} \cdot \underbrace{\ln x}_{f}\,dx = \left[x \cdot \ln x\right]_{\frac{1}{e}}^{1} - \int_{\frac{1}{e}}^{1} \underbrace{\frac{1}{x} x}_{1}\,dx = \left[x \cdot \ln x - x\right]_{\frac{1}{e}}^{1} = 1\ln 1 - 1 - \left(\frac{1}{e} \cdot \ln\frac{1}{e} - \frac{1}{e}\right) =$

$T-\left(\frac{1}{e}\right)$

$f = \ln x \xrightarrow{dr} f' \frac{1}{x}$

$g' = 1 \xrightarrow{int} g = x$

$= -1 - \left(-\frac{1}{e} - \frac{1}{e}\right) = -1 + \frac{2}{e}$

$I_2 = \int_1^e \ln x\,dx =$ ua.

3, $\begin{cases} y^2 = x & y = \sqrt{x} \quad x \in \mathbb{R}_+ \\ y = x^2 \end{cases}$

1, lépés metszéspont meghatározás

$$\left.\begin{aligned} y &= \sqrt{x} \\ y &= x^2 \end{aligned}\right\}$$

$$\sqrt{x} = x^2 \to x = x^4$$

$$1 = x^4 \quad (x \in \mathbb{R}_+)$$

$x = 1$ $\qquad x^4 - x = 0$

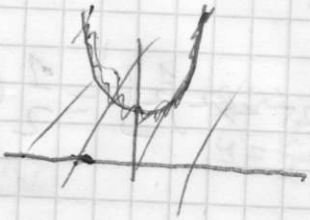
$x = 0$ $\qquad x(x^3 - 1) = 0$

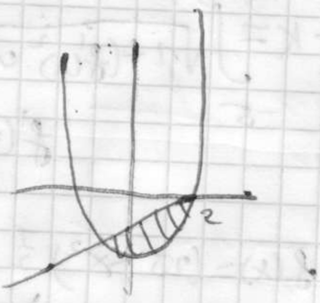
2. lépés

$$T = \int_0^1 \underbrace{\sqrt{x}}_{x^{\frac{1}{2}}} dx - \int_0^1 x^2\,dx = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1 = \frac{1^{\frac{3}{2}}}{\frac{3}{2}} - 0 - \left(\frac{1^3}{3} - 0\right) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

4, $f, g : \mathbb{R} \to \mathbb{R}$ $\quad f(x) = x^2 - 4$

$\qquad\qquad\qquad\quad g(x) = x - 2$





metszéspont meghatározás

$$\left.\begin{aligned} y &= x^2 - 4 \\ y &= x - 2 \end{aligned}\right\}$$

$$x^2 - 4 = x - 2$$

$$x^2 - x - 2 = 0$$

$\cancel{x_{12}}$

$$x_{12} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \begin{matrix} \nearrow 2 \\ \searrow -1 \end{matrix}$$

$$T = \int_{-1}^{2} x^2 - 4\,dx - \int_{-1}^{2} (x-2)\,dx = \left[\frac{x^3}{3} - 4x - \frac{x^2}{2} - 2x\right]_{-1}^{2} =$$

$=$ $\qquad - \qquad =$

$F(2)$ $\qquad F(-2)$

~~Görbe~~

Görbe ívhossza $l(\Gamma)=\int_{\alpha}^{\beta}\sqrt{1+[f'(t)]^2}\,dt$

Forgástest térfogata

$$V(H)=\pi\int_{\alpha}^{\beta} f^2(x)\,dx$$

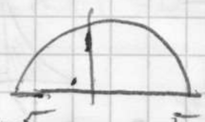
Forgástest felszíne

$$F(H)=2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$$

1, Szám. ki az 5 sugarú kör kerületét (ívhosszát)!

$x^2+y^2=25$

$y>0$



$\boxed{y=\sqrt{25-x^2}}$

$$\frac{1}{2}K=\int_{-5}^{5}\sqrt{1+(f'(x))^2}\,dx$$

$$f'(x)=\left[(25-x^2)^{\frac{1}{2}}\right]'=\frac{1}{2}(25-x^2)^{-\frac{1}{2}}\cdot(-2x)=\frac{-\cancel{2}x}{\cancel{2}\sqrt{25-x^2}}$$

$f(x)=(25-x^2)^{\frac{1}{2}}$

$r(x)=x^{\frac{1}{2}}$ $\qquad$ $l(x)=25-x^2$

$r'(x)=\frac{1}{2}\cdot x^{-\frac{1}{2}}$ $\qquad$ $l'(x)=-2x$

$f'(x)=\frac{1}{2}(25-x^2)^{-\frac{1}{2}}\cdot(-2x)$

$$\int_{-5}^{5}\sqrt{1+\frac{x^2}{25-x^2}}\,dx=\int_{-5}^{5}\sqrt{\frac{25\cancel{-x^2}\cancel{+x^2}}{25-x^2}}\,dx=\int_{-5}^{5}\frac{5}{\sqrt{25-x^2}}\,dx=\left[5\cdot\arcsin\frac{t}{5}\right]_{-5}^{5}=$$

$$=5\cdot\arcsin\frac{5}{5}-5\cdot\arcsin\frac{-5}{5}=\qquad =5\pi$$

$K=10\pi$

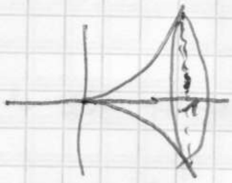
2, Az $R=5$ sugarú gömb térfogata

$f(x)=\sqrt{25-x^2}$

$$V(H)=\pi\int_{-5}^{5}\left(\sqrt{25-x^2}\right)^2 dx=\pi\int_{-5}^{5}(25-x^2)\,dx=\pi\left[25x-\frac{x^3}{3}\right]_{-5}^{5}=$$

$$=\pi\left[\underbrace{25\cdot5-\frac{5^3}{3}}_{F(5)}-\left(25\cdot(-5)-\frac{(-5)^3}{3}\right)\right]=\frac{4\pi\cdot5^3}{3}$$

3, $f(x)=x^3$ $\quad x\in[0,1]$



$f'(x)=3x^2$

$$A=F(H)=2\pi\int_0^1\frac{36}{36}x^3\cdot\sqrt{1+9x^4}\,dx=\frac{2\pi}{36}\int\sqrt{t}\,dt=\frac{2\pi}{36}\left[\sqrt{1+9x^2}\right]_0^1=\frac{2\pi}{36}(1+9-1)$$

$1+9x^4=t$

$36x^3\,dx=dt$

Differenciálegyenlete:

$y(x)$

$y'=\frac{dy}{dx}$

I Szétválasztható változójú / normális

$$f_1(x)\,g_1(y)\,dx+f_2(x)\cdot g_2(y)\cdot d(y)=0$$

$$\frac{f_1(x)}{f_2(x)}dx+\frac{g_2(y)}{g_1(y)}dy=0$$

1, $x^3dx+(y+1)^2dy=0$

$\int x^3dx+\int(y+1)^2dy=C$ $\qquad y+1=t$

$dy=dt$

$\frac{x^4}{4}+\int t^2dt=C$

$\frac{x^4}{4}+\frac{(y+1)^3}{3}=C$

$\frac{y+1}{3}=C-\frac{x^4}{4}$

$y=\sqrt[3]{3\left(C-\frac{x^4}{4}\right)}-1$

2) $9\cdot(9+4x^2)y' = 1$

$$y' = \frac{dy}{dx}$$

$$9(9+4x^2)\,dy = dx$$

$$y\,dy = \frac{1}{9+4x^2}\,dx$$

$$\int y\,dy = \int \frac{1}{9+4x^2}\,dx + C$$

$$\frac{y^2}{2} = \frac{1}{4}\int \frac{1}{\frac{9}{4}+x^2}\,dx$$

$$\frac{y^2}{2} = \frac{1}{4}\cdot\frac{1}{\frac{9}{4}}\operatorname{arctg}\frac{x}{\frac{9}{4}} + C \qquad \cdot 2 \quad \sqrt{\ }$$

$$y = \pm\sqrt{\frac{2}{9}\operatorname{arctg}\frac{4x}{9} + C}$$

3) $y' = \dfrac{4y}{x\cdot(y-3)}$

$$\frac{dy}{dx} = \frac{4y}{x(y-3)}$$

$$\frac{y-3}{4y}\,dy = \frac{1}{x}\,dx$$

$$\frac{1}{4}\int\frac{y-3}{y}\,dy = \int\frac{1}{x}\,dx$$

$$\frac{1}{4}\int\left(1-3\frac{1}{y}\right)dy = \int\frac{1}{x}\,dx$$

$$\frac{1}{4}\cdot(y-3\cdot\ln|y|) = \ln|x| + C$$

4) $x^2(y+1)\,dx + y^2\cdot(x-1)\,dy = 0$

$$\frac{x^2}{x-1}\,dx + \frac{y^2}{y+1}\,dy = 0$$

$$\int\frac{x^2}{x-1}\,dx + \int\frac{y^2}{y+1}\,dy = C$$

$$\int\frac{x^2}{x-1}\,dx = \int\frac{x^2-1+1}{x-1}\,dx = \int\left(\frac{(x-1)(x+1)}{x-1} + \frac{1}{x-1}\right)dx = \int\left(x+1+\frac{1}{x-1}\right)dx =$$

$$= \frac{x^2}{2} + x + \ln|x-1| + C$$

$$\int \frac{y^2-1+1}{y+1}\,dy = \int \left(y-1+\frac{1}{y+1}\right)dy = \frac{y^2}{2} - y + \ln|y+1| + C$$

$$\frac{x^2}{2} + x + \ln|x-1| = \frac{y^2}{2} - y + C \quad \ln|y+1| \cancel{=} c$$

II. elsőrendű lineáris diff.-egyenletek $\quad y' + f(x)\cdot y = g(x)$

→ Homogén $g(x)=0$

→ Inhomogén $g(x) \neq 0$

1, $y' - \frac{y}{x} = x^3+3x-2$

I. első lépés megoldjuk a homogén párját

$$y' - \frac{y}{x} = 0$$

$$\frac{dy}{dx} = \frac{y}{x} \qquad \frac{1}{y}\,dy = \frac{1}{x}\,dx \qquad \int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

$$\ln|y| = \ln|x| + \underbrace{C}_{\ln c}$$

$$\ln|y| = \ln|x|\cdot c \quad / e^{()}$$

$$\boxed{y_H = c\,x}$$

második lépés $y_0 = ?$

$y = C(x)\cdot x$

(keresett $C(x)$-et írok)

$y' = C'(x)\cdot x + C(x)\cdot 1$

behelyettesítem az eredeti diff.-egyenletbe

$\cancel{C'(x)\cdot x + C(x) - C(x)}$

$$\overbrace{C'(x)\cdot x + C(x)}^{y'} - \overbrace{\frac{C(x)\,x}{x}}^{-\frac{y}{x}} = x^3+3x-2$$

$=0$

$$C'(x)\cdot x = x^3 - 3x - 2 \qquad \Big/ \frac{1}{x}$$

$$C'(x) = x^2 + 3 - \frac{2}{x}$$

$$C(x) = \int \left(x^2+3-\frac{2}{x}\right)dx = \frac{x^3}{3} + 3x - 2\ln|x| + C$$

$$y_0 = C(x)\cdot x = \frac{x^4}{3} + 3x^2 - 2x\ln|x| + cx$$

3. lépés $\quad y = y_0 + y_H \ldots$

2, $(x-2)y' - y = 2\cdot(x-2)^3$

1 lépés $(x-2)y' - y = 0$

$(x-2)\frac{dy}{dx} = y$

$\frac{1}{y}\,dy = \frac{1}{x-2}\,dx \qquad \int\frac{1}{y}\,dy = \int\frac{1}{x-2}\,dx$

$\ln|y| = \underbrace{\ln|x-2| + \underset{\ln c}{\overset{\shortparallel}{c}}}_{\ln|x-2|\cdot c}$

$\boxed{y_H = (x-2)\cdot c}$

2 lépés $\left.\begin{array}{l} y_0 = (x-2)\cdot c(x) \\ y' = 1\cdot c(x) + (x-2)\cdot c'(x) \end{array}\right\} \Rightarrow$ behelyettesítés a rendszerbe

$(x-2)\cdot\underbrace{[c(x) + (x-2)\cdot c'(x)]}_{y'} - \underbrace{(x-2)\cdot c(x)}_{y} = 2\cdot(x-2)^3$

$\cancel{(x-2)c(x)} + (x-2)^2\cdot c'(x) - \cancel{(x-2)c(x)} = 2(x-2)^3$

$(x-2)^2\cdot c'(x) = 2\cdot(x-2)^3$

$c'(x) = 2(x-2)$

$c(x) = 2\cdot\left(\frac{x^2}{2} - 2x\right)$

$y_0 = (x-2)\cdot 2\cdot\left(\frac{x^2}{2} - 2x\right)$

3. lépés $y = y_H + y_0 \ldots$