

Programozáselmélet dolgozat

2008. március 19.

1a. feladat (5 pont)

Adja meg a következő Hoare-formula levezetését, ahol $D(x, y)$ az x egész szám y -nál nem nagyobb, pozitív osztóinak a számát jelöli, azaz

$$D(x, y) = |\{d : 0 < d \leq y \wedge d | x\}|.$$

$\{0 \leq n\} x := 1; y := 0; \mathbf{while} \ x \leq n \ \mathbf{do} \ \mathbf{if} \ x | n \ \mathbf{then}$
 $\quad \{y = D(n, x - 1) \wedge 0 < x \leq n + 1 \wedge x \leq n \wedge x | n\}$
 $\quad \quad y := y + 1 \ \mathbf{else} \ \mathbf{skip} \ \mathbf{fi}; \ x := x + 1 \ \mathbf{od} \ \{y = D(n, n)\}$

Megoldás:

$$\begin{aligned} & \cup \{0 \leq n\} \\ & \cup \{0 = D(n, 1 - 1) \wedge 0 < 1 \leq n + 1\} \\ & \quad x := 1; \\ & \quad \{0 = D(n, x - 1) \wedge 0 < x \leq n + 1\} \\ & \quad y := 0; \\ & \quad \{y = D(n, x - 1) \wedge 0 < x \leq n + 1\} \\ & \quad \mathbf{while} \ x \leq n \ \mathbf{do} \\ & \quad \quad \{y = D(n, x - 1) \wedge 0 < x \leq n + 1 \wedge x \leq n\} \\ & \quad \quad \mathbf{if} \ x | n \ \mathbf{then} \\ & \quad \quad \quad \cup \{y = D(n, x - 1) \wedge 0 < x \leq n + 1 \wedge x \leq n \wedge x | n\} \\ & \quad \quad \quad \cup \{y + 1 = D(n, x) \wedge 0 < x + 1 \leq n + 1\} \\ & \quad \quad \quad \quad y := y + 1 \\ & \quad \quad \quad \quad \{y = D(n, x) \wedge 0 < x + 1 \leq n + 1\} \\ & \quad \quad \mathbf{else} \\ & \quad \quad \quad \cup \{y = D(n, x - 1) \wedge 0 < x \leq n + 1 \wedge x \leq n \wedge x \not| n\} \\ & \quad \quad \quad \cup \{y = D(n, x) \wedge 0 < x + 1 \leq n + 1\} \\ & \quad \quad \quad \mathbf{skip} \\ & \quad \quad \quad \{y = D(n, x) \wedge 0 < x + 1 \leq n + 1\} \\ & \quad \quad \mathbf{fi}; \\ & \quad \quad \{y = D(n, x) \wedge 0 < x + 1 \leq n + 1\} \\ & \quad \quad \quad x := x + 1 \\ & \quad \quad \quad \{y = D(n, x - 1) \wedge 0 < x \leq n + 1\} \\ & \quad \mathbf{od} \\ & \quad \cup \{y = D(n, x - 1) \wedge 0 < x \leq n + 1 \wedge x > n\} \\ & \quad \cup \{y = D(n, n)\} \end{aligned}$$

1b. feladat (5 pont)

Adja meg a következő Hoare-formula levezetéséhez szükséges ciklusinvariánst:

$\{\top\} z := x; y := \lambda; \mathbf{while} \ z \neq \lambda \ \mathbf{do} \ \mathbf{if} \ f(z) = f(t(z))$
 $\quad \mathbf{then} \ y := yf(z) \ \mathbf{else} \ \mathbf{skip} \ \mathbf{fi}; \ z := t(z) \ \mathbf{od} \ \{y = \text{dec}(x)\}$

Megoldás: $y \text{ dec}(z) = \text{dec}(x)$

2a. feladat (5 pont)

Adja meg a következő Hoare-formula levezetését (a hatvány csak nem negatív kitevő esetén értelmezett):

$$[n \geq 0] \ x := n; \ y := m; \ z := 1; \ \mathbf{while} \ x > 0 \ \mathbf{do}$$

$$[z y^x = m^n \wedge x > 0 \wedge x = k] \ \mathbf{if} \ 2 \mid x$$

$$\mathbf{then} \ x := x/2; \ y := y^2 \ \mathbf{else} \ x := x - 1; \ z := yz \ \mathbf{fi} \ \mathbf{od} \ [z = m^n]$$

Megoldás:

$$\cup [n \geq 0]$$

$$\cup [1 m^n = m^n]$$

$$x := n;$$

$$[1 m^x = m^n]$$

$$y := m;$$

$$[1 y^x = m^n]$$

$$z := 1;$$

$$[z y^x = m^n]$$

$$\mathbf{while} \ x > 0 \ \mathbf{do} \quad z y^x = m^n \wedge x > 0 \supset x > 0$$

$$[z y^x = m^n \wedge x > 0 \wedge x = k]$$

$$\mathbf{if} \ 2 \mid x \ \mathbf{then}$$

$$\cup [z y^x = m^n \wedge x > 0 \wedge x = k \wedge 2 \mid x]$$

$$\cup [z (y^2)^{x/2} = m^n \wedge x/2 < k]$$

$$x := x/2;$$

$$[z (y^2)^x = m^n \wedge x < k]$$

$$y := y^2$$

$$[z y^x = m^n \wedge x < k]$$

$$\mathbf{else}$$

$$\cup [z y^x = m^n \wedge x > 0 \wedge x = k \wedge 2 \nmid x]$$

$$\cup [yz y^{x-1} = m^n \wedge x - 1 < k]$$

$$x := x - 1;$$

$$[yz y^x = m^n \wedge x < k]$$

$$z := yz$$

$$[z y^x = m^n \wedge x < k]$$

$$\mathbf{fi}$$

$$[z y^x = m^n \wedge x < k]$$

$$\mathbf{od}$$

$$\cup [z y^x = m^n \wedge x \leq 0]$$

$$\cup [z = m^n]$$

2b. feladat (5 pont)

Adja meg a következő Hoare-formula levezetéséhez szükséges ciklusinvariánst és ciklusszámlálót:

$$[0 < y] \ \mathbf{while} \ x < 3 \ \mathbf{do} \ x := x + y; \ y := y + 1 \ \mathbf{od} \ [\top]$$

Megoldás: ciklusinvariáns: $0 < y$, ciklusszámláló: $-x + 3$.

3. feladat (10 pont)

Határozza meg a következő leggyengébb előfeltételt, használja az $m(x, y)$ jelölést a két szót összefésülő függvényre (pl. $m(\text{abc}, \text{efghi}) = \text{aebfcghi}$):

$$\text{wp}(\text{ while } xy \neq \lambda \text{ do } z := zf(x)f(y); x := t(x); y := t(y) \text{ od}, z = w)$$

Megoldás:

$$\begin{aligned} & \text{wp}(\text{ while } xy \neq \lambda \text{ do } z := zf(x)f(y); x := t(x); y := t(y) \text{ od}, z = w) = \exists i \geq 0. P_i, \text{ ahol} \\ P_0 &= xy = \lambda \wedge z = w, \\ P_1 &= xy \neq \lambda \wedge \text{wp}(z := zf(x)f(y); x := t(x); y := t(y), xy = \lambda \wedge z = w) = \\ & \quad xy \neq \lambda \wedge \text{wp}(z := zf(x)f(y), \text{wp}(x := t(x), \text{wp}(y := t(y), xy = \lambda \wedge z = w))) = \\ & \quad xy \neq \lambda \wedge \text{wp}(z := zf(x)f(y), \text{wp}(x := t(x), xt(y) = \lambda \wedge z = w)) = \\ & \quad xy \neq \lambda \wedge \text{wp}(z := zf(x)f(y), t(x)t(y) = \lambda \wedge z = w) = \\ & \quad xy \neq \lambda \wedge t(x)t(y) = \lambda \wedge zf(x)f(y) = w = \\ & \quad \max(|x|, |y|) = 1 \wedge zm(x, y) = w \\ P_2 &= xy \neq \lambda \wedge \text{wp}(z := zf(x)f(y), \text{wp}(x := t(x), \text{wp}(y := t(y), \max(|x|, |y|) = 1 \wedge zm(x, y) = w))) = \\ & \quad xy \neq \lambda \wedge \text{wp}(z := zf(x)f(y), \text{wp}(x := t(x), \max(|x|, |t(y)|) = 1 \wedge zm(x, t(y)) = w)) = \\ & \quad xy \neq \lambda \wedge \text{wp}(z := zf(x)f(y), \max(|t(x)|, |t(y)|) = 1 \wedge zm(t(x), t(y)) = w) = \\ & \quad xy \neq \lambda \wedge \max(|t(x)|, |t(y)|) = 1 \wedge zf(x)f(y)m(t(x), t(y)) = w = \\ & \quad \max(|x|, |y|) = 2 \wedge zm(x, y) = w \\ P_i &= \max(|x|, |y|) = i \wedge zm(x, y) = w, \text{ ha } i \geq 0 \\ \exists i \geq 0. P_i &= \exists i \geq 0. (\max(|x|, |y|) = i \wedge zm(x, y) = w) = \\ & \quad \exists i \geq 0. \max(|x|, |y|) = i \wedge zm(x, y) = w = \top \wedge zm(x, y) = w = zm(x, y) = w \end{aligned}$$

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1a. feladat (5 pont)

Adja meg a következő Hoare-formula levezetését:

$$\{0 \leq x < 4^k\} y := 0; z := x; \mathbf{while} \ k > 0 \ \mathbf{do} \ k := k - 1; y := 2y;$$

$$\{x = 4^k y^2 + z \wedge 0 \leq z < 4^{k+1}(y+1)\} \mathbf{if} \ z \geq 4^k(2y+1) \ \mathbf{then} \ z := z - 4^k(2y+1); y := y + 1$$

$$\mathbf{else} \ \mathbf{skip} \ \mathbf{fi} \ \{x = 4^k y^2 + z \wedge 0 \leq z < 4^k(2y+1)\} \ \mathbf{od} \ \{y = \lceil \sqrt{x} \rceil\}$$

Megoldás:

$$\cup \{0 \leq x < 4^k\}$$

$$\cup \{x = 4^k 0^2 + x \wedge 0 \leq x < 4^k\}$$

$$y := 0;$$

$$\{x = 4^k y^2 + x \wedge 0 \leq x < 4^k(2y+1)\}$$

$$z := x;$$

$$\{x = 4^k y^2 + z \wedge 0 \leq z < 4^k(2y+1)\}$$

$$\mathbf{while} \ k > 0 \ \mathbf{do}$$

$$\cup \{x = 4^k y^2 + z \wedge 0 \leq z < 4^k(2y+1) \wedge k > 0\}$$

$$\cup \{x = 4^{k-1} (2y)^2 + z \wedge 0 \leq z < 4^k(2y+1)\}$$

$$k := k - 1;$$

$$\{x = 4^k (2y)^2 + z \wedge 0 \leq z < 4^{k+1}(2y+1)\}$$

$$y := 2y;$$

$$\{x = 4^k y^2 + z \wedge 0 \leq z < 4^{k+1}(y+1)\}$$

$$\mathbf{if} \ z \geq 4^k(2y+1) \ \mathbf{then}$$

$$\cup \{x = 4^k y^2 + z \wedge 0 \leq z < 4^{k+1}(y+1) \wedge z \geq 4^k(2y+1)\}$$

$$\cup \{x = 4^k (y+1)^2 + z - 4^k(2y+1) \wedge 0 \leq z - 4^k(2y+1) < 4^k(2(y+1)+1)\}$$

$$z := z - 4^k(2y+1)$$

$$\{x = 4^k (y+1)^2 + z \wedge 0 \leq z < 4^k(2(y+1)+1)\}$$

$$y := y + 1;$$

$$\{x = 4^k y^2 + z \wedge 0 \leq z < 4^k(2y+1)\}$$

$$\mathbf{else}$$

$$\cup \{x = 4^k y^2 + z \wedge 0 \leq z < 4^{k+1}(y+1) \wedge z < 4^k(2y+1)\}$$

$$\cup \{x = 4^k y^2 + z \wedge 0 \leq z < 4^k(2y+1)\}$$

$$\mathbf{skip}$$

$$\{x = 4^k y^2 + z \wedge 0 \leq z < 4^k(2y+1)\}$$

$$\mathbf{fi}$$

$$\{x = 4^k y^2 + z \wedge 0 \leq z < 4^k(2y+1)\}$$

$$\mathbf{od}$$

$$\cup \{x = 4^k y^2 + z \wedge 0 \leq z < 4^k(2y+1) \wedge k \leq 0\}$$

$$\cup \{y = \lceil \sqrt{x} \rceil\}$$

1b. feladat (5 pont)

Adja meg a következő Hoare-formula levezetéséhez szükséges ciklusinvariánst, ahol a $\text{red}(x)$ függvény az x szóból eltávolítja a betűismétléseket:
$$\{\top\} z := x; y := \lambda; \mathbf{while} \ z \neq \lambda \ \mathbf{do} \ \mathbf{if} \ f(y) = f(z) \ \mathbf{then}$$

$$z := t(z) \ \mathbf{else} \ y := f(z)y \ \mathbf{fi} \ \mathbf{od} \ \{y = \text{red}(x^{-1})\}$$
Megoldás: $\text{red}(z^{-1}y) = \text{red}(x^{-1}) \wedge \text{red}(y) = y$.

2a. feladat (5 pont)

Adja meg a következő Hoare-formula levezetését, ahol $\alpha(n)$ az $n(> 0)$ osztóinak a számát, $A(n, x)$ az $n(> 0)$ $x(> 0)$ -nél kisebb osztóinak a számát jelöli:

$$[n > 0] \ x := 2; \ y := 1; \ \mathbf{while} \ x \leq n \ \mathbf{do}$$

$$\quad \mathbf{if} \ x | n \ \mathbf{then} \ y := y + 1 \ [y = A(n, x + 1) \wedge n - x < k] \ \mathbf{else} \ \mathbf{skip} \ \mathbf{fi};$$

$$\quad \quad \quad x := x + 1 \ \mathbf{od} \ [y = \alpha(n)]$$

Megoldás:

$$\cup [n > 0]$$

$$\cup [1 = A(n, 2)]$$

$$x := 2;$$

$$[1 = A(n, x)]$$

$$y := 1;$$

$$[y = A(n, x)]$$

$$\mathbf{while} \ x \leq n \ \mathbf{do} \quad \quad \quad y = A(n, x) \wedge x \leq n \supset n - x + 1 > 0$$

$$\quad [y = A(n, x) \wedge x \leq n \wedge n - x + 1 = k]$$

$$\quad \mathbf{if} \ x | n \ \mathbf{then}$$

$$\quad \quad \cup [y = A(n, x) \wedge x \leq n \wedge n - x + 1 = k \wedge x | n]$$

$$\quad \quad \quad y := y + 1$$

$$\quad \quad \quad [y = A(n, x + 1) \wedge n - x < k]$$

$$\quad \mathbf{else}$$

$$\quad \quad \cup [y = A(n, x + 1) \wedge x \leq n \wedge n - x + 1 = k \wedge x \not| n]$$

$$\quad \quad \quad [y = A(n, x + 1) \wedge n - x < k]$$

$$\quad \quad \quad \mathbf{skip}$$

$$\quad \quad \quad [y = A(n, x + 1) \wedge n - x < k]$$

$$\quad \quad \quad \mathbf{fi};$$

$$\quad [y = A(n, x + 1) \wedge n - (x + 1) + 1 < k]$$

$$\quad x := x + 1$$

$$\quad [y = A(n, x) \wedge n - x + 1 < k]$$

$$\quad \mathbf{od}$$

$$\cup [y = A(n, x) \wedge x > n]$$

$$\cup [y = \alpha(n)]$$

2b. feladat (5 pont)

Adja meg a következő Hoare-formula levezetéséhez szükséges ciklusinvariánst és ciklusszámlálót:

$$[\top] \ y := 0; \ \mathbf{while} \ x + 1 < 0 \ \mathbf{do} \ \mathbf{if} \ 3 | x \ \mathbf{then} \ x := x/3 \ \mathbf{else} \ y := y + 1; \ x := x + y \ \mathbf{fi} \ \mathbf{od} \ [\top]$$

Megoldás: ciklusinvariáns: $y \geq 0$, ciklusszámláló: $-x$.

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1a. feladat (5 pont)

Adja meg a következő Hoare-formula levezetését, ahol a $\text{col}(x, y)$ függvény kigyűjti az y szóból az x szó első betűjét (pl. $\text{col}(\text{abc}, \text{alma}) = \text{aa}$), de $\text{col}(\lambda, y)$ nem definiált:

$$\{x \neq \lambda\} z := y; w := f(x); \text{ while } z \neq \lambda \text{ do}$$

$$\quad \text{if } f(z) \neq f(w) \text{ then skip else } w := wf(z) \text{ fi}; z := t(z)$$

$$\quad \{w \text{ col}(w, z) = f(x) \text{ col}(x, y)\} \text{ od}; w := t(w) \{w = \text{col}(x, y)\}$$

Megoldás:

$$\cup \{x \neq \lambda\}$$

$$\cup \{f(x) \text{ col}(f(x), y) = f(x) \text{ col}(x, y)\}$$

$$z := y;$$

$$\{f(x) \text{ col}(f(x), z) = f(x) \text{ col}(x, y)\}$$

$$w := f(x);$$

$$\{w \text{ col}(w, z) = f(x) \text{ col}(x, y)\}$$

$$\text{ while } z \neq \lambda \text{ do}$$

$$\quad \{w \text{ col}(w, z) = f(x) \text{ col}(x, y) \wedge z \neq \lambda\}$$

$$\quad \text{if } f(z) \neq f(w) \text{ then}$$

$$\quad \cup \{w \text{ col}(w, z) = f(x) \text{ col}(x, y) \wedge z \neq \lambda \wedge f(z) \neq f(w)\}$$

$$\quad \cup \{w \text{ col}(w, t(z)) = f(x) \text{ col}(x, y)\}$$

$$\quad \text{skip}$$

$$\quad \{w \text{ col}(w, t(z)) = f(x) \text{ col}(x, y)\}$$

$$\quad \text{else}$$

$$\quad \cup \{w \text{ col}(w, z) = f(x) \text{ col}(x, y) \wedge z \neq \lambda \wedge f(z) = f(w)\}$$

$$\quad \cup \{wf(z) \text{ col}(wf(z), t(z)) = f(x) \text{ col}(x, y)\}$$

$$\quad w := wf(z)$$

$$\quad \{w \text{ col}(w, t(z)) = f(x) \text{ col}(x, y)\}$$

$$\quad \text{fi};$$

$$\quad \{w \text{ col}(w, t(z)) = f(x) \text{ col}(x, y)\}$$

$$\quad z := t(z)$$

$$\quad \{w \text{ col}(w, z) = f(x) \text{ col}(x, y)\}$$

$$\text{ od};$$

$$\cup \{w \text{ col}(w, z) = f(x) \text{ col}(x, y) \wedge z = \lambda\}$$

$$\cup \{t(w) = \text{col}(x, y)\}$$

$$w := t(w)$$

$$\{w = \text{col}(x, y)\}$$

1b. feladat (5 pont)

Adja meg a következő Hoare-formula levezetéséhez szükséges ciklusinvariánst, ahol f egy adott egész függvény, $M(x) = \max\{0, f(1), f(2), \dots, f(x)\}$, ha $x > 0$, különben $M(x) = 0$.

$$\{\top\} x := 1; y := 0; \text{ while } x \leq n \text{ do}$$

$$\quad \text{if } f(x) > y \text{ then } y := f(x) \text{ else skip fi}; x := x + 1 \text{ od } \{y = M(n)\}$$
Megoldás: $(y = M(x - 1) \wedge x \leq n + 1) \vee (x = 1 \wedge y = 0 \wedge n < 0)$

3. feladat (10 pont)

Határozza meg a következő leggyengébb előfeltételt:

$$\text{wp}(y := 1; \text{while } y \leq 2n \text{ do } x := x + y; y := y + 2 \text{ od}, y = 2n + 1 \wedge x = n^2)$$

Megoldás:

$$\begin{aligned} & \text{wp}(y := 1; \text{while } y \leq 2n \text{ do } x := x + y; y := y + 2 \text{ od}, y = 2n + 1 \wedge x = n^2) = \\ & \text{wp}(y := 1, \text{wp}(\text{while } y \leq 2n \text{ do } x := x + y; y := y + 2 \text{ od}, y = 2n + 1 \wedge x = n^2)) \\ \text{wp}(\text{while } y \leq 2n \text{ do } x := x + y; y := y + 2 \text{ od}, y = 2n + 1 \wedge x = n^2) &= \exists i \geq 0. P_i, \text{ ahol} \\ P_0 = y > 2n \wedge y = 2n + 1 \wedge x = n^2 &= y = 2n + 1 \wedge x = n^2, \\ P_1 = y \leq 2n \wedge \text{wp}(x := x + y; y := y + 2, y = 2n + 1 \wedge x = n^2) &= \\ & y \leq 2n \wedge \text{wp}(x := x + y, \text{wp}(y := y + 2, y = 2n + 1 \wedge x = n^2)) = \\ & y \leq 2n \wedge \text{wp}(x := x + y, y + 2 = 2n + 1 \wedge x = n^2) = \\ & y \leq 2n \wedge y = 2n - 1 \wedge x + y = n^2 = y = 2n - 1 \wedge x + y = n^2 = \\ & y = 2n - 1 \wedge x + 2n - 1 = n^2 = y = 2n - 1 \wedge x = (n - 1)^2 \\ P_2 = y \leq 2n \wedge \text{wp}(x := x + y, \text{wp}(y := y + 2, y = 2n - 1 \wedge x = (n - 1)^2)) &= \\ & y \leq 2n \wedge \text{wp}(x := x + y, y + 2 = 2n - 1 \wedge x = (n - 1)^2) = \\ & y \leq 2n \wedge y = 2n - 3 \wedge x + y = (n - 1)^2 = y = 2n - 3 \wedge x = (n - 2)^2 \\ P_i = y = 2(n - i) + 1 \wedge x = (n - i)^2, \text{ ha } i \geq 0 & \\ \exists i \geq 0. P_i = \exists i \geq 0. (y = 2(n - i) + 1 \wedge x = (n - i)^2) &= \\ \exists i \geq 0. (y - 1 = 2(n - i) \wedge 4x = (2(n - i))^2) &= \exists i \geq 0. 2i = 2n - y + 1 \wedge 4x = (y - 1)^2 = \\ 2n - y + 1 \geq 0 \wedge 2 \mid 2n - y + 1 \wedge 4x = (y - 1)^2 &= 2n \geq y - 1 \wedge 2 \mid y + 1 \wedge 4x = (y - 1)^2 \\ \text{wp}(y := 1, 2n \geq y - 1 \wedge 2 \mid y + 1 \wedge 4x = (y - 1)^2) &= 2n \geq 1 - 1 \wedge 2 \mid 1 + 1 \wedge 4x = (1 - 1)^2 = \\ & n \geq 0 \wedge x = 0. \end{aligned}$$

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1a. feladat (5 pont)

Adja meg a következő Hoare-formula levezetését, ahol (x, y) , ill. $[x, y]$ jelöli az $x(> 0)$ és $y(> 0)$ legnagyobb közös osztóját, ill. legkisebb közös többszörösét:

$$\{a > 0 \wedge b > 0\} x := a; y := b; u := b; v := 0; \text{ while } x \neq y \text{ do} \\ \text{ if } x > y \text{ then } x := x - y; v := u + v \text{ else } y := y - x; u := u + v \\ \{ (x, y) = (a, b) \wedge ux + vy = ab \} \text{ fi od } \{ u + v = [a, b] \}$$

Megoldás:

$$\cup \{a > 0 \wedge b > 0\} \\ \cup \{(a, b) = (a, b) \wedge ba = ab\} \\ x := a; \\ \{(x, b) = (a, b) \wedge bx = ab\} \\ y := b; \\ \{(x, y) = (a, b) \wedge bx = ab\} \\ u := b; \\ \{(x, y) = (a, b) \wedge ux = ab\} \\ v := 0; \\ \{(x, y) = (a, b) \wedge ux + vy = ab\} \\ \text{ while } x \neq y \text{ do} \\ \{(x, y) = (a, b) \wedge ux + vy = ab \wedge x \neq y\} \\ \text{ if } x > y \text{ then} \\ \cup \{(x, y) = (a, b) \wedge ux + vy = ab \wedge x \neq y \wedge x > y\} \\ \cup \{(x - y, y) = (a, b) \wedge u(x - y) + (u + v)y = ab\} \\ x := x - y; \\ \{(x, y) = (a, b) \wedge ux + (u + v)y = ab\} \\ v := u + v \\ \{(x, y) = (a, b) \wedge ux + vy = ab\} \\ \text{ else} \\ \cup \{(x, y) = (a, b) \wedge ux + vy = ab \wedge x \neq y \wedge x \leq y\} \\ \cup \{(x, y - x) = (a, b) \wedge (u + v)x + v(y - x) = ab\} \\ y := y - x; \\ \{(x, y) = (a, b) \wedge (u + v)x + vy = ab\} \\ u := u + v \\ \{(x, y) = (a, b) \wedge ux + vy = ab\} \\ \text{ fi} \\ \{(x, y) = (a, b) \wedge ux + vy = ab\} \\ \text{ od} \\ \cup \{(x, y) = (a, b) \wedge ux + vy = ab \wedge x = y\} \\ \cup \{u + v = [a, b]\}$$

1b. feladat (5 pont)

Adja meg a következő Hoare-formula levezetéséhez szükséges ciklusinvariánst, ahol a $\text{red}(x)$ függvény az x szóból eltávolítja a betűismétléseket:

$$\{\top\} z := x; y := \lambda; \text{ while } z \neq \lambda \text{ do if } f(z) = f(t(z)) \\ \text{ then skip else } y := yf(z) \text{ fi; } z := t(z) \text{ od } \{y = \text{red}(x)\}$$

Megoldás: $y \text{ red}(z) = \text{red}(x)$.

2a. feladat (5 pont)

Adja meg a következő Hoare-formula levezetését:

$$[\top] y := 0; \text{ while } x > 0 \text{ do if } x = y \text{ then } x := x - y; y := y - 3 \\ [-3 < y \wedge x + y + 2 < z] \text{ else } x := x - 2; y := y + 1 \text{ fi od } [\top]$$

Megoldás:

$$\begin{aligned} & \cup \begin{array}{l} [\top] \\ [-3 < 0] \\ y := 0; \\ [-3 < y] \end{array} \\ & \text{while } x > 0 \text{ do} \qquad \qquad \qquad -3 < y \wedge x > 0 \supset x + y + 2 > 0 \\ & \quad [-3 < y \wedge x > 0 \wedge x + y + 2 = z] \\ & \quad \text{if } x = y \text{ then} \\ & \quad \quad \cup \begin{array}{l} [-3 < y \wedge x > 0 \wedge x + y + 2 = z \wedge x = y] \\ [-3 < y - 3 \wedge x - y + y - 3 + 2 < z] \end{array} \\ & \quad \quad \quad x := x - y; \\ & \quad \quad \quad [-3 < y - 3 \wedge x + y - 3 + 2 < z] \\ & \quad \quad \quad y := y - 3 \\ & \quad \quad \quad [-3 < y \wedge x + y + 2 < z] \\ & \quad \text{else} \\ & \quad \quad \cup \begin{array}{l} [-3 < y \wedge x > 0 \wedge x + y + 2 = z \wedge x \neq y] \\ [-3 < y + 1 \wedge x - 2 + y + 1 + 2 < z] \end{array} \\ & \quad \quad \quad x := x - 2; \\ & \quad \quad \quad [-3 < y + 1 \wedge x + y + 1 + 2 < z] \\ & \quad \quad \quad y := y + 1 \\ & \quad \quad \quad [-3 < y \wedge x + y + 2 < z] \\ & \quad \text{fi} \\ & \quad [-3 < y \wedge x + y + 2 < z] \\ & \text{od} \\ & \cup \begin{array}{l} [-3 < y \wedge x \leq 0] \\ [\top] \end{array} \end{aligned}$$

2b. feladat (5 pont)

Adja meg a következő Hoare-formula levezetéséhez szükséges ciklusinvariánst és ciklusszámlálót:

$$[y = 0] \text{ while } x \leq 0 \text{ do } x := x + 2y; y := 1 - y \text{ od } [\top]$$

Megoldás: ciklusinvariáns: $y = 0 \vee y = 1$, ciklusszámláló: $2 - y - x$.

3. feladat (10 pont)

Határozza meg a következő leggyengébb előfeltételt:

$$\text{wp}(\text{while } |x| \leq 3 \text{ do } x := x - 1; y := y + 2 \text{ od}; y := 1 - y, y < 0)$$

Megoldás:

$$\begin{aligned} \text{wp}(\text{while } |x| \leq 3 \text{ do } x := x - 1; y := y + 2 \text{ od}; y := 1 - y, y < 0) &= \\ \text{wp}(\text{while } |x| \leq 3 \text{ do } x := x - 1; y := y + 2 \text{ od}, \text{wp}(y := 1 - y, y < 0)) &= \\ \text{wp}(\text{while } |x| \leq 3 \text{ do } x := x - 1; y := y + 2 \text{ od}, 1 - y < 0) &= \end{aligned}$$

$\exists i \geq 0. P_i$, ahol

$$P_0 = |x| > 3 \wedge 1 < y,$$

$$\begin{aligned} P_1 = |x| \leq 3 \wedge \text{wp}(x := x - 1; y := y + 2, |x| > 3 \wedge 1 < y) &= \\ |x| \leq 3 \wedge \text{wp}(x := x - 1, \text{wp}(y := y + 2, |x| > 3 \wedge 1 < y)) &= \\ |x| \leq 3 \wedge \text{wp}(x := x - 1, |x| > 3 \wedge 1 < y + 2) &= \\ |x| \leq 3 \wedge |x - 1| > 3 \wedge 1 < y + 2 = x = -3 \wedge -1 < y & \end{aligned}$$

$$\begin{aligned} P_2 = |x| \leq 3 \wedge \text{wp}(x := x - 1, \text{wp}(y := y + 2, x = -3 \wedge -1 < y)) &= \\ |x| \leq 3 \wedge \text{wp}(x := x - 1, x = -3 \wedge -1 < y + 2) &= \\ |x| \leq 3 \wedge x - 1 = -3 \wedge -3 < y) = x = -2 \wedge -3 < y & \end{aligned}$$

$$P_i = \begin{cases} |x| > 3 \wedge 1 < y, & \text{ha } i = 0 \\ x = i - 4 \wedge 1 - 2i < y, & \text{ha } i = 1, \dots, 7 \\ \perp, & \text{ha } i = 8, \dots \end{cases}$$

$\exists i \geq 0. P_i =$

$$\begin{aligned} (|x| > 3 \wedge 1 < y) \vee \exists 1 \leq i \leq 7. (i = x + 4 \wedge 1 - 2(x + 4) < y) &= \\ (|x| > 3 \wedge 1 < y) \vee (1 \leq x + 4 \leq 7 \wedge 1 - 2(x + 4) < y) &= \\ (|x| > 3 \wedge 1 < y) \vee (|x| \leq 3 \wedge 0 < y + 2x + 7) & \end{aligned}$$