APPLIED GAME THEORY

Jozsef Zoltan Malik

BUDAPEST METROPOLITAN UNIVERSITY

Budapest, 2022

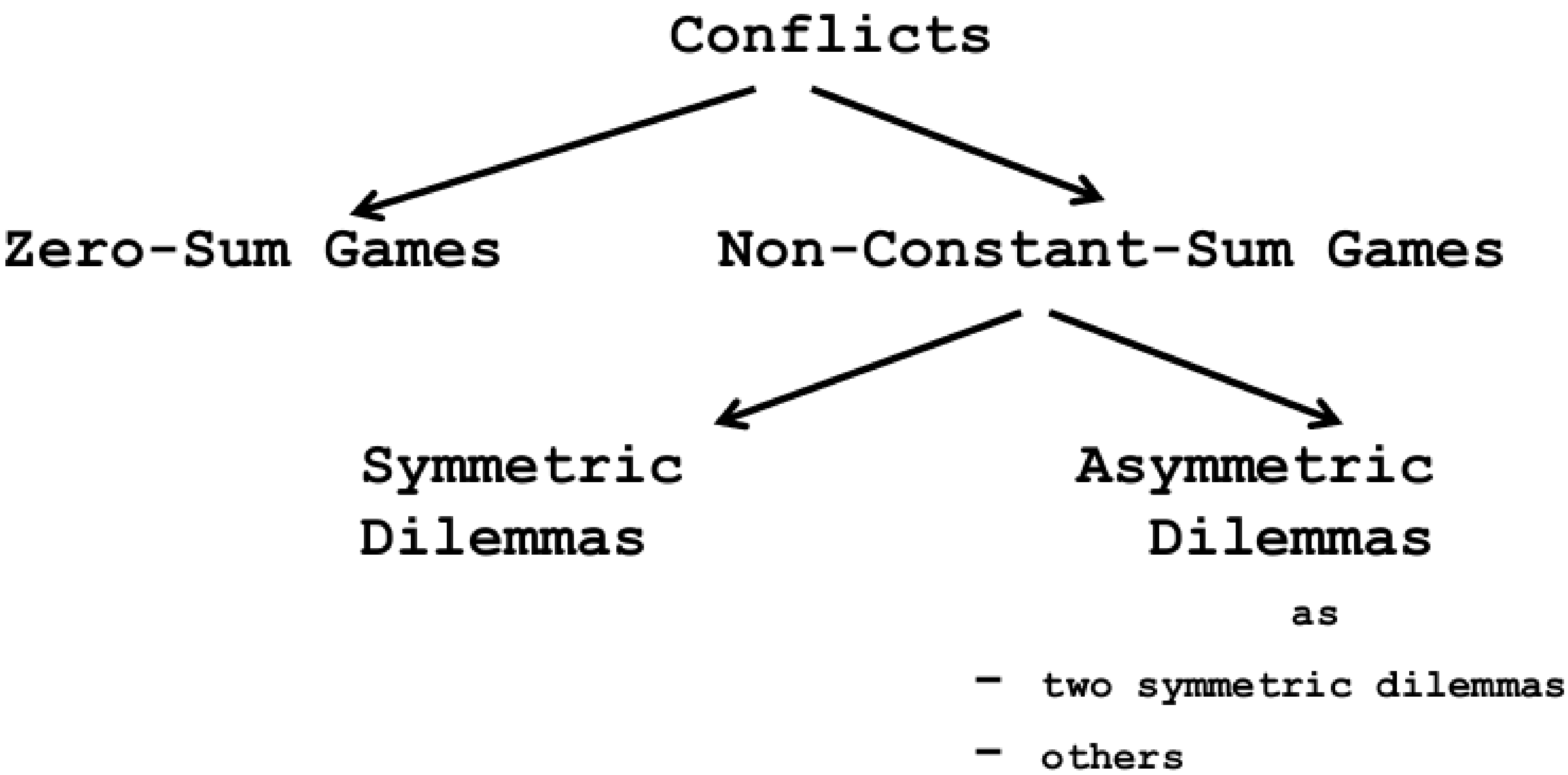
An Introductory Course to Game Theory
Part II: Application

IV. Conflicts

TYPES OF POLITICAL VIOLENCE

		Target	
Source		**Individual Groups**	**State**
	Individual Groups	Crime, Terrorism, Social conflicts	Riots, Rebellion Coup, Revolution
	State	Establishment violence State-sponsored terrorism Order maintenance	International Conflicts War

IV.2. Conflicts in Game Theory

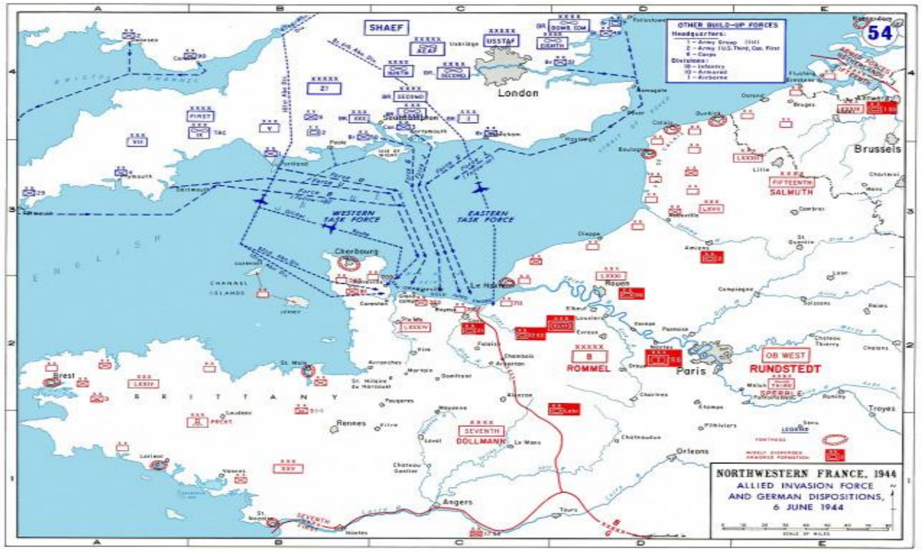


IV.3. The D-Day as Zero-Sum Game

- **Strategies:**

  The Allies can invade either Calais or Normandy.

  Germans can choose between concentrating their forces at Calais or at Normandy.



- **Payoff matrix:**

		Allies	
		Calais	Normandy
Germans	Calais	(1; -1)	(-1;1)
	Normandy	(-1;1)	(1;-1)

- **Assessment:**

  ✓ This is a zero-sum game, the logic of the game is similar to the Matching Pennies game. And so is its payoff matrix.

IV.4. The Crisis of Sarajevo, 1914

HISTORICAL BACKGROUND

The assassination by Serbian nationalists in Sarajevo in 1914, which caused the death of the Archduke, Francis Ferdinand, of the Dual Monarchy of Austria-Hungary, is perhaps the most important event in the history of the twentieth century. This was the immediate cause of World War I, and as a consequence of power vacuum in Central Europe after the disintegration of the Monarchy in 1918, this gave to rise national confrontations,nationalistic (Hungarian Horthy's regime, 1920-44) and autocratic governing (nazism, communism), several crises and genocides (Holocaust in 1940s and Yugoslavian civil wars in 1990s). Serbia was a thorn in the Monarchy's side. In the Balkan war in 1912, Serbia destabilized the region. Together with Bulgaria and Greece they were at war with Turkey, Serbia wanted a large part of Albania and agitated against the Monarchy in Bosnia and Herzegovina that was annexed by the Monarchy in 1908.

In the conflict both parties had a patronage with her own selfish political ambition: Germany backed up the Monarchy and Serbia was supported by Russia. We investigate the conflict between the Monarchy and Russia, the two empires that competed over the Balkan and East Europe.

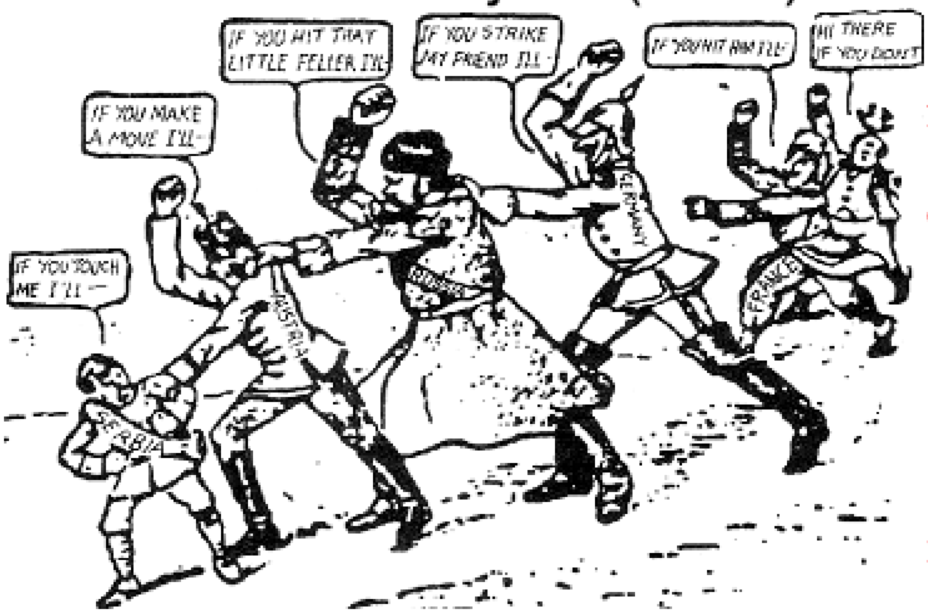
IV.4. Conflict as Zero-Sum Game

Reverse Preferences

Preferences:

- Monarchy: Q (2) > S (1) > P (-1) > R (-2)
- Russia : R (2) > P (1) > S (1) > Q (-2)

The Crisis of Sarajevo (1914)



Outcomes:

P: To make a compromise with Serbia. The influence of Russia is increasing in the Balkan.

Q: The loss of Russia's prestige in the Balkan, including that Serbia get the control over the Monarchy (it would be a new failure for Tsarist Russian Empire after the Russo-Japanese war over Manchuria in which a victorious Japan forced Russia to abandon its expansionist policy in the Far East, becoming the first Asian power in modern times to defeat a European power.)

R: Serbia escapes from the war, the influence of Russia remains, and Serbia continues the agitation

S: Both parties are ready to fight.

Russia (columns) / Monarchy (rows)

	C	D
C	P -1, 1	R -2, 2
D	Q 2, -2	S 1, -1

Russia (columns) / Monarchy (rows)

	C	D	MIN
C	-1	-2	-2
D	2	1	1
MAX	2	1	

IV.5. The Types of Symmetric 2x2 Dilemmas

1. Battle of Sexes – Hero**:** if W is indulgent, she remunerates both herself and her partner, but her partner is better off. **Their preferences are DC CD DD CC**

		H	
		D	C
W	D	**-1, -1**	2, 1
	C	1, 2	-2, -2

Wife
D: Concert
C: Football

Husband
C: Concert
D: Football

The game has several versions. Imagine a couple that agreed to meet this evening, but cannot recall if they will be attending the opera or a football match (and the fact that they forgot is common knowledge). The husband would most of all like to go to the football game. The wife would like to go to the opera. Both would prefer to go to the same place rather than different ones. If they cannot communicate, where should they go?

2. Apology – Leader**:** if row player defects, he remunerates both of them, but he is better off than his opponent. **Their preferences are DC CD CC DD**

	C	D
C	**-1, -1**	1, 2
D	2, 1	-2, -2

D: Waiting
C: Walking out

In an absolute polite society two persons arrive at a revolving door, and they stick to the rule that the other walks out first. The cooperative is the one in this situation who undertakes the rudeness to walk out first, since the other is able to leave the place as a moral winner. What should they do?

3. Game of Chicken – Exploiter**:** if row player defects, he remunerates himself, but punishes the other. **Their preferences are DC CC CD DD**

	C	D
C	**1, 1**	-1, 2
D	2, -1	-2, -2

D: Remain in the game
C: To swerve off the road

The basic idea of the story goes back to a James Dean's cult classic movie „Rebel Without a Cause", and the name of the game is from Bertrand Russell. Two guys compete with each other: they drive their stolen cars to a precipice in a narrow path. The one who swerves off the road is the chicken, and the other going straight on is the winner. What should they do?

IV.6. The Types of Symmetric 2x2 Dilemmas

***4. Prisoner's dilemma – Martyr*:** if row player defects, he injures himself and remunerates the other. **Their preferences are DC CC DD CD**

	D	C
D	**-1, -1**	2, -2
C	-2, 2	1, 1

D: Confess
C: Deny

Two prisoners are suspected of taking part in a serious crime and shut up in separate jails. The punishment depends on whether or not they confess. If both confess, they will be sentenced to five years. If neither confesses, both will get a sentence to one year on account of a lesser guilt. If one confesses and the other does not, the former will be free, while the other will receive a severe sentence of twenty years. What should they do?

5. Deadlock **Their preferences are DC DD CC CD**

	C	D
C	-1, -1	-2, 2
D	2, -2	1, 1

D: Pose
C: Drop out

Two male animals compete over the favor of a female. Both are robust and if they were in a fight, they could be wounded what they don't want. That's why they start posing, that is to say, trying to do something to deter the other. The looser is the one who drops out.

6. Security Dilemma or Stag Hunt **Their preferences are CC DC DD CD**

	C	D
C	2, 2	-2, 1
D	1, -2	-1, -1

D: Rabbit shoot
C: Stag hunt

The idea of the game is from one of the books of J. J. Rousseau. We are on a mega-rich party and we are hunting. The target is a stag, but if we fail, we don't want to go home with „empty hands", and so if there is no stag, some rabbits are enough. The crux is that if we are shooting rabbits, they scare off stags. So what should we do?

IV.7. Symmetric 2x2 Dilemmas: A characterisation

- **What is symmetric 2x2 dilemmas?**

  The situation and the positions of the players are the same but they arrive at different circumstances by choosing their strategies.

- **Their preferences ranked in a 4-stage scale:**

  Battle of Sexes: DC CD DD CC

  Prisoner's dilemma: DC CC DD CD

  Apology: DC CD CC DD

  Game of Chicken: DC CC CD DD

  Deadlock: DC DD CC CD

  Security dilemma: CC DC DD CD
  (or Stag Hunt)

2

# 1		D	C
	D	-1, -1	2, -2
	C	-2, 2	1, 1

DC CC DD CD

2 > 1 > -1 > -2

IV.8. Symmetric 2x2 Dilemmas: Another Characterisation

- **The Principle of Maximin**
  To play their most cautios strategy, players can reach a natural outcome (the left-upper cells) but they may diverge from this strategy as a consequence of psychological pressure.

- **Archetypes**
  Hero: Battle of Sexes (Me: +, Others: ++)
  Martyr: Prisoner's dilemma (Me: –, Others: +)
  Leader: Apology (Me: ++, Others: +)
  Exploiter: Game of Chicken (Me: +, Others:–)

- **Characterization of two conditions**
  Conjuctive: 1) CC ≻ CD 2) DC ≻ DD
  Disjunctive: 3) DC ≻ CC 4) DD ≻ CD

  Prisoner's dilemma, Deadlock: conditions 1-4
  Game of Chicken: conditions 1-3
  Security Dilemma: conditions 1,2 and 4

HERO

	D	C
D	-1, -1	2, 1
C	1, 2	-2, -2

MARTYR

	D	C
D	-1, 1	2, -2
C	-2, 2	1, 1

LEADER

	C	D
C	-1, -1	1, 2
D	2, 1	-2, -2

EXPLOITER

	C	D
C	1, 1	-1, 2
D	2, -1	-2, -2

IV.9. A "Battle of Sexes" Dilemma

- **Question: Which human rights are worthy of defending?**

Where should we draw a line between freedom of speech and freedom of religion including the revelation of sacrilege? Current issues:

- Muslim ladies should wear veil (*hijab*) at work?
- May a caricature on Mohamed (Jesus) be published?
- Debate over Creationism vs. Evolutionism

Liberals: maximal freedom of speech (D) or to put some limit up considering personal right and dignity (C).
Defenders of Religion: a fundamentalist standpoint (D) or to budge the expansion of secularization (C).

Defenders of Religion

Liberals	D	C
D	-1, -1	2, 1
C	1, 2	-2, -2

DC CD DD CC

Who will be the hero?

No accessible

No democratic equilibria

Possible Outcomes:

CC ("the failure of tolerance"): the limitation of personal autonomy from the side of Liberals, and as to the defenders of religion, to make norms and values relative and empty.

DD (the natural outcome): mutual battle may as well take upon segregation and terrorism.

DC, CD (no democratic equilibria): to expect or to make unilateral concessions by giving up personal autonomy.

IV.10. Liberal Paradox

Liberal paradox is an example that refuses the liberal claim that a collective choice always ought to be reconcilable with individual liberties.

- A naughty novel by Lawrence: "Lady Chatterley`s Lover"
  - Players:

    Prude: „stop publishing a book like this in the society"

    Lewd: „everybody has to read it"
  - Alternatives: d_o: no one reads it, d_P: the Prude reads it, d_L: the Lewd reads it.

(1) The Requirement of Personal Autonomy for Prude

- Individual preferences:

  Prude: $d_o \succ d_P \succ d_L$ → the Prude is in a decision position as for d_o and d_P he does not wish to read it and there must not be one who forces him to do it

  Lewd: $d_P \succ d_L \succ d_0$ → the Lewd is in a decision position as for d_L and d_0 he wants to read it, and there must not be one who prohibits him from doing this

(2) The Requirement of Personal Autonomy for Lewd

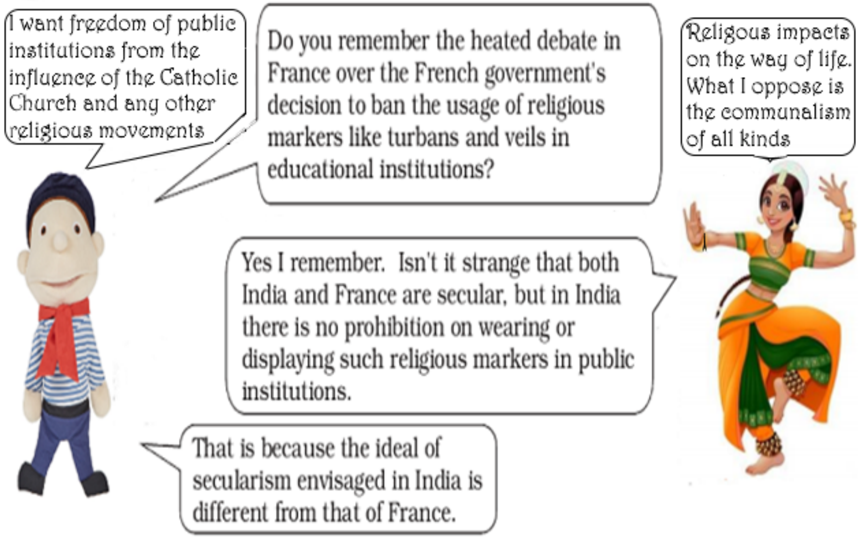
- Social preference:

  $$d_L \overset{(2)}{\succ} d_0 \underset{(1)}{\succ} d_P \iff d_P \succ d_L \text{ in both individual preferences}$$

- The solution might be the assignment of rights of which precondition is
  - to rule out certain people of those concerned in the choice (autocratic solution)
  - to persuade some people to change their preferences (motivational mechanism)
- In the sphere of economy they are ok, but at a social level they are not

IV.11. Interlude: Secularism and Public Life

- **What do we mean by secularism exactly?**
  - **Secularism** is a normative doctrine which seeks to realize a secular society that promotes freedom and equality between, as well as within, religions.

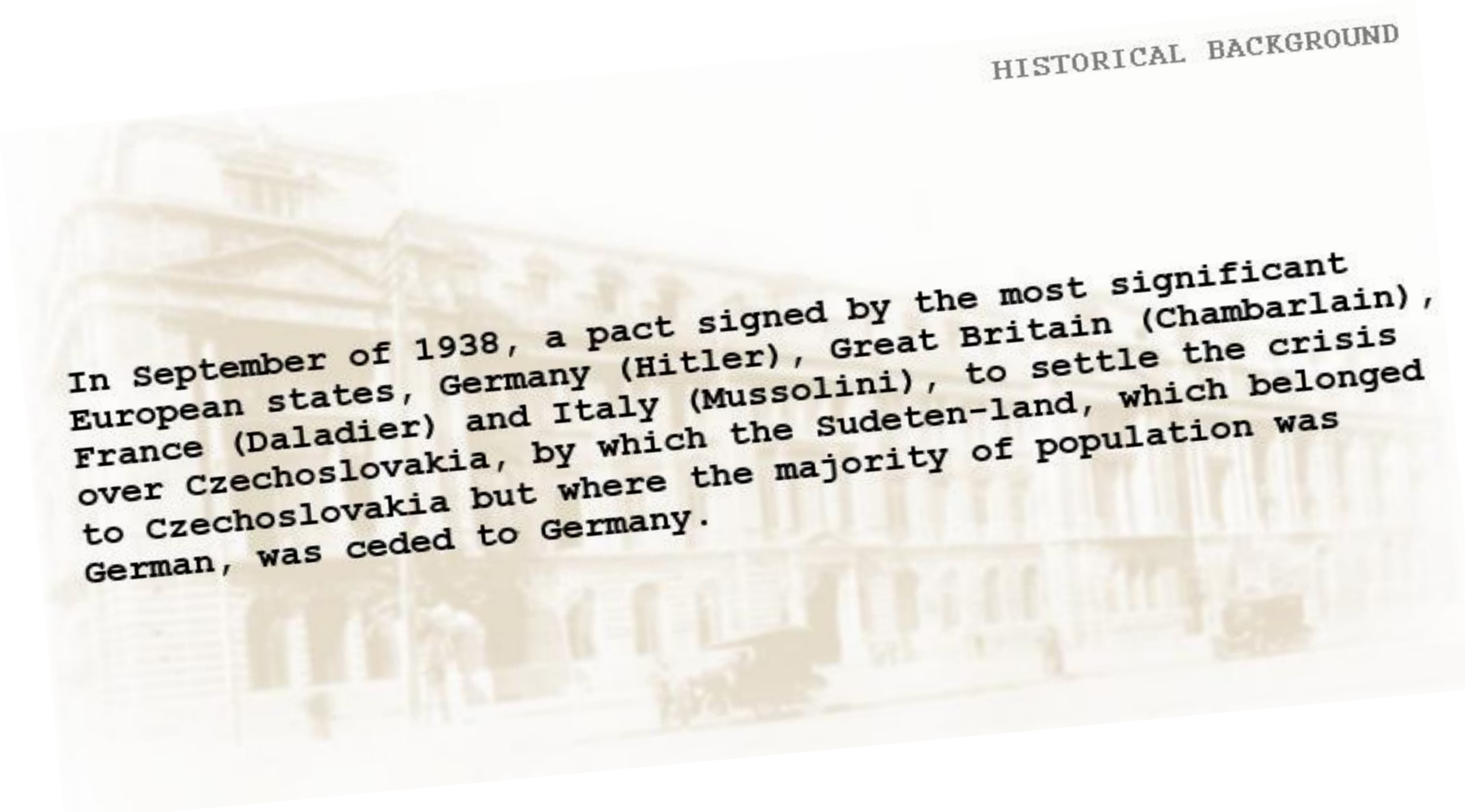


- **How should a state prevent domination by any religious group?**
  - **Non-Theocratic State:** a state must not be run by the heads of any particular religion (counter-examples: Vatican, Iran).
  - Nevertheless, **many states** which are non-theocratic continue to **have a close alliance with a particular religion** (e.g.: England – Anglican State; Denmark – Lutheran Church; Greece – Eastern Orthodox Church; Pakistan – Sunni Islam).
  - A secular state must be committed to principles and goals which are at least partly derived from non-religious sources. (These ends should include peace, religious freedom, freedom from religiously grounded oppressions, discrimination and exclusions).
  - However, the nature and extent of separation may take different forms, depending upon several traditional, historical and other cultural factors.
  - A secular state may interfere in matters of religion to regulate the religious impact on public life.

Kemal Ataturk's reforms in Turkey

- ✓ The Fez (a traditional cap worn by Muslims), was banned by the Hat Law.
- ✓ Western clothing was encouraged for men and women.
- ✓ The Western (Gregorian) calendar replaced the traditional Turkish calendar.
- ✓ In 1928, the new Turkish alphabet (in a modified Latin form) was adopted.

IV.12. Munich Pact, 1938



IV.13. Munich Pact (cont.)

The Dilemma of Hero and Leader

- These two dilemmas are often occur between allies. In a crisis the allies prefer different strategies against the opponent, but they are willing to keep the allies at a cost of giving up the application of their own strategies.
- Both dilemma require an actor moving the situation from the natural outcome (left-upper cells) with efforts: a hero who makes sacrifice (C), and hence the partner get more advantage for her efforts than herself (it is a Battle of Sexes dilemma); or a leader who rules over the situation: she shows determination (D) to make the position better, but thus she arrives at a better position than her partner (it is an Apology dilemma).

HERO

	D	C
D	-1, -1	2, 1
C	1, 2	-2, -2

DC CD DD CC

LEADER

	C	D
C	-1, -1	1, 2
D	2, 1	-2, -2

DC CD CC DD

IV.14. Munich Pact (cont.)

Czechoslovakia

GB and France	C	D
C	-1, -1	1, 2
D	2, 1	-2, -2

GB as Leader
(in an Apology situation)

Czechoslovakia: DC(2) > CD (1) > CC (-1) > DD (-2)

Germany

GB	D	C
D	-1, -1	2, 1
C	1, 2	-2, -2

GB as Hero
(in a Battle of sexes situation)

Outcomes from the Czechoslovakian point of view:

DC: top preference of Czechoslovakia, she can refuse Hitler forcible policy and her allies support her.
CD (one of equilibria): she yields Hitler because of leaving her alone, but avoids the total breakdown.
CC: this is impossible outcome (though she is supported by GB and France, she yields).
DD: her worst outcome is to refuse Hitler, no matter she is not supported. This involves a sure defeat and a possible revenge.

To sum up Chamberlain's pace policy:
Due to international law and Hitler`s forcible policy, the natural outcome would be DD but Chambarlain's pace policy enforces GB to do the DD → CD transmission.

The PM's strategy is this: by territorial arrangement in which GB plays a leader role, both defending the interest of government in Prague and saving the pace in Europe, GB can eventually perform a „hero" on the stage of history. (Refusal: Munich Syndrome – in March 1939 Hitler's troops invades Czechoslovakia.)

IV.15. The Cuban Missile Crisis, 1962

HISTORICAL BACKGROUND

In 1959 F. Batista, who was the president of Cuba from 1940 to 1944 and, after returning to power in 1952 by means of a military coup, from 1954 to 1958 was overthrown by F. Castro. Relations between Castro`s socialist government and the US were increasingly strained and Cuba moved closer in international relations to the Soviet Union. In 1961 an invasion by Cuban exiles with US support was defeated at the Bay of Pigs, and in 1962 the Soviet installation of nuclear missile bases in Cuba resulted in a US naval blockade. The Cubanian missile crises aroused world-wide fear of nuclear catastrophe when the US president J. F. Kennedy put pressure on Khrushchev who was the premier of the Soviet Union (1953-64) after the death of Stalin Finally, the Soviets agreed to remove the basis and thus the crisis was resolved.

This was the first situation to analyse by game-theoretical means from the US side, and the experts pointed out the crisis can be considered as a Game of Chicken dilemma.

IV.16. The Cuban Missile Crisis (cont.)

Strategies:

USA: - Cooperative: a naval blockade to avoid the Soviet arms transport, which is followed by a strict action to persuade the Soviet Union to remove the missile basis;
- Defecting: air attack against the Cuban basis, and then may as well be an invasion of the island;

Soviet Union: - Cooperative: to remove the missile basis and stop to the arms transport under certain conditions (the US does not attack Cuba and to moderate her naval policy in Turkey);
- Defecting: to leave the missiles in Cuba.

Outcomes from the US point of view:

DC and CC: These are the two outcomes in which the Kennedy`s expert team was thinking. CC is the natural outcome, and the fear of nuclear catastrophe (DD) makes the Soviets yield.
CD: This means the unacceptable outcome for the US to agree to building up the missile basis in Cuba.
DD: The risk of nuclear war if the Soviet reply is D to the US behaviour tending to D .

Munich Syndrome

USA: DC(2) > CC (1) > CD (-1) > DD (-2)

Nuclear Doom

The US behave in this situation as an Exploiter: she does not make concessions, will defend "the door of America", and to avoid nuclear conflict the Soviet Union is obliged to compromise. So the US gives her advantage and harms the Soviets by threatening with her own D strategy (CC → DC) .

Soviet Union

USA	C	D
C	1, 1	-1, 2
D	2, -1	-2, -2

US as exploiter:

CC (US: 1, Soviet: 1) → DC (US: 2, Soviet: -1)

IV.17. Asymmetric 2x2 Dilemmas

- In an asymmetric dilemma the players are in the same situation but their position are different.
- One type of 2x2 asymmetric dilemmas is the situation where two players are in two different 2x2 symmetric dilemmas:
  - Called Bluff: Prisoner's Dilemma + Game of Chicken
  - Perceptional dilemma: Prisoner's Dilemma + Stag Hunt or Deadlock + Security Dilemma
  - Bully: Deadlock + Game of Chicken
  - Protector: Deadlock + Game of Chicken or Prisoner's Dilemma + Battle of Sexes
- The "Prisoner's Dilemma + Deadlock" asymmetric situation is so intensive conflict that it is a zero-sum game as to the evaluation of the outcomes
- However there are several asymmetric dilemmas that cannot satisfy the conjunctive conditions of symmetric dilemmas and so they cannot be considered as two symmetric dilemmas, e.g.:
  - Big Bully
  - "To be the partner's cat's paw"
  - Forced Identical Interest

SOLOMONIC DECISION AS CALLED BLUFF

Real mother

Fake mother		C	D
	C	1, 1	-2, 2
	D	2, -1	-1, -2

Real mother: DC CC CD DD (Game of Chicken)

Fake mother: DC CC DD CD (Prisoner's Dilemma)

BIG BULLY

2#

1#		D	C
	D	2, -2	1, -1
	C	-2, 2	-1, 1

1#: DD DC CC CD (NS.)

2#: DC CC CD DD (Game of Chicken)

"TO BE A CAT'S-PAW"

1#		D	C
	C	-2, -2	-1, -1
	D	1, 2	2, 1

1#: DC DD CC CD (Deadlock)

2#: DD CD CC DC (NS.)

FORCED IDENTICAL INTEREST

1#		D	C
	D	-1, -1	2, 1
	C	1, 2	-2, -2

1#: DC CC DD CD (Prisoner's Dilemma)

2#: DD CC CD DC (NS.)

IV.18. The Cold War as Perceptional Dilemma

- **Perceptional dilemma** (from the US point of view) =

Security Dilemma + Prisoner's dilemma

(USA in the arms race) (USSR in the arms race)

CC DC DD CD DC CC DD CD

$$\begin{pmatrix} 2 & -2 \\ \uparrow & \downarrow \\ 1 & -1 \end{pmatrix} \qquad \begin{pmatrix} 1 & \rightarrow & 2 \\ & & \\ -2 & \rightarrow & -1 \end{pmatrix}$$

	USA	USSR
Both are disarming (CC)	7,97 (2)	5,88 (1)
US arming, USSR disarming (DC)	0,97 (1)	-7,31 (-2)
Both keep on arming (DD)	-5,31 (-1)	-0,91 (-1)
US disarming, USSR arming (CD)	-6,66 (-2)	6,92 (2)

Security Dilemma

USSR

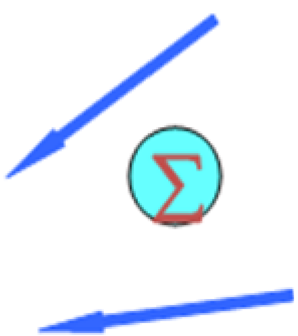
USA	C	D
C	2, 2	-2, 1
D	1, -2	-1, -1

CC DC DD CD

ARMS RACE as PERCEPTIONAL DILEMMA

USSR

USA	C To stop the armament	D To build missiles
C To stop the armament	2, 1	-2, 2
D To build missiles	1, -2	-1, -1



Prisoner's Dilemma

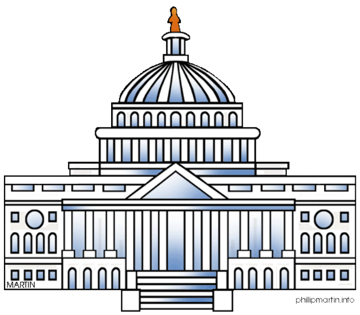
USSR

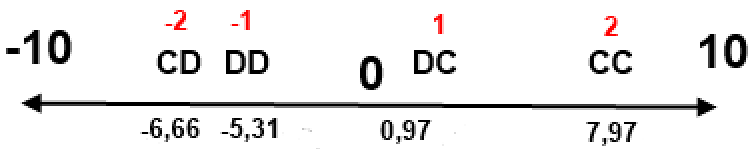
USA	D	C
D	-1, 1	2, -2
C	-2, 2	1, 1

DC CC DD CD

IV.19. The Cold War as Perceptional Dilemma (#2)

	USA	USSR
Both are disarming (CC)	7,97 (2)	5,88 (1)
US arming, USSR disarming (DC)	0,97 (1)	-7,31 (-2)
Both keep on arming (DD)	-5,31 (-1)	-0,91 (-1)
US disarming, USSR arming (CD)	-6,66 (-2)	6,92 (2)



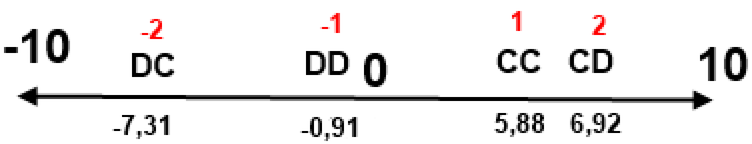


Security Dilemma

USSR

USA	C	D
C	2, 2	-2, 1
D	1, -2	-1, -1

CC DC DD CD



Prisoner's Dilemma

USSR

USA	D	C
D	-1, 1	2, -2
C	-2, 2	1, 1

DC CC DD CD

IV.20. The Conflict over Falkland Islands, 1982

HISTORICAL BACKGROUND

In the middle of economic crisis at the beginning of the 1980s, Leopold Galtieri, who was the leader of the Argentinean military junta, to recall national feelings over Falkland Islands (a national proud arisen from the fact that Argentina won the world championship of soccer in 1978), which had been under British supervision since 1833, wanted to reoccupy the islands. He thought that the Thatcher cabinet could not take the war upon on account of the prime minister's liberal economic package which caused heated debates and strikes in GB, and the Falkland is too far from GB to enter into a war. So he guessed GB was likely to make a compromise with Argentina. But Thatcher was not chickened out, and the Queen supported her cabinet...the result: the Falkland War in April of 1982.

IV.21. The Conflict over Falkland Islands (cont.)

		GB	
		C	D
Argentina	C	1, 1	-2, 2
	D	2, -1	-1, -2

Called Bluff: Prisoner's Dilemma (Argentina) + Game of Chicken (GB)

- The outcomes from Argentinean point of view:

  DC: to storm Falkland, a quick win, minimal loss because of the logistic problems of British military forces and crisis of internal politics in GB.

  CC: Great Britain tries to tackle the conflict in a diplomatic way, the public opinion will predict the Thatcher cabinet as weak. This implies a crises of government including the opportunity for Argentina to straighten out the territorial debate.

  DD: the war;

  CD: to withdraw the troops, a military defeat.

Assessment:

Argentina puts up a prisoner's dilemma: DC CC DD CD in which the Galtieris think that GB will chicken out, so for GB the situation is a Game of Chicken: DC CC CD DD. However, in reality, the Thatcher cabinet proved to be strong and convert the situation into a symmetric Prisoner's Dilemmain which the equilibrium is DD.

IV.22. North Korea as Bully in the Far East

The communist regime of North Korea regularly blackmails the US to aid her with financial packages in economic desperations occurred over and over in the country because of excessive military lavish spending, otherwise she as nuclear nation will follow an aggressive policy. Hence, the communist regime can comfortably keep up her power.

A BULLY DILEMMA

USA

North Korea		C	D
	C	-1, 1	-2, 2
	D	2, -1	1, -2

D

North Korea: DC(2) > DD (1) > CC (-1) > CD (-2)

Deadlock

Game of Chicken

USA: DC(2) > CC (1) > CD (-1) > DD (-2)

C enforced

IV.23. Big Bully Dilemma

The Big Bully conflict differs from the Bully Dilemma only in that the mutual defecting behaviour means the best outcome for player A. The equilibrium is (D, C) again. Player A is not willing to make a compromise, he does not primarily want player B to capitulate in the situation because his demand is only a pretence to resort to force.

Example: Monarchy-Serbia (1914) and Germany-Czechoslovakia (1938)

Serbia

Monarchy	C	D
C	-1, 1	-2, 2
D	**1, -1**	2, -2

D

Monarchy: DD **DC** CC CD (NS.)

Serbia: DC CC **CD** DD (Game of Chiken)

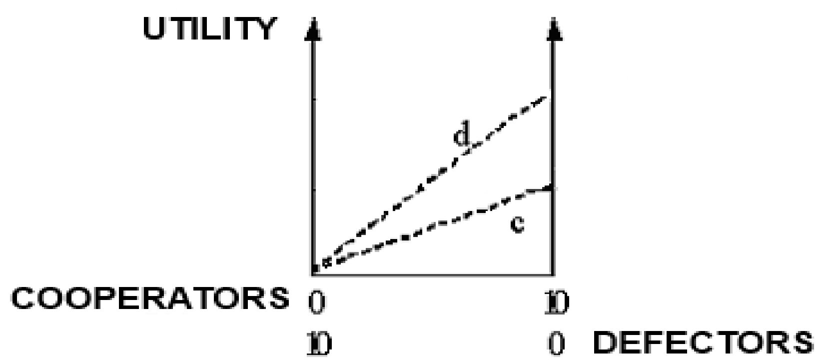
IV.24. Multi-Player 2x2 Dilemmas

- **Multi-player 2x2 dilemma** can be considered as an n-person 2x2 dilemma where a representative player as Ego, who is in the same situation and has the same strategies as the other n-1 players, plays the game against the others identified as Alters (from "alter ego").

Alters

Ego	C	D
C	CC	CD
D	DC	DD

- The situation can be represented by a **Schelling diagram** after Th. Schelling who proposed it. In Schelling diagram we can visualize the expected utility of both the cooperative (c-line or c-curve) and defecting (d-line or d-curve) players.

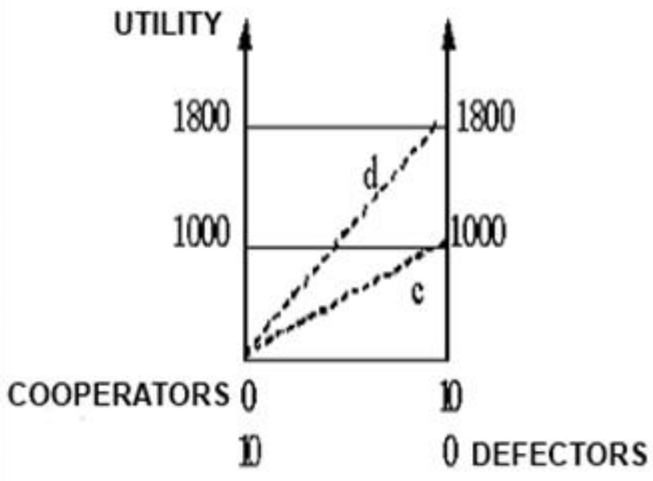


- For example, in a multi-player Prisoner's dilemma each player is worth defecting, and so d-line is always above c-line regardless of how many players are willing to cooperate. It is the only opportunity for the society to avoid or to escape from the dilemma that the expected utility of c(n-1) from the collective interest is higher than the expected utility of the defecting, individual interest at the beginning, d(0).

IV.25. The Tradegy of the Commons

Imagine a rural pasture on which 10 farmers graze one cow each by the habits of the village. For the sake of simplicity, suppose each cow weighing 1000 pounds, so the total weight of theirs is 10000 pounds. Once upon a time one of the farmers (the "defector") in order to get twice more profit, breaks the habits by sending one more cow to the pasture. From this time on, there are 11 cows on the pasture, and since each has a bit of less grass to eat, they are able to put on weight up to 900 pounds. However, the defector whose has two cows on the pasture is a better position than the others who cooperate: he has two cows weighing 900 instead of one weighing 1000 pounds. Each farmer who is willing to cooperate has a loss of 100 pounds, and the village as a whole loses 100 pounds, too, because the total weight of the 11 cows weighing 900 pounds is 9900 pounds instead of the original 10000 pounds. This is not a big problem otherwise, but what happens if more and more farmers think that they want more profit and also send another cow each to the pasture.

THE NUMBER OF COWS	THE GAIN OF COOPERATORS	THE GAIN OF DEFECTORS	THE TOTAL WEIGHT OF COWS	THE TOTAL LOSS OF WEIGHT
10	1000	0	10000	0
11	900	1800	9900	100
12	800	1600	9600	400
:	:	:	:	:
19	100	200	1900	8100
20	0	0	0	0



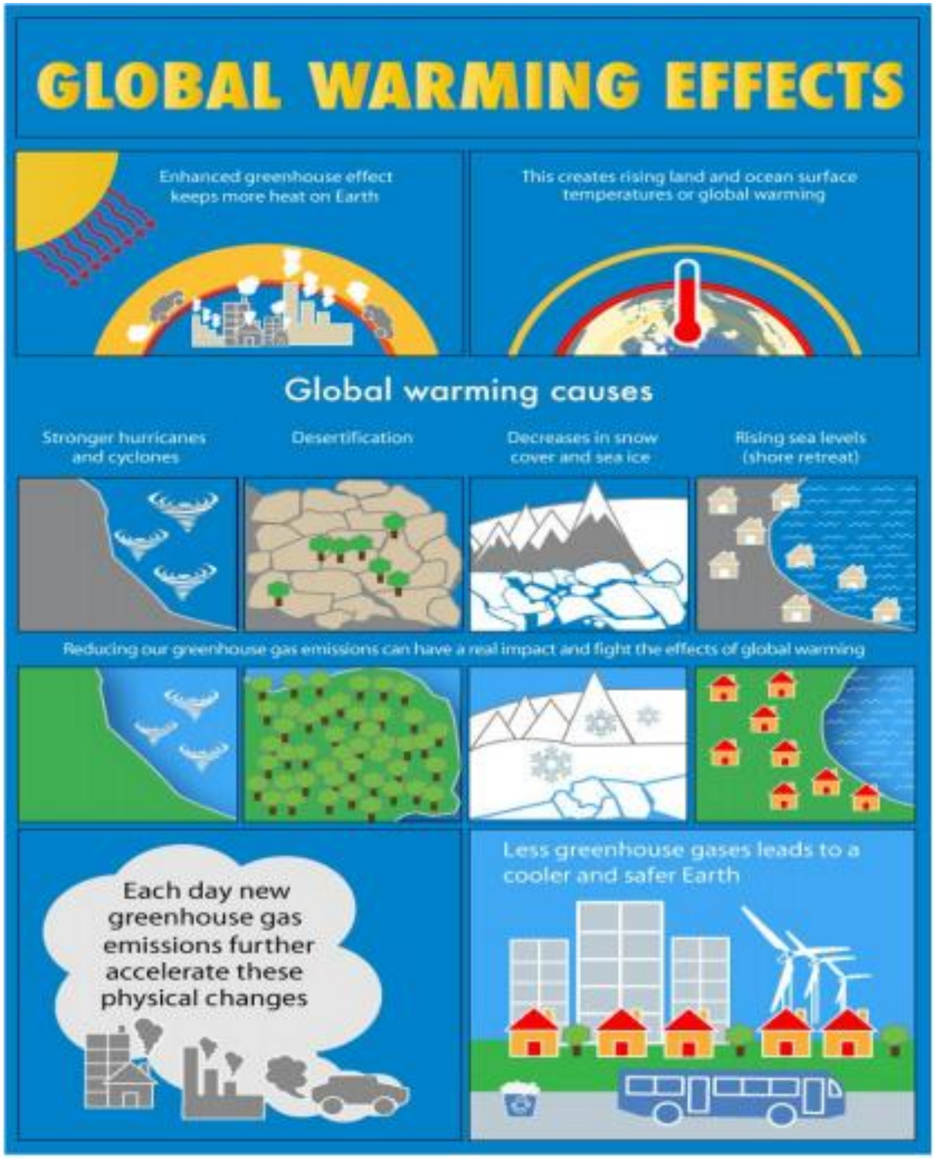
Tradegy of the Commons:

- **Individual Rationality:** Individual users acting independently according to their own self-interest over shared-resources system (e.g. soils, rivers, oceans, freshwater, fishstocks, etc.)
- **Common Rationality:** Set up coercive regulations that restrict over-exploitation of common resources.
- **The Boiled Frog Syndrome:**

Individual rationality →
(Finite Horizon of Time)

← Common Rationality
(Infinite Horizon of Time)

Anecdote: If a frog is placed in boiling water, it will jump out, but if it is placed in cold water that is slowly heated, it will not perceive the danger and will be cooked to death.



IV.27. Multi-Player Dilemma of Hero

- The dramatic consequence of Missing Hero in our everyday life is well illustrated by a tragic case in 1970s when a poor young girl was raped and killed before the pedestrians and residents ("the allies") very eyes in a no deserted environs of New York, and there was no one who called the police (C). During the official investigation it turned out that the reason for the incomprehensible indifference was that everybody shifted the blame and showed indetermination (D): there were ones who thought that someone had certainly called the police, others feared of getting vengeance from the criminals, etc. There was no a hero who would have called the cops.

- The many-player Missing Hero Dilemma pays our attention to a new interpretation of general social problems. Say, if an extreme, armed group is attacking a man in the street, and the people only see the atrocity with shock and they are hesitating to help him at once, this behaviour can be somewhat comprehensible. But, the people in the street not to get in a Missing Hero Dilemma, the society must behave strict, uniform and show determination against aggressive and excluding views and behaviours (up to zero tolerance?… This question also might lead to a dilemma.)

V. Coordination Mechanisms

V.1. The Types of Coordination Mechanisms

1. **Bureaucratic coordination**: individuals are obliged to coordinate their actions by a coordinator accepted by individuals, and the coordinator has a title to govern and to control individual actions.
2. **Spontaneous coordination**: the cooperation of individuals is governed and oriented by payoffs under the compulsion of utility-maximization.
3. **Ethical coordination**: coordination is induced by reciprocal altruism under iterated interactions.
4. **Aggressive coordination**: how effective is it? → Evolutionary Game Theory ("Hawks and Doves" game)

V.2. The ”Hungarian Black Coffee”

“Hungarian black coffee” is a game-theoretical model by Hungarian economists for corruption, and it is actually a multi-person prisoner's dilemma.

Suppose there is only one coffee bar in a little town situated far from other settlements. The price of a cup of espresso is $1. But the barmaid is corrupt: if you pay $.1 tip to her, she gives you a better $1.5 espresso. In order not to fire her because of her cheating, she has to save up the coffee bean from the espresso of another client who thus get a $.5 espresso for $1.

V.2. The "Hungarian Black Coffee" (cont.)

- The situation without corruption is the cell ($1, $1). If corruption is occurred, the payment of tip, i.e. the price $1.1, dominates the normal price $1, and corruption might start up a chain of new corruptions, because you get a $1.5 espresso for a moderate, $1.1 price. However, beyond a certain degree, this is the general defecting behaviour that displays in cell ($1.1, $1.1), the clients in the situation do not get better espresso for their higher price.

The quality of the espresso

Alters (columns), Ego (rows)

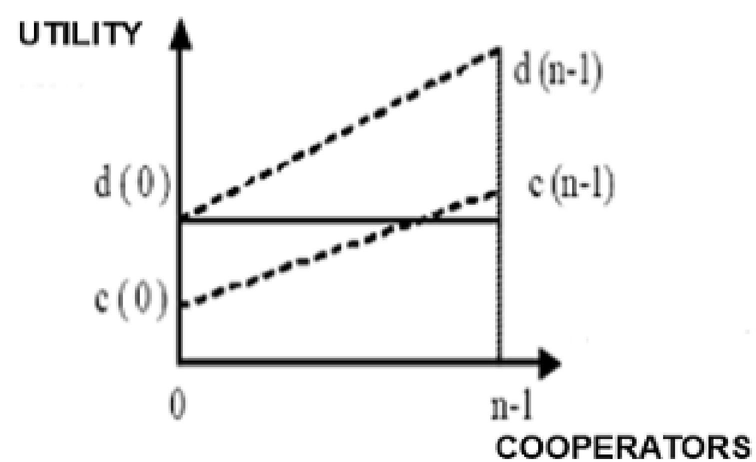
	$1.1 (D)	$1 (C)
$1.1 (D)	$1, $1	$1.5, $ .5
$1 (C)	$ .5, $ 1.5	$1, $1

The utility of transaction

Alters (columns), Ego (rows)

	$1.1 (D)	$1 (C)
$1.1 (D)	-$ .1, -$ .1	$ .4, -$ .5
$1 (C)	-$ .5, $ .4	$0, $0

- This is a multi-player Prisoner's Dilemma, and it clearly illustrates what occurs in Schelling diagram that the utility of cooperators who pay the normal price is always lower than that of those who pay the tips up to a certain degree when everybody arrives at a worse situation than the original one.



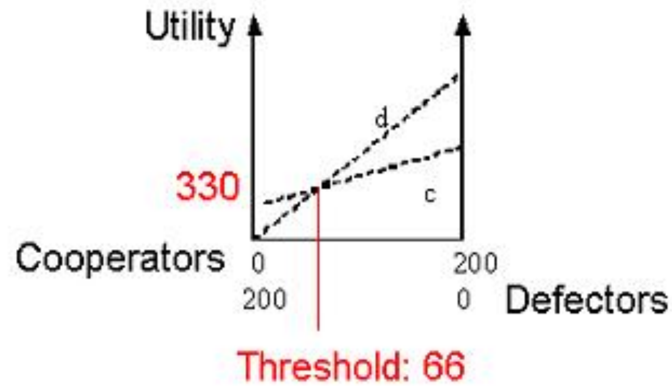
V.3. The "Ingenious" Taxi Driver

The case of ingenious taxi driver as a Game of Chicken:

DC CC CD DD

- This case study is a many-player Game of Chicken situation. Imagine that you are in a metropolis and are about to get to the airport in peak time. You are sitting in a cab going in a major road where there is a traffic jam. The driver who knows well enough the roads of the metropolis, makes a detour and save up a lot of time, but the crux is that you need to return later in the busy major road. After some waiting and throng you are able to do it. Of course, several car drivers may do the same, and there are two strategies:
  - Defecting: to turn to the slip road that is also blocked, and back to the avenue will be very difficult because a lot of cars are not easily let in by the cars going along in the major road.
  - Cooperative: No other way, go along in the busy major road and look at your watch.

V.3. The ”Ingenious” Taxi Driver (cont.)



Preference: $v(DC)=5 > v(CC)=3 > v(CD)=1 > v(DD)=0.$

$$Eu_C=(N_C-1)\cdot v(CC)+N_D\cdot v(CD),$$

$$Eu_D=(N_D-1)\cdot v(DD)+N_C\cdot v(DC).$$

$$Eu_{C+D}=N_C\cdot Eu_C+N_D\cdot Eu_D.$$

Suppose the density of traffic is 200 cars/quarter

Thus:

$$Eu_C=(N_C-1)\cdot 3+(200-N_C)\cdot 1=2\cdot N_C+197;$$

$$Eu_D=(199-N_C)\cdot 0+N_C\cdot 5=5\cdot N_C;$$

The threshold: $Eu_C=Eu_D$

$$Eu_C=Eu_D \to 2\cdot N_C+197=5\cdot N_C \to N_C\approx 66.$$

- To take a look at Schelling diagram, we can see that
  (1) if there are relatively large number of cars, the expected utility of cooperators is higher than the defectors otherwise it is lower.
  (2) the more car driver cooperates, the more higher is the common expected utility of the drivers as a whole. If everybody is willing to cooperate, the common expected utility (E_{C+D}) is maximal, and if everybody defects, E_{C+D} is minimal.

V.4. When Hierarchy is needed

The Problem of Controlling

	D	C
D	-1, -1	2, -2
C	-2, 2	1, 1

The Assignment of Rights

	D	C
D	-1, -1	2, 1
C	1, 2	-2, -2

Normative Hobbes Theorem:
Structure the law so as to minimize the harm caused by failures in private agreements.

Sen`s Impossibility Theorem:
There is no a coordinaton mechanism satisfying the conditions of (U): universality, (P): Pareto and (L*): minimal liberalism.

To the explanation of Sen's Theorem:

(U) – Universality: coordination mechanism can be used under any preferences of actors.

(P) – Pareto condition: if all the actors prefer a to b, this will be true at collective level.

(L) – the condition of Minimal Liberalism: any actor's choice is decisive between two alternatives.

V.5. When Bargaining is possible

- **Normative Coase Theorem:**
  Structure the law so as to remove the impediments to private agreements.

- If the rights of the usage of a double bed room in a dormitory are clearly fixed, the amount of smoke will be the same either smoking is possible or not in the room.
  Just the deal will be different:
  1) If smoking is prohibited, the smoker has to make a deal with her flatmate to let her light the smoking lamp.
  2) If smoking is not prohibited, the non-smoker has to ask his flatmate not to smoke.

- **Bargaining vs. Hierarchy:** Bargaing is better off because
  - more flexible and more effective (cheaper, more quickly and Pareto optimal);
  - ends up with an outcome that is mutually acceptable by both.

V.6. Coordination Failures

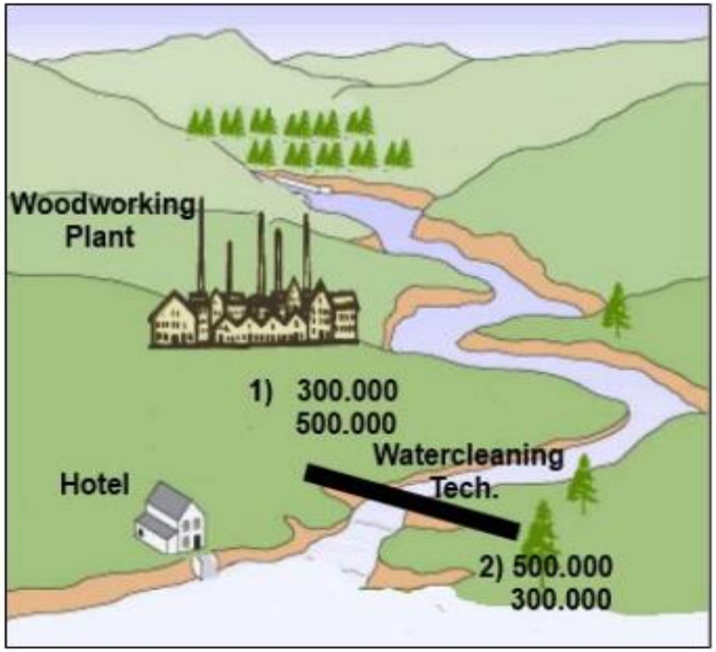
Transaction Costs (in the most general sense): **the costs of exchange.**

Property Rights: the ability to freely exercise a choice over a good or service.

Transaction Costs #1: the costs establishing and maintaining property rights.

Transaction Costs #2: the costs resulting from the transfer of property rights.

Lower	**Higher**
Standardized goods/services	Unique goods/servicies
Clear, simple rights	Uncertain, complex rights
Few parties	Many parties
Friendly (familiar) parties	Hostile (unfamiliar) parties
Reasonable behaviour	Unreasonable behaviour
Prompt exchange	Delayed exchange
Low costs of monitoring	High costs of monitoring
Cheap punishment	Costly punishment



They float trees on the river, but to avoid microorganisms induced by water, they cover the trees some impregnated stuff. The trouble is that it is going into the water, and this is a bad advertising for anglers and tourists.

Proposals (2x2):

	#1:	**#2:**
WP.:	**300.000**	**500.000**
H:	**500.000**	**300.000**

V.6. Coordination Failures (cont.)

The reason for the failures of spontaneous coordination:

- information asymmetry,
- high transaction costs,
- strategic behaviour,
- monopoly,
- externalities.

The reason for the failures of bureaucratic coordination at collective level:

The impediments of spontaneous coordination bring out the failures of voting as coordination mechanism, too → a coordinator that is usually chosen in a suboptimal way will coordinate in a suboptimal manner.

V.7. The Preconditions of Confidence

1. **I will be cooperative if I am not going to be a looser**
   - Battle of Sexes: „You don't make use of my service!"
   - The case of Ingenious Taxi Driver: „You may exploit me if it does not come expensive!"
   - Failure: Prisoner's dilemma

2. **Reciprocity**
   - Battle of Sexes: „Pay your way or leave my way!"
   - The case of Ingenious Taxi Driver: „You have been a situation like this!"
   - Failure: Prisoner's dilemma

3. **Iterated Interactions with Indetermined (Infinite) Horizon of Time**
   - **The Parable of the Good Samaritan in the Bible (Luke, 10): there were once a Jewish man who was going down from Jerusalem to Jericho when robbers attacked him, stripped him, and beat him up, leaving him half dead. It so happened that a priest, then a Levite came there, but when they saw the man, they walked on by on the other side. But a Samaritan who was travelling that way came upon the man, and when he saw him, he went over to him, gave him meal and drink, then he put the man on his own animal. He did it in spite of the fact that the Jews and the Samaritans hated each other.**

V.8. Intertemporal Choices

- Most choices require a player as decision-maker to trade-off costs and benefits at different points in time.
  - Players prefer to receive \$1 now, invest it at interest rate r and receive $\$1 \cdot (1+r) = \$(1+r)$ one round later, rather than wait and receive \$1 later.

- Discount factor, $0 < d < 1$, provides a mean of evaluating future money amounts in terms of current equivalent money amounts:

  $d = 1/(1+r)$, since $\$1 \cdot d = \$1/(1+r)$ now = \$1 later on.

  PV (Future money is discounted to Present Value)

- Consider an infinitely repeated game. Suppose an outcome of this game is that a player receives \$p in every future round of the game. Then the value of this stream of payoffs is:

  $$PV = \$p \cdot (1 + d + d^2 + \ldots).$$

- Since $0 < d < 1$, it is a mathematical fact that the geometric series $1 + d + d^2 + \ldots$ converges to $1/(1-d)$. Thus $PV = \$p/1-d$.

V.8. Intertemporal Choices (supplement)

Present Value (PV) Calculation:

$PV_0 = \$1$

$PV_1 = \$1 \times d = \$0.9804$

$PV_2 = \$1 \times d^2 = \$0.9612$

$PV_3 = \$1 \times d^3 = \$0.9424$

....

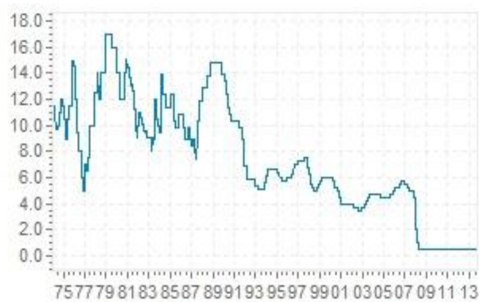
$$PV_n = \underbrace{\underbrace{(\$1 \times d) \times d}_{\$1 \times d^2} \times d}_{\$1 \times d^3} \times \ldots \times d = \$1 \times d^n$$

$r = 0.02 \leftarrow 2\%$ (Poland or South Korea, 2014)

$$d = \frac{1}{1+0.02} = \frac{1}{1.02} = 0.9804$$

FED interest rates, long-term

Bank of England interest rates, long-term



Russian CBR interest rates, short-term

Turkish CBRT interest rates, short-term

(Real) Interest rate (r) – October, 2014:

Name of interest rate	country/region	current rate	direction	previous rate	change
American interest rate FED	United States	0.250 %	↓	1.000 %	12-16-2008
Australian interest rate RBA	Australia	2.500 %	↓	2.750 %	08-06-2013
Banco Central interest rate	Chile	3.000 %	↓	3.250 %	10-16-2014
Bank of Korea interest rate	South Korea	2.000 %	↓	2.250 %	10-15-2014
Brazilian interest rate BACEN	Brazil	11.000 %	↑	10.750 %	04-02-2014
British interest rate BoE	Great Britain	0.500 %	↓	1.000 %	03-05-2009
Canadian interest rate BOC	Canada	1.000 %	↑	0.750 %	09-08-2010
Chinese interest rate PBC	China	6.000 %	↓	6.310 %	07-06-2012
Czech interest rate CNB	Czech Republic	0.050 %	↓	0.250 %	11-01-2012
Danish interest rate Nationalbanken	Denmark	0.200 %	↓	0.300 %	05-02-2013
European interest rate ECB	Europe	0.050 %	↓	0.150 %	09-04-2014
Hungarian interest rate	Hungary	2.100 %	↓	2.300 %	07-22-2014
Indian interest rate RBI	India	8.000 %	↑	7.750 %	01-28-2014
Indonesian interest rate BI	Indonesia	7.500 %	↑	7.250 %	11-12-2013
Israeli interest rate BOI	Israel	0.250 %	↓	0.500 %	08-25-2014
Japanese interest rate BoJ	Japan	0.100 %	↓	0.100 %	10-05-2010
Mexican interest rate Banxico	Mexico	3.000 %	↓	3.500 %	06-06-2014
New Zealand interest rate	New Zealand	3.500 %	↑	3.250 %	07-24-2014
Norwegian interest rate	Norway	1.500 %	↓	1.750 %	03-14-2012
Polish interest rate	Poland	2.000 %	↓	2.500 %	10-08-2014
Russian interest rate CBR	Russia	8.000 %	↑	7.500 %	07-25-2014
Saudi Ariabian interest rate	Saudi Arabia	2.000 %	↓	2.500 %	01-19-2009
South African interest rate SARB	South Africa	5.750 %	↑	5.500 %	07-17-2014
Swedish interest rate Riksbank	Sweden	0.250 %	↓	0.750 %	07-03-2014
Swiss interest rate SNB	Switzerland	0.250 %	↓	0.500 %	03-12-2009
Turkish interest rate CBRT	Turkey	8.250 %	↓	8.750 %	07-18-2014

- Now let p be the probability of interactions between the players in an infinitely repeated game. Since $0 < p < 1$, p can be considered as a discount factor of the payoffs of the players.

- To apply this considerations to the payoff matrix of an infinitely repeated prisoner's dilemma
  - Cooperation gives PV_C = {"C now"} + {"CC" later on}= 3 + 3/(1-p),
  - Defecting serves PV_D = {"D now"} + {"DD" later on} = 4 + 1/(1-p).

	D	C
D	1, 1	4, 0
C	0, 4	3, 3

- The expected utility of mutual cooperation and of mutual defecting are

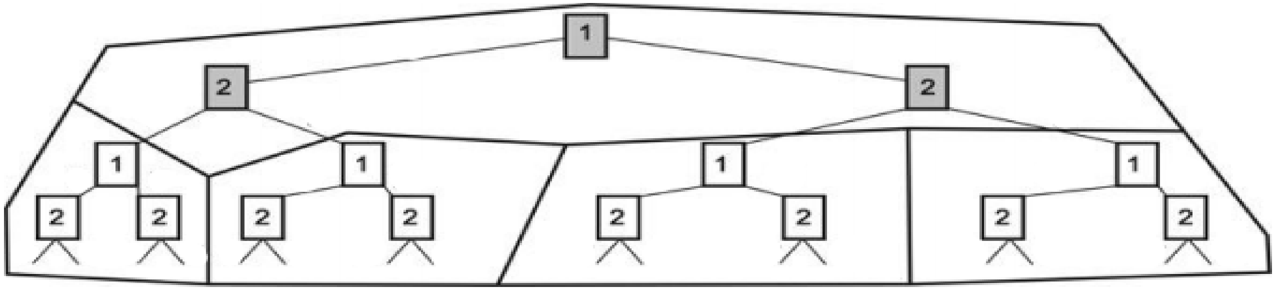
$$Eu_{cc} = 3+(3/(1-p)) \cdot p$$

$$Eu_{DD} = 4+(1/(1-p)) \cdot p$$

  - And now if the probability of interactions is large (p=0,9), Eu_{cc}= 30 and Eu_{DD} = 22, so the mutual cooperation is better.
  - But if the probability of interactions is small (p= 0,1), Eu_{cc}= 3,33 and Eu_{DD} = 4,11, it is worth defecting.

V.10. Finitely Repeated Games

- **Backward induction paradox**: Prisoner's dilemma is a simultaneous move game. But if we play over and over a prisoner's dilemma, we have a sequential move game in which
  - the whole game is a union of simultaneous move subgames, namely, a lot of prisoner's dilemmas;
  - in the n^{th} round of the game the players take move knowing the moves of the players up to the $(n-1)^{th}$ stages of the game.



- We can apply the backward induction method to this sequential game. In the last round both players know that the game will not continue further. They will therefore both play their dominant strategy of defecting. But knowing that the results of the last, n^{th} round are mutual defecting (DD), there are no benefits for players to cooperating in the $(n-1)^{th}$ round. Hence, both players are defecting in this round as well… and so on up to the first round of the game. As long as there is a known, finite end, there will be no change in the equilibrium outcome of a game with a unique equilibrium.

- **Selten`s Theorem**: The equilibrium of the basic game, which is repeated, will be the subgame perfect equilibrium of the whole finitely repeated game.

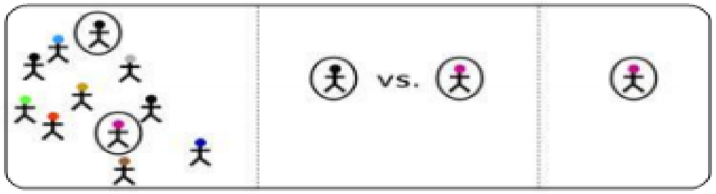
- However, we cannot apply this reasoning to the Good Samaritan dilemma as infinitely repeated Prisoner's dilemma, because in this situation there is no "last" dilemma to reason backward from. In their ignorance,players have more reason to cooperate.

V.11. Axelrod's Tournament and Reciprocity

Tit for tat is an English saying meaning "equivalent retaliation "

In Western culture a handshake when meeting someone is an example of initial cooperation

- **Axelrod's tournament**: Each strategy was paired with each other strategy for 200 iterations of a Prisoner's Dilemma game, and scored on the total points accumulated through the tournament.



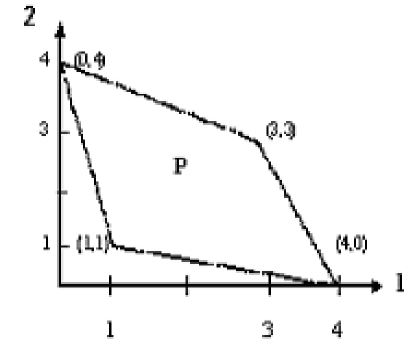
- The winner was a very simple strategy submitted by Antol Rapaport called TFT that cooperates on the first move, and subsequently echoes (reciprocates) what the other player did on the previous move. The main properties of this evolutionary success are:
  - Don't be yellow (envious, jealous)!
  - Don't be the first to defect!
  - Be foresightful and think of the next interaction!
  - Be reciprocative! Welcome to nice gestures, gun for unfairness!

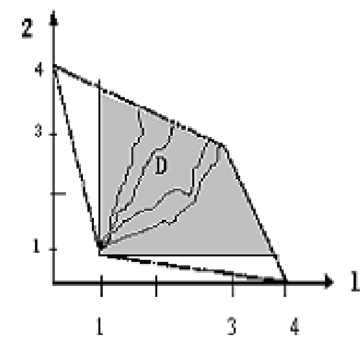
- We can graphically display the payoff matrix of a game like this, and thus we have a convex polygon P whose points are a vector constituted by the payoffs. We must cut the points out from the area of P, which are outside the minimal guaranteed payoffs of players what they can obtain by using their minimax strategies. We thus get the area of D inside P, which is the set of the individual rational payoffs.
- Any course of strategies can be imaginable, which remains inside D. Is there a course of strategies that is good enough to use?
- The outcome CC forever, yielding payoffs (3,3) can be a subgame perfect equilibrium (SPE) of the infinitely prisoner's dilemma provided

  i) the probability of interactions between players is sufficiently large, and

  ii) each player uses some kind of trigger courses of strategies. For example, Grim strategy in which first the player plays C, and so long as the history of play has been CC in every round, he plays C; otherwise he plays D unconditionally and forever.
- There are nicer courses of strategies that will support CC as an equilibrium (though these are not SPE). Consider TFT ("tit-for-tat"): first play C, and if the history from the latest round is CC or DC, play C, and if the history from the latest round is CD or DD, play D.

	D	C
D	1, 1	4, 0
C	0, 4	3, 3





- Axelrod`s computer simulations: Nicer courses of strategies are better in a competition. Hence, TFT is also better than Grim, but it is not SPE: if the opponent deviates only once, best response is not to punish.

V.12. Infinitely Repeated Games (cont.)

Time Horizon of Games

Finite	Infinite (unspecified)
Original Equilibrium of the Game.	Overvaluation of Cooperation.
In PD: D	In PD: TFT

Under Finite Time Horizon

Selten's Theorem: The equilibrium of the basic game will be the SPE of the whole finitely repeated game also. In prisoner's dilemma this equilibrium is D.

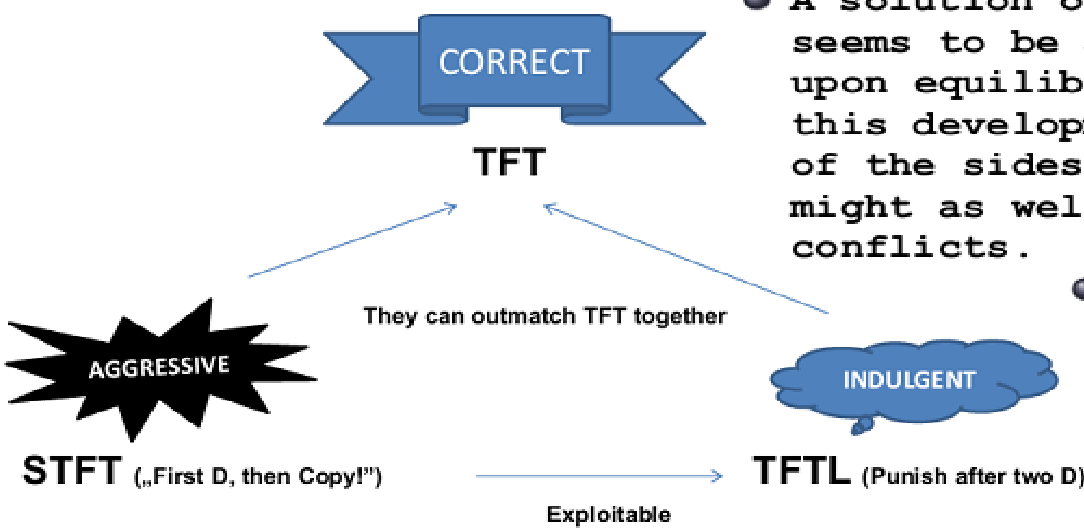
Under Infinite Time Horizon

Folk theorem: If a one-shot game is infinitely repeated, then essentially all distributions of individual rational payoffs can be equilibria of the whole infinitely repeated game. The best equilibrium candidate is TFT.

V.13. Tolerance as Social Interplaying

- Everyday actions based upon reciprocal altruism and evergreen "deep" problems of society can be considered as infinitely repeated games, which are always on agenda, and cannot be resolved while the citizens are in the play.

- By Folk Theorem, we may hope that social courses of strategies lead up to the efficient point of the representative members of society. History also shows a direction to the extension of liberalisation, though it was not continuous and is still not in full in the world. Long centuries needed to accept the tolerance of faith, to condemn slavery, to accept the equality of the political rights of women and not to consider some behaviour as deviation, etc.



- A solution of a "deep" problem of society seems to be a stable status qou built upon equilibrium. The precondition of this development might be the tolerance of the sides, but says Folk Theorem, it might as well be realized by social conflicts.

- Though democracy is more successful than autocracy, "aggressive strategies" are always about to undermine "civic friendship", and are able to spread in society in which mutual respect is not strong enough.

V.14. Aggressive Behaviour in Evolutionary Games

During the Vietnamese war, the journalists began to use two adjectives to describe the politicians` ability of tackling conflicts: hawks who were to escalate the war and doves who were against the war. The "Hawks and Doves" game generalizes this, and as a model it can be applied both in political theory and in mathematical biology. In a population there are two phenotypes: hawk and dove. In conflicts Hawks uninhibitedly fight and stand off only if they got injured seriously, and doves are to make an effort for threatening the opponent but avoid serious battles.

Pure Dove population:

$$Eu_D = \frac{50 - 10}{2} = 20$$

Pure Hawk population:

$$Eu_H = \frac{50 - 100}{2} = -25$$

Payoff matrix of the „H&D" game

#1 (rows) / #2 (columns)

	Dove	Hawk
Hawk	50, 0	50, -100 or -100, 50
Dove	-10, 50 or 50, -10	0, 50

A mixed population where the proportion of Hawks and Doves is 90% and 10%:

$$Eu_H = (50) \cdot 0{,}45 + (-100) \cdot 0{,}45 + (50) \cdot 0{,}1 = -17{,}5$$

$$Eu_D = (0) \cdot 0{,}9 + (50) \cdot 0{,}05 + (-10) \cdot 0{,}05 = 2$$

Equilibrium: $$Eu_H = (50 - 100) \cdot \frac{p}{2} + (50) \cdot (1 - p) = (0) \cdot p + (50 - 10) \cdot \frac{1 - p}{2} = Eu_D$$

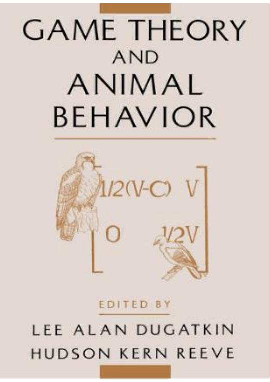
$$p = \frac{6}{11} \longrightarrow \textbf{Hawks : Doves} = 6 : 5$$

V.14. Asymmetric ”H&D” Games in Animal Life

1. Territorial Behaviour Games:

There are three strategies:

- **Hawk**
- **Dove**
- **Retaliator:** it fights as a Hawk on its own territory but behaves as a dove if it is an intruder



	H	D	R
H	$\frac{v-c}{2}$	v	$\frac{v-c}{2}$
D	0	$v/2$	$v/2$
R	$\frac{v-c}{2}$	$v/2$	$v/2$

- **Evolutionarily Stable Strategies (ESS):** The strategy adopted by a population in a given environment, cannot be invaded by any alternative strategy.
- **In Territorial Behaviour Game**, it is
  - **H** if V > C
  - **R** if C > V

V.14. Asymmetric "H&D" Games (cont.)

❷ Paternal Care Behaviour Games:

- **Strategies: G** as Guarding, and **D** as Defecting;
- **Probabilities** P_0, P_1, P_2 that the offspring will survive if they are guarded by 0,1, or 2 parents;
- Probabilities p^G, p^D are **the likelihood of the fact that the male is able to find a new couple** in case of guarding its own offspring and in case of deserting them ($p^G < p^D$);
- V^G and V^D are **the expected number of offspring** if the female is guarding or deserting them ($V^G < V^D$).

EUMale

♂ \ ♀	G	D
G	$V^G P_2 (1+p^G)$	$V^D P_1 (1+p^G)$
D	$V^G P_1 (1+p^D)$	$V^D P_0 (1+p^D)$

EUFemale

♀ \ ♂	G	D
G	$V^G P_2$	$V^G P_1$
D	$V^D P_1$	$V^D P_0$

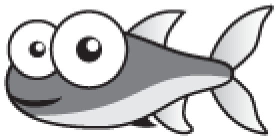
- **There are four possible ESS in this game:**
  - ✓ **Both parents provide parental care:** the two parents are able to bring up more offspring than one parent, and strategy D will not give the chance for the male to have more offspring.
    $$P_2 (1 + p^G) > P_1 (1 + p^D) \ \& \ V^G P_2 > V^D$$
  - ✓ **Father alone cares the offspring:** the two parents are unable to bring up more offspring than one parent, and the female will have more offspring in case of defecting.
    $$P_1 (1 + p^G) > P_0 (1 + p^D) \ \& \ V^D P_1 > V^G P_2$$
  - ✓ **Mother alone cares the offspring:** the two parents are unable to bring up more offspring than one parent, and strategy D will give chance for the male to have more offspring.
    $$P_1 (1 + p^D) > P_2 (1 + p^G) \ \& \ V^G P_1 > V^D P_0$$
  - ✓ **Neither parent provides care:** one parent is unable to bring up more offspring than no one, and strategy D will give chance for both parents to have more offspring.
    $$P_0 (1 + p^D) > P_1 (1 + p^G) \ \& \ V^D P_0 > V^G P_1$$

Swans

Northen Jacanas

Sea lions

A lot of fish but not all



VI. Cooperative Games

VI.1. Why to Cooperate?

- Imagine Robinson and Friday doing two of vital importance actions: they are either building a shack or fishing and cooking. Suppose Robinson is better than Friday in both works. Is Robinson worth cooperating with Friday?

	Robinson	Friday
Shack	20 hours	45 hours
Fishing&Cooking	10 hours	15 hours
Total working hours in a year	2000 hours	3600 hours

Their performance if they work separately:

Robinson: Building in 1000 h: 50 shacks; Fishing & cooking in 1000 h: 100 dishes

Friday: Building in 1800 h: 40 shacks; Fishing & cooking in 1800 h: 120 dishes

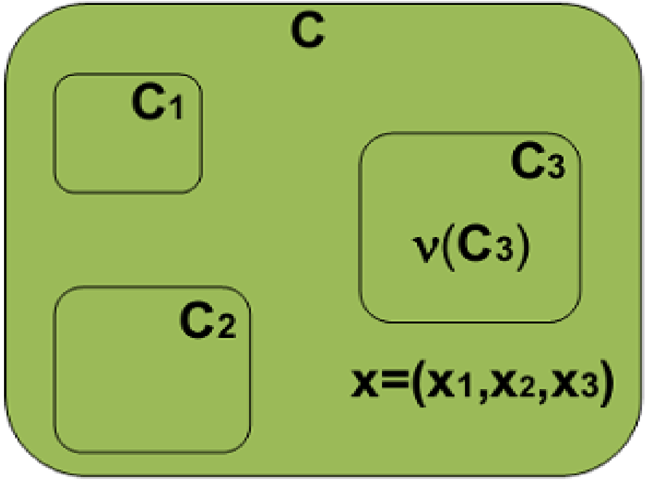
Their performance together

Robinson is building in 2000h: 100 shacks

Friday is fishing & cooking in 3600 h: 240 dishes

- Investigate their performance if they work separately and if they are in a cooperation: Robinson is solely building shacks and Friday is solely fishing and cooking.
- We can see that their performance is increasing due to the cooperation.

- In an n-person cooperative game, the selection of strategies is no longer the main problem to be considered. The point to be emphasized is the formation of coalitions.
- After a coalition is formed by some players, it as a whole will strive to gain as large a total payoff as possible. This payoff is a function of the coalition C, which is called the characteristic function, and is denoted by v(C).



- The total payoff to a coalition of players may be divided in any way among the these players. This condition is called as the side payments condition.
- Each player in a cooperative game has a right to receive his share from the total payoff v(C) available. A division of v(C) among the players belonging to coalition C can be represented by an n-dimensional vector $x=(x_1,\dots,x_n)$ where x_i is the amount received by player i. This vector is called as an imputation.

The Properties of Characteristic Function:

- To an empty coalition in which there is no player the characteristic function serves zero payoff: v(C) = 0;
- The complement of a coalition: $v(n - C_i) = v(n) - v(C_i)$ for any coalition C;
- Superadditivity: If C and G two different coalitions, $v(C + G) \geq v(C) + v(G)$ for any two coalitions C and G;
- Essential cooperative games for which the characteristic function satisfies the condition $v(C) > \Sigma_i\, v(\{i\})$ for any i=1,…,n.

VI.3. Solving Cooperative Games

- **Dominant Inputation (through coalition C)**
  In an n-person cooperative game the imputation $x=(x_1,\dots,x_n)$ is dominated by another imputation $y=(y_1,\dots,y_n)$ through coalition C if
  - each member of C prefers y to x, and
  - the members of C can obtain the income $y_1,\dots,y_n$ from the imputation y. This means that $\Sigma_{i\in C}\, y_i \le v(C)$.

  The fact of domination is denoted by $y \overset{C}{\leftarrow} x$.

- The **core** of an n-person cooperative game is the set of the non-dominated imputations of the game. If the core is empty, the pre-play agreement from the emerging coalition structure is not stable.

- **Empty core:**
  Suppose you have three children, Cynthia, Robert and Steve. One of them is cutting the cake, and the other two children are forming a coalition. They begin to put forward a proposal for the distribution of the parts of the cake. Review the possible proposals:

	a_1	a_2	a_3
C	2	-4	1
S	1	2	-4
R	-4	1	2
TOTAL	-1	-1	-1

(2, 1, -4) (-4, 2, 1) (1,-4, 2)

only one proposal: {C, S}: a1 → {S, R}: a2 → {C, R}: a3

two proposals toward C: (a1, a3), but then C: 3, S: -3 and R: -2

three proposals together: (a1, a2, a3), then C, S, R: -1

We arrive at a never-ending cycle of proposals, and one of the children can put forward a better proposal dominating the prior proposals in each round of bargaining (see the arrows). This game has an empty core.

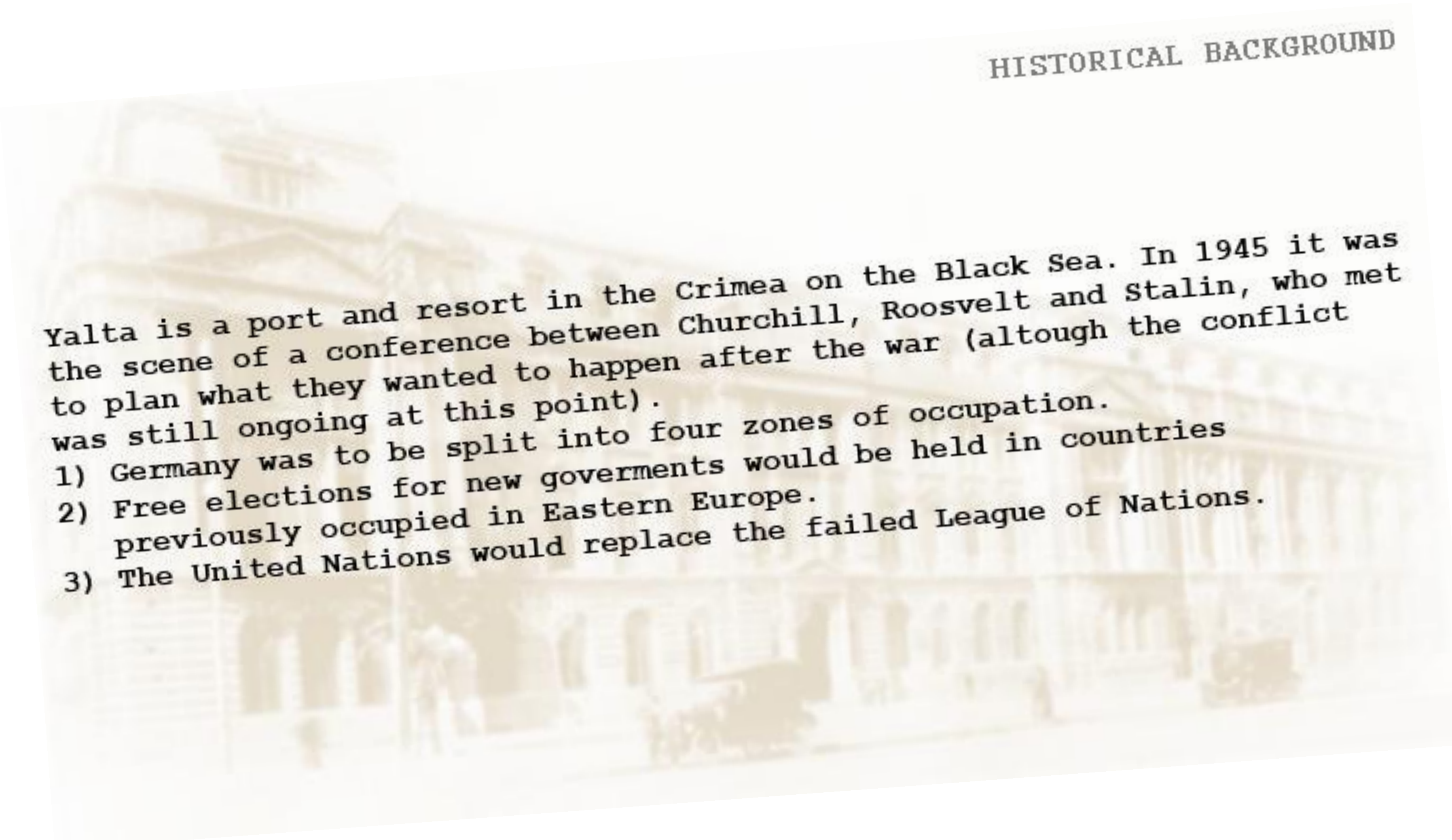
VI.4. Some Theorems on Empty-Core Cooperative Games

- **Theorem**: If G at least a three-player, superadditive, essential and constant-sum game described by characteristic function, i.e.,
  (1) C_1, C_2, . . ., C_n are the coalitions of game G;
  (2) In the possible allocations of utility there is a maximal value of utility, $v(C_i)$, for any coalition C_i;
  (3) $v(C_i) + v(n - C_i) = k$ (G is constant-sum game);
  (4) G is superadditive: there is at least one coalition with different players i and j, $(C_i + C_j)$ in which the members obtains more utility: $v(C_i + C_j) \geq v(C_i) + v(C_j)$;
  (5) G is essential: $v(C) > \Sigma_i v(\{i\})$ for $i=1,\dots,n$;
  G has an empty core.
- The pre-play agreement from the emerging coalition structure is not stable, which means that the situation can be strategically manipulated.

- **Mckelvey`s Chaos Theorem**: If there are at least three issues on agenda, anything can happen in a decision mechanism, e.g. in a voting (because no Condorcet winner), except some special situations. Whoever controls the decision mechanism, say, the order of voting, can determine the final outcome.
- **Shapley-Shubik`s Power Index (SSPI)**: it is an attempt to measure mathematically how much power a player has to control a decision mechanism regardless of the opinions of the members.
- **Pivot member i** is the last member who is chronologically joining to the winning coalition.

- Now, SSPI measures the voting power of i such a way that we consider how many cases player i is pivot member (p_i) in all the possible permutations of the members (n!) establishing a winning coalition: $\varphi_i=p_i/n!$. In the sense of McKelvey`s Theorem, considering the opinions of the members, this value is shifted to a player (called "manipulator" or "dictator" in Social Choice Theory) determining the final outcome.
- In our 3-player "cutting the cake" game, all the possible permutations are 3! = 1·2·3 = 6, and each player is a pivot member in 2 cases because each child has one vote for the possible distributions and thus the winning coalition depends on a majority voting: 123, 132, 213, 231, 312, 321. So SSPI = 2/6 = 1/3, is equal for each, as we suppose it intuitively.

VI.5. Yalta Talks, 1945



VI.5. Yalta Talks (cont.)

There were three fundamental proposals on agenda:

a_1: to keep up the influence on the occupied territories during the settlement of the situation after the war. (Stalin at Postdam Meeting: as first step, territorial revision in Poland and to establish a temporary government in Poland, which can be followed by elections);

a_2: Each actor of the conference keeps up the influence on the territories she has occupied;

a_3: to establish an international organization by which the peace in the world can be guaranteed.

- By their preferences, the imputations of the actors are the same as in the "cutting the cake" game in Slide VI.3., just the initials of the players are now the famous premiers of the victorious, i.e., C as Churchill, R as Roosevelt and S as Stalin.
- Though the SSPI is also equal in this situation, but Stalin as "pivot player" cut the Geordies knot of the game as so to manipulate Roosevelt by Churchill's proposal with a long-run opportunity of his real ambition: he proposes a_1 as the intension of the USSR instead of her honest intention a_2.
- Churchill was distrustful of the ambition of Stalin, but was obliged to allow him especially because Roosevelt was tending to relying to the USSR in order to be able to soothe the life after the war in Europe. The US and the USSR formed a coalition in the Crimea, but shortly after the iron curtain was falling down in Europe, as Churchill said in his famous Fulton Speech (or "The Sinews of Peace" Speech), and the members of the coalition got into a prisoner's dilemma.

VII. Negotiations

VII.1. Bargaining Games

- Basic bargaining situation: Two (or more) players bargain over how to divide the gains from a trade. The gains are represented by a sum of money, *M, that is on the table.*
  - If the players make an agreement, player A receives u and player B achieves v from the trade. If they disagree, then each has a disagreement value: let u^* be the disagreement value to the Player A and let v^* be the disagreement value to Player B. The disagreement value is known by BATNA, as the best alternative to negotiated alternative. Remark that in many cases: $u^*= v^*=0$.
  - By gains from trade we mean that $M > u^*+v^*$, and the excess value, or the surplus, from the bargaining negotiation is: $s = M - (u^*+v^*) > 0$.
- Bargaining negotiations attempt to divide the surplus that is available to the parties if an agreement can be reached. Bargaining can be analyzed as cooperative game in which parties find a solution jointly. Obviously, this game cannot be a zero-sum game, because without excess value, the negotiation would be pointless. On the other hand, the total payoff from the negotiated agreement should be greater than the sum of individual payoffs they achieve separately. This is Nash's cooperative bargaining concept in which bargaining is considered as a special kind of bimatrix games.
- In another concept, which is called Rubinstein's non-cooperative bargaining concept, parties choose strategies separately and attempt to reach an equilibrium, we have a non-cooperative setting of alternating offers. This bargaining game is sequential, and a rollback reasoning (backward induction) is used to find an equilibrium.

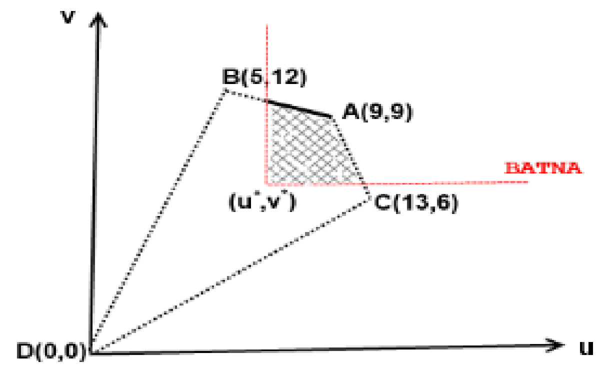
VII.2. Cooperative Bargaining Games

- In this type of bargaining there are three stages:
  - It is necessary for the players to fix their BATNA;
  - They need to specify the cooperative excess value;
  - They should negotiate over the division of the surplus.
- Nash's cooperative solution is based on some plausible principles:
  - Individual rationality: the result of agreement (u^0, v^0) is better than the status quo (u^*, v^*);
  - Possibility: there exists a solution of the bargaining game, i.e., $(u^0, v^0) \in P$;
  - Pareto efficiency: there is no points in P, which dominates the agreement;
  - Invariance to linear changes in the payoff scale and to removal of irrelevant outcomes: either the shrinking or magnifying of P or the removing of dominated pointes does not change the solution of the negotiation;
  - Symmetry: the players are different only in their payoff functions.

Let the payoff matrix of the bargaining game be:

$$\begin{pmatrix} (9,9) & (5,12) \\ (13,6) & (0,0) \end{pmatrix},$$

and display it. Thus we have a convex polygon P whose points are a vector constituted by the payoffs:



Parties worth making an agreement if both achieve a grater gain than their disagreement values guaranteed by (u^*, v^*). So (u^*, v^*) is the *status quo payoff* under which they will not come to an agreement. So, the solution will be inside the green area of the polygon. But the points which are not dominated by another point of the green area lie in the bold part of line BA. This is called the *line of agreement*.

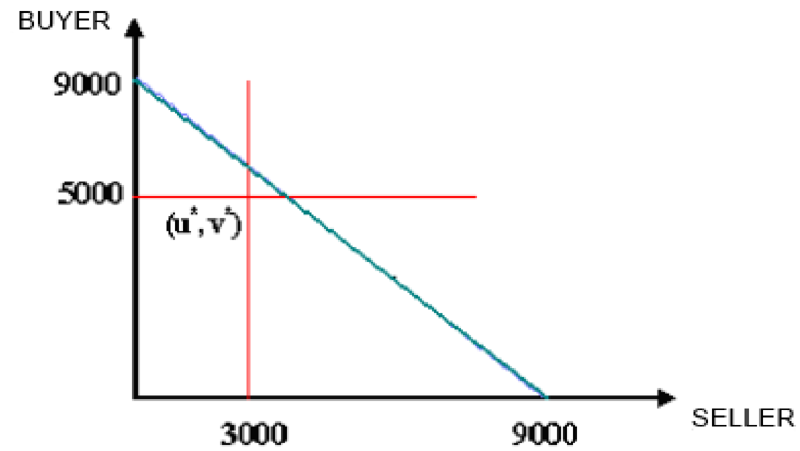
VII.3. Solution Conceptions of Bargaining Games

- **Nash's theorem**: Suppose there are $u > u^*$ and $v > v^*$, and function $g(u, v) = (u - u^*) \cdot (v - v^*)$ takes its maximum at point (u^0, v^0). Then $\varphi(u^*, v^*) = (u^0, v^0)$ is the only function that satisfies the principles of Nash's solution for a cooperative bargaining game, and thus (u^0, v^0) is the solution to the bargaining problem.
- Based upon Nash's theorem, there are several methods to specify (u^0, v^0).
  - **The solution conception of Nash**:
    - First, both parties in the bargaining threaten each other. They choose a threatening strategy (u^*, v^*) what they play if the negotiation is failed. This (u^*, v^*) will be the status quo payoff of the bargaining game.
    - Both parties announce their threatening status quo.
    - If the other is willing to accept this, the solution of the bargaining problem will be (u^0, v^0) provided $u^0 > u^*$ and $v^0 > v^*$.
  - **The solution conception of Shapley**:
    - In this conception the negotiation is no so aggressive than in that of Nash. Parties choose a security status quo (u^*, v^*), which guarantees them a payoff they want to achieve.
    - To find solution (u^0, v^0), you should calculate u^* and v^* by applying formula $\dfrac{a \cdot d - b \cdot c}{(a + d) - (b + c)}$ to the payoff matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ of both players. Then, you should find the maximum of function $g(u, v)$ on the line of agreement.

VII.4. A "used car" example

- Seller will not sell his car for a price less than \$3000, because this is the "real" price of the car what Seller certainly gets for the car. Buyer has \$5000 to buy a car, and is willing to pay a maximum price \$4000 for the seller's car.

  Their disagreement values: $u^* = \$3000$ to the Seller and $v^* = \$5000$ to the Buyer. The total of the disagreement values is: $u^*+v^* = \$8000$.
- Suppose Buyer proposes \$3500 and the seller accepts it. Thus the gain from the trade is: M = \$4000 (what the car is worth for the Buyer) + \$1500 (which remains at the Buyer after the business) + \$3500 (what the Seller gets for his car) = \$9000. And the surplus is \$9000 – \$8000 = \$1000.
- The result of the negotiation is a fair distribution over the surplus, \$500 for both parties: Seller will have \$3500 without a car, and the Buyer will get the car what he assess at \$4000 and will have \$1500 in cash.
- Checking back the result by applying cooperative bargaining model (see on the right): $u^0 = \$3500$ and $v^0 = \$5500$.
- Owning to the bargaining situation now, the solution may go along in Nash or Shapley fashion, that is, (u^*, v^*) is both security and threatening status quo now.



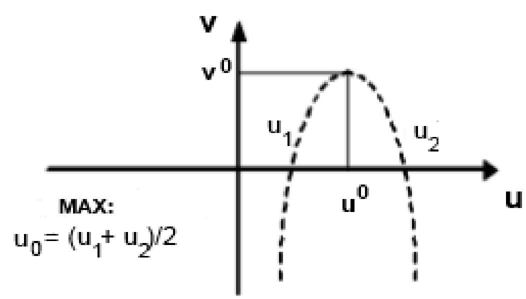
$v = -u + 9000$

$g(u, v) = (u - u^*) \cdot (v - v^*) \Rightarrow \max\{g\}$

$g(u, v) = (u - 3000) \cdot (v - 5000)$

$g(u) = (u - 3000) \cdot (4000 - u) = -u^2 + 7000u - 12{,}000{,}000$

$\Downarrow$



$u_1 = 3000,\ u_2 = 4000$

$u^0 = (u_1 + u_2)/2 = 3500$

$v^0 = -u^0 + 9000 = 5500$

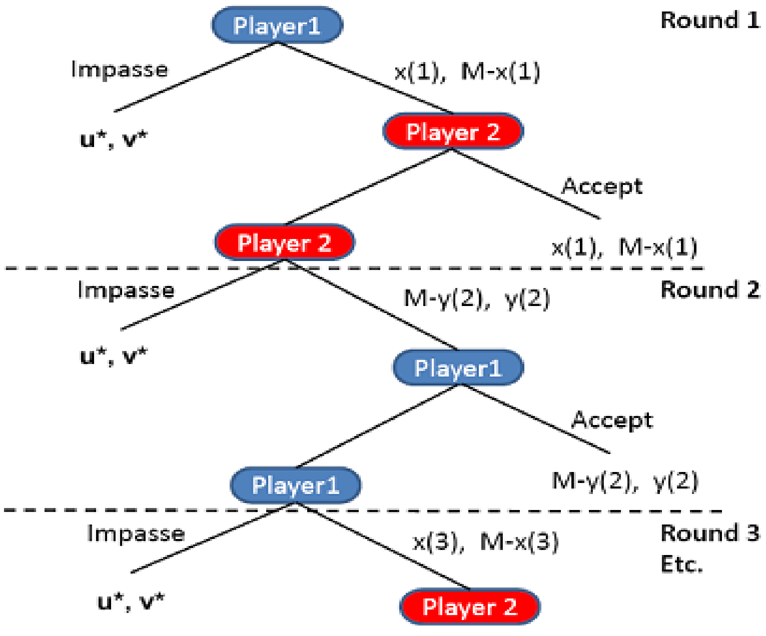
Remark: To find max{g}, we may use the derivative of g(u):

$g'(u) = -2u + 7000 = 0 \Rightarrow u^0 = 3500$

$v^0 = -u^0 + 9000 = 5500$

VII.5. Non-Cooperative Bargaining Games

- A non-cooperative bargaining game is an alternating offers model of negotiation. It is a sequential move game where players have perfect information at each move. Players take turns making alternating offers, with one offer per round. Round numbers: t=1,2,3,…
- Let $x(t_1)$ be the amount that player 1 ask for in bargaining round t_1, and let $y(t_2)$ be the amount that player 2 asks for in bargaining round t_2.
  - Player 1 begins in the first round by proposing to keep x(1) for himself and giving Player 2 M−x(1). If Player 2 accepts, the deal is struck. If she rejects, another bargaining round may be played.
  - In round 2, Player 2 proposes to keep y(2) for herself, and M−y(2) for Player 1. If player 1 accepts, the deal is struck, otherwise, it is round 3 and Player 1 gets to make another proposal.
- Bargaining continues in this way until a deal is struck or no agreement is reached. Then, both players earn their disagreement values, $u^*$and v^*.
- When does this bargaining game end, as it happens in reality? How can the parties put an end to the game?
  - Both sides have agreed to a deadline in advance (or M = 0 at a certain date).
  - The gains from the trade, M, diminish in value over time, and may follow $u^* + v^*$.
  - The players are impatient, because time is money.



VII.6. Impatience in Bargaining

- Players are impatient if they prefer money now to money received later in later rounds. Impatience can be due either to *the time of money* or to *uncertainty* that the game continues. Impatience can be considered as a discount factor:
  - As for the time of money: players prefer \$1 now, invest it at interest rate r and receive \$1·(1+r) = \$(1+r) one round later, rather than wait and receive \$1 next round.
  - As for uncertainty: if $0 < p < 1$ is the probability that the bargaining game continues from one round to the next, then \$1 next round is worth only \$1· p = \$p.
- "Bargaining over a house" example: Suppose the minimum price a seller will sell her house for is \$150,000, and the maximum price the buyer will pay for the house is \$160,000. Therefore, M = \$10,000.
  - Suppose both players have the exact same discount factor, d = .80, i.e., r = .25, since $d=1/(1+r)$.
- Finitely repeated analysis: Suppose there are only two rounds of bargaining, because the buyer has to start a new job in another state and needs a another house. Suppose the buyer makes a proposal in the first round, and the seller makes a proposal in the second round.
  - In the second and final round, the Seller will propose a price of \$160,000, and so to keep all of M = \$10,000 for herself.
  - Move now to the first round. In the first round, the Buyer will have to offer the Seller the equivalent of \$10,000 at the end of the second round. This amount is: \$10,000· d = \$8000.
  - If the Buyer offers anything less, the Seller will reject the proposal, as she will do better as the proposer in the second and last round.
  - Therefore, the Buyer offers \$8000 in round 1 and the seller accepts. The purchase price of the home is \$150,000 + \$8000 = \$158,000.
  - Note that while there are two rounds, bargaining ends after the first round.

VII.7. Infinitely Repeated Analysis

- Suppose now there is no end to the number of bargaining rounds.
  - When Buyer`s proposal in round t of $x(t)\cdot M$ for himself, must give Seller an amount that is equivalent to the need the Seller can get in the next round $y(t+1)\cdot M$. The equivalent amount now is $d\cdot y(t+1)\cdot M$.
  - Buyer offers $(1-x)\cdot M = (d\cdot y)\cdot M$ to the seller, thus $x = 1-d\cdot y$.
  - By a similar argument, Seller must offer $(1-y)\cdot M = d\cdot x\cdot M$ to the Buyer, thus $y = 1-d\cdot x$.
  - $x = 1-d\cdot(1-d\cdot x)$, $x=(1-d)/(1-d^2)$ and $y = 1-d\cdot(1-d\cdot y)$, $y = (1-d)/(1-d^2)$.
  - x is the amount the Buyer gets if he makes the first proposal in the very first round, and y is the amount the Buyer gets if he makes the first proposal in the very first round. Recall $x = y = (1-d)/(1-d^2)$.
  - If the Buyer is the first proposer, he gets $x\cdot M$ and the seller gets $(1-x)\cdot M$. Price is $\$150,000 + (1-x)\cdot M$. If the Seller is the first proposer, she gets $y\cdot M$ and the buyer gets $(1-y)\cdot M$. Price is $\$150,000 + y\cdot M$.
  - In our "Bargaining over a house" example, the Buyer was the first proposer. So $x = (1-.8)/(1-.8^2) = .566$. The Seller gets $(1-x)\cdot M = (1-.566)\cdot M = 0.444\cdot M$. Since M = \$10,000, the price of the house is \$154,440.
- Differing Discount Factors: Suppose $d_B < d_S$, i.e., the Buyer`s discount factor is less than the Seller`s discount factor, that is, the Buyer is less patient than the Seller.
  - When Buyer is the first mover, his new offer is $(1-x)\cdot M = d_S\cdot y\cdot M$ to the Seller, and if Seller is the first, she offers $(1-y)\cdot M = d_B\cdot x\cdot M$ to the Buyer. $\Rightarrow x = 1-d_S\cdot y$ and $y = 1-d_B\cdot x$. The new offer is: $x^{new} = (1-d_S)/(1-d_S\cdot d_B)$. It is easy to see that $(1-d_S)/(1-d_S\cdot d_B) = x^{new} < x = (1-d)/(1-d^2)$.
- Practical lesson: Bargainers do not know one another's discount factors, but may try to guess these values. The more patient player gets the higher fraction of the amount M that is on the table. So bargainers signal they are patient even if they are not. They have a "poker face".

VII.8. Auctions

- An **auction** is a negotiation (bargaining) mechanism where
  - the mechanism is well-specified (it runs according to *explicit* rules);
  - the negotiation is mediated;
  - exchanges are market/currency-based;

- **Mediation:**
  - In a traditional auction, the mediator is the auctioneer, who
    - manages communication and information exchange between participants;
    - provides structure and enforcement of rules.
  - The auctioneer is *not* an agent or a participant in the negotiation.

- **Types of Autions:**
  - ❖ Open vs. sealed-bid (Question: do you know what other participants are bidding?)
  - ❖ One-sided vs. two-sided (Question: do buyers and sellers both submit bids, or just buyers?)
  - ❖ Clearing policy (Question: when are winners determined; occasionally, continuously, once?)
  - ❖ Number of bids allowed (Question: One, many?)

VII.8. Some Classic Auction Types

- **English Auctions**
  - These are the most common auctions in practice. Bids ascend, winner gets the item at the price he bid.
  - Optimal strategy: bid $0.01 more than the next highest person.

Fish Market, Tokyo, JAP

- **Dutch-style Auctions**
  - Used to sell tulips in Dutch flower markets.
  - One-sided (only buyers bid), bids are publicly known and must be decreasing.
  - Auction closes when anyone accepts (usually closes quickly).

Auction Hall, Aalsmeer, NED

- **Vickery Auctions**
  - One-sided (only buyers bid), bids are publicly known (turns out not to matter whether bids are secret).
  - Highest bid receives the good, pays second-highest bid.
  - Optimal strategy: bidding your actual valuation, that is, truth-telling is a *dominant strategy*.