War and Chaos

Complexity theory may be useful in modeling how real-world situations get out of control

Alvin M. Saperstein

At first glance, physics may not appear to share much with the social sciences. In fact, both disciplines are concerned to some degree with the creation and dissolution of order. The physicist, for example, might ask how ordered crystals form from disordered fluids and how the crystals later fall back into the disorder of liquids and gases. On the other hand, the social scientist might be concerned with the formation of international alliances and their collapse into the disorder of war.

Although the questions asked by practitioners of both disciplines appear to be similar, the methods each science uses to answer them differ. The physicist often has the advantage of working with systems where what is put in has a direct relation to what comes out. For this kind of question, the physicist can construct models that allow him or her to predict the future activities of the system in question under various conditions. It is the consistent relation between input and output that allows for predictability.

In contrast, when dealing with human systems, social scientists cannot assume a direct relation between input and output. In fact, human history is filled with instances where dramatic consequences have resulted from fairly minor actions. Or even more perplexing, identical actions can lead to dramatically different results, depending on the context. Such systems do not lend themselves well to the kinds of linear models that are so predictive in physics. Nevertheless, many other scientists have attempted to emulate the methods of the physical sciences, without much success.

But there are modes of physics that have come to prominence in the latter half of this century under the rubric of "complexity" that may be of greater use to the social sciences than is the predictive mode. The rules of complexity theory may provide more useful models of unpredictable activities, such as issues of international relations and the problem of war.

Complexity theory is an old physical tool, newly named. But it allows scientists to make mathematical models of events in which the inputs do not necessarily have a direct, or linear, relation to the outcome. In this way complexity provides an enormous potential advantage over linear analysis in describing international relations. In the linear models previously devised, the situation never becomes unpredictable, or chaotic, no matter how much international hostility and arsenals seem to escalate. It was my hope that a model derived from complexity theory would be able to predict when a situation would become unpredictable, chaotic and possibly end in war.

Predictive Models

In order to understand the limits of the predictive model, it is important to understand what it can and cannot do. The predictive model is based on a deterministic mathematical theory in which all variables are uniquely and precisely defined and in which, given values for all required parameters, the values of the variables at each instant in time are uniquely related to their values at an immediately previous instant. A rule connects successive values of any of the variables. The mathematical structure of the theory *theoretically* iterates the rule repeatedly, allowing the values of the theory's variables, at some future time, to be ascertained, from their values at the present time. Comparison of the predicted values with those actually observed at the future time for some "simple" system allows us to reject, modify or keep the theory. Disagreement between observed and calculated values forces us to change the theory in some way. Agreement, however, does not "prove" the theory; it just means that, so far, the theory—or its specific representation as a model—is compatible with the real system under observation. Successful tests of various simple, idealized, testable subsets of a theory allow the theories to be combined (it is always assumed that their combination will not alter their individual successes) into a grand theory of some complicated system with practical purposes. For example, Newton's laws of force and motion can be combined into a theory for a working automobile. The success of the theory in making predictions about a system indicates that we "understand" that system; the theoretical model of the system is our understanding.

Prediction implies that knowledge of the future is no more uncertain than knowledge of the present—that the range of output values from the theory is of the same order as the range of input values. This predictive approach requires that similar inputs will lead to outputs

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Figure 1. Few political assassinations have had as great an effect on world events as the murder of the Austrian Archduke Ferdinand and his wife in Sarajevo on June 28, 1914, which is generally thought to have precipitated World War I. In human affairs, relatively small events, such as the Archduke's murder, illustrated here in a wash drawing by an unknown artist, can have enormous and unpredictable consequences, which makes human systems difficult to model. The author argues that complexity theory, which has been applied successfully to explain the creation and dissolution of order in physical systems, may provide models for unpredictable social situations.

that are also similar to each other. Otherwise prediction would be impossible, since there is always a range of inputs: Observations of a real system, required to establish the values of the input and model parameters, are always accompanied by observational uncertainties, meaning that only ranges of values, not precise parameter values, can be put into the calculations. This requirement is fine for physics, where one atom or one car should behave quite the same as other atoms or cars of the same type.

For Want of a Nail … a Kingdom Is Lost

The details of context or history in which the atom or car finds itself are usually not a major determinant of its subsequent behavior. This is not true in the social sciences, where contextual details are a major factor. In international politics, similar events or contexts can lead to very dissimilar outcomes. The 1994 shooting down of a plane in central Africa led to the massacre of hundreds of thousands of lives beyond those of the two presidential passengers; the shooting down of the Korean airliner in the 1980s stopped with the death of its few hundred occupants; the assassination of

Figure 2. Order and its dissolution are subjects of interest to both physical and social scientists. Physicists might be interested in the organization of molecules. Molecules in a gas *(left)* are dispersed randomly in space. The space therefore is considered perfectly symmetric in that each point of space carries the same probability of being occupied by a molecule. When the molecules are arranged into the ordered ranks of a crystal *(upper part of center panel)*, the spatial symmetry is broken, and all occupied points are related to each other by definite rules. Now certain points in space carry a near-100-percent probability of being occupied by a molecule, whereas the probability of other points being occupied is near zero. If the crystal is melted, the molecules start to disperse into increasingly random arrangements, as is the case as you move to the bottom of this panel. Molecules in clumps *(right)* define another kind of organization. The molecules may be randomly distributed within the clumps, and the clumps may be randomly located with respect to each other. However, certain regions of the space are definitely not occupied. Therefore all points are not equivalent, so there is a lesser symmetry. The transition from left panel to right may represent the organization of a randomly dispersed population into communities, a social-science application of the same symmetry concepts.

a duke in Serbia early this century led to the downfall of many of the "high and mighty" and the loss of millions of lives throughout the world; the current assassinations in Serbia's neighborhood seem to be confined to the ordinary people of that neighborhood.

The approach called "complexity," old and familiar in the physical sciences but newly named, brings context to the fore. It should thus be a very suitable paradigm for exploring some social questions. Complexity may be defined as the set of deterministic theories that do not necessarily lead to long-term prediction. Such theories are still mathematical and deterministic. The numerical variables are still uniquely related to each other locally in space and time. But the structure of the mathematics is such that we cannot obtain the future values implied by the theory just as a result of a compact, well-defined manipulation of the present values. The calculation requires the *actual* computational stepping through of all intermediate values of the system variables between "now" and "then." Complexity theories thus depend on the complete "path" taken by the system between its beginning and end points. As such, they are sensitive to all perturbations that may have an impact on the system as it evolves in time. Every intermediate instant of time may see the theoretical system diverted from the path it might have taken in the absence of perturbations, which are always present. Hence minute changes in the input parameters may lead to large, incalculable changes in the output. Prediction is no longer possible. The system is extremely context-dependent.

Testing such a theory depends on statistics in physical science or "plausibility" in the social sciences. Physicists would compare the statistical distribution of outcomes *calculated*, given a randomly distributed small set of initial conditions, with the distribution of outcomes *observed*. Agreement between the two distributions means the theory "works," and the physicist has achieved understanding. Of course, in physics, experiments can be set to simulate the conditions of the system being studied. In international politics, where ensembles of identical systems are not available, understanding is obtained if the range of theoretical output possibilities are plausible—if some of them conform to the behavior of actual systems as given by historical observation or "common sense."

It is known in physics that such theories can lead to the formation of structures out of less structured constituents through a process called "bifurcation": a situation that has a binary, either-or solution, the outcome of which cannot be predicted beforehand. A coin toss is an example of a bifurcation. These theories can also lead to the dissolution of an ordered structure into a disordered melange of constituents constituting a transition to "chaos." Through my work, I have shown how these ideas may be applied to international politics.

Structure and Its Causes

Before international relations can be broken, they must be formed. And by analogy to the physical sciences, alliances between nations constitute a kind of structure. By structure I mean the arrangement of a system's elements in the system's space such that the elements have definite relations to each other, breaking the initial symmetry of the space.

To give an example from physics, the elements of ordinary matter are molecules. In the usual gas, these molecules are randomly distributed throughout the three-dimensional volume they occupy. Any point in the space is equally likely to be momentarily occupied by a molecule. There is no correlation between the molecules; knowing that one point in space is occupied tells you nothing about whether a different point is also occupied by a molecule. There is no "structure" to such a gas. The space is perfectly symmetric—every point is as likely to be occupied as every other one.

Consider now a crystal lattice of molecules resulting from the condensation of this gas into a solid. All points of space are not now equally likely to be occupied by molecules. If you know where some molecules are, you know the distances and directions to the location of others. Points not at these locations will not be occupied. The symmetry of the space is broken. You may not know where the first few molecules will locate themselves as the gas begins to condense (the initial space is still symmetric), but the location of all subsequent molecules condensing into the crystal is preordained once the initial ones are fixed.

Or consider groups of people spread uniformly over an extensive landscape. No location is favored; symmetry exists. But then towns spring up. People move to them. You are more likely to find people in the towns than at other locations, so the symmetry is broken, and instead, a structure has been impressed on the space. Some of those towns eventually amalgamate into cities, and a different structure is now evident. Later still, the collections of cities have become nations that in turn form the structure we call the "international system."

How do such structures arise? Consider, for example, the motion of a single planet in the given gravitational force field of a fixed sun. Newton's laws of motion completely specify the acceleration of the planet in terms of its position. Given the acceleration, the position and velocity of the planet are uniquely determined by its position and velocity at an immediately prior instant. Thus we have a logical-deterministic theory expressed as a set of second-order differential equations. The structure to be determined is the position and velocity of the planet in some future epoch, given its present position and velocity. The solution of the differential equations in closed form gives the structural variables—position and velocity—at any time, in terms of well-determined trigonometric functions. Plug in the time you want, and out come the desired structural variables. *There is no need to compute them at intermediate times*. Modify the initial conditions slightly and the output variables come out slightly differently. Thus the entire future of the system, to any specified accuracy, is completely contained in, and obtainable from, the initial parameters and the theory via finite explicitly specified numerical procedures. The closed continuous mathematical form of the solutions to the theory's equations means that output is continuously related to input—there can be no surprises, no disorder and therefore no complexity.

A biological analogue of this completely predictive deterministic system is the very old idea of biological development that held that the complete prescriptive rules for the future organism were contained in its genetic material. Everything was specified once the egg was fertilized. If only biologists could read the biological rules as well as the physicists could read their rules, there would be no surprises in the future. (A political analogue of this model of biological growth, in which everything was to be specified and pre-ordained from the center, would be the extreme Leninist state.) Of course this model of biological growth is extreme. It requires too much information to be contained in the genes. It implies that identical twin babies should grow into identical adults, contrary to observation. We know that contingency is a powerful factor in biological phenomena.

Noncontingent models have also been quite prevalent in theories of international structure. For example, at-

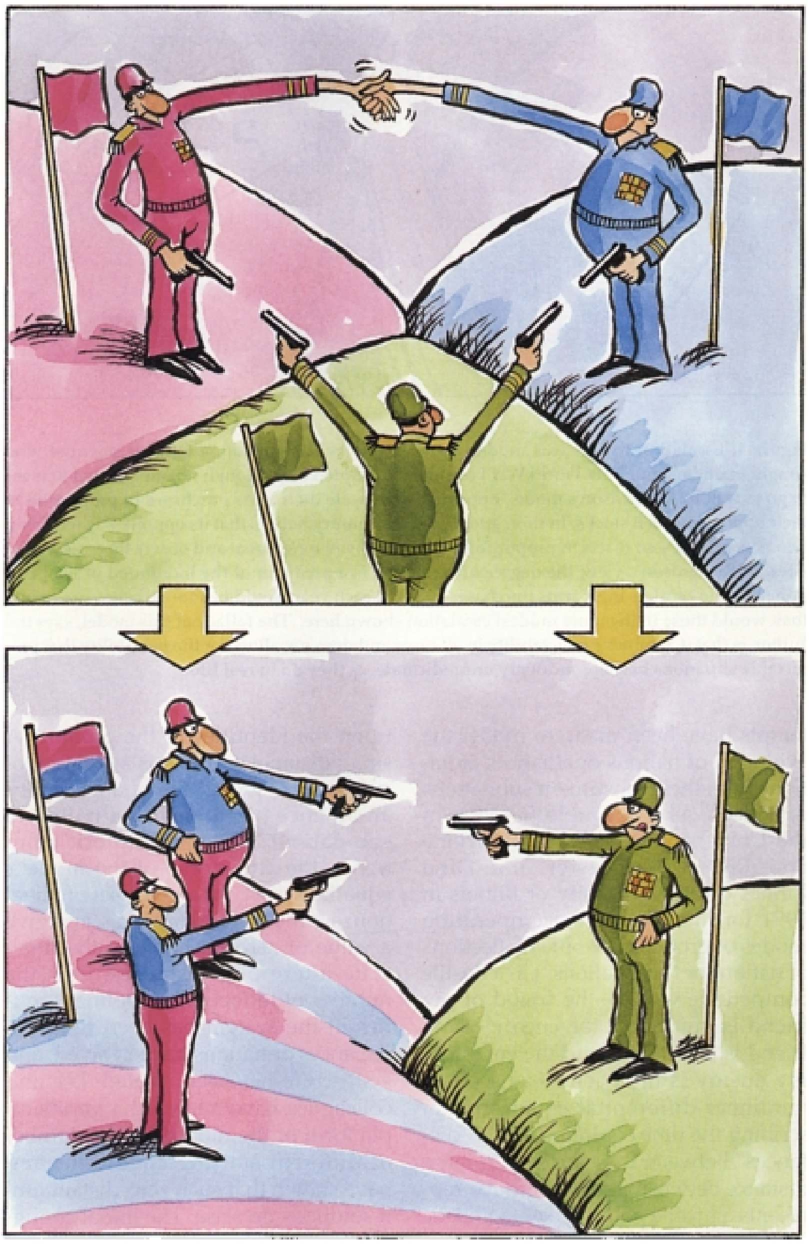

Figure 3. Nations organize themselves into structures by forming and destroying alliances. One organizing principle in international relations is based on common goals and common enemies. Alliances can be formed between groups fighting a common enemy *(top)*. Sometimes allied groups coalesce into a single nation.

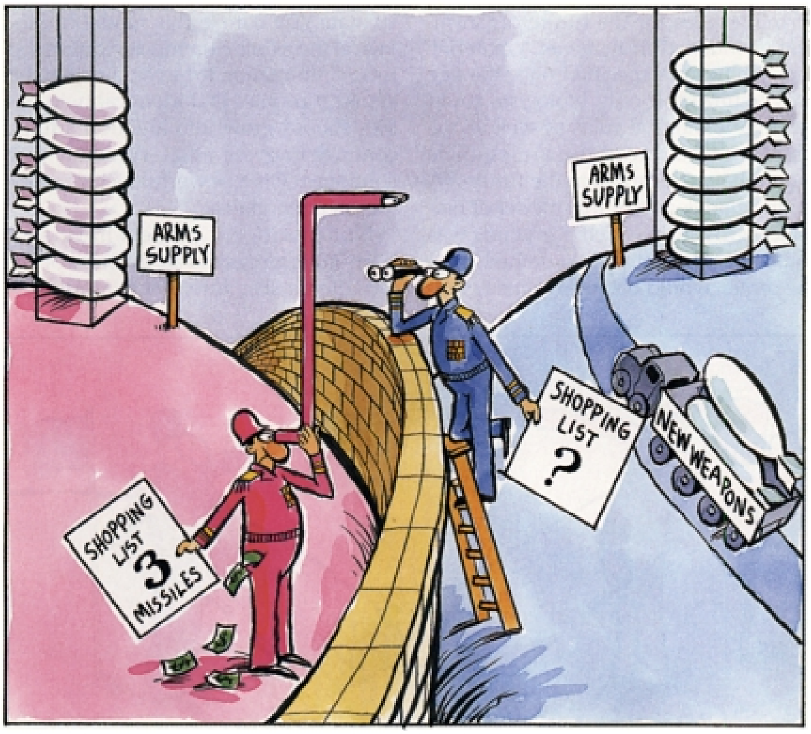

Figure 4. Escalation to war was modeled by Lewis Fry Richardson, a Quaker scientist who sought to understand how World War I could have erupted even though no one seemed to want to go to war. In Richardson's model, opposing sides scale their arms purchases in proportion to their opponent's total stocks. In this cartoon, the red country notices that its opponent is receiving two new missiles, so it acts in proportion to the perceived new threat and orders three new missiles. In Richardson's view, the degree of escalation is a predictor of the likelihood of war. Opponents who escalate their arms purchases 100-fold each year would be more likely to go to war than would those in the more modest escalation shown here. The fallacy of this model, says the author, is that it assumes predictability in all cases and does not allow for the possibility that predictable situations become suddenly unpredictable, as they do in real life.

tempts have been made to model the evolution of nations or alliances of nations from their constituent subnations or nations. As an example, S. C. Lee of the University of California at Irvine and Robert Muncaster and Dina Zinnes of the University of Illinois in 1991 formulated the basic operating modes of groups of people (collections of nations or "sub-nations") in a hostile competitive world—the friend of my friend is my friend, the enemy of my friend is my enemy, and the enemy of my enemy is my friend—as a set of nonlinear differential equations describing the time evolution of the "distances" between these groups. A large distance between a pair of groups represents a hostile pair or an antagonistic relation between two alliances, two nations or two other groupings of people. (The differential equations, modeling the time evolution of the relative distances between groups, do not depend upon the identities of the groups.) A small distance represents a coming together of the groups into a single entity, an alliance of nations or a nation of sub-nations. If the asymptotic solutions—the attractors—of the model's equations exist, for a given set of initial nonzero intergroup distances, and give a value of zero for some of the final-state intergroup distances, then the number of independent political entities in the system has been reduced. Alliances or nations have evolved, and a structure has been created. Lee and colleagues have analyzed a very simple form of the model from a number of different starting conditions and have shown that such zero-distant limit solutions do arise. The initial configurations can be put into continuous classes, and all members of a given class evolve to the same final configuration. Hence by knowing the initial state of the system to some reasonable accuracy, the outcome is foreordained. All information is already present in the initial information. There are no surprises and no contingency.

Contingency

Some theories of structure formation do exhibit contingency. To understand how that can be, let us take the fictional example of a circular table, set for dinner. There are dinner plates symmetrically placed around the circumference of the table, and midway between each pair of plates is a wine glass *(see Figure 5)*. Nothing is specified as to whether the glass to the left or right of a given plate belongs to that plate. The initial symmetry is that either is possible. Once a person sits before one plate at the table and selects one of the two optional neighboring wine glasses, say the left one, the symmetry is broken. The only way the table can be filled is if all subsequent people sitting at the table also select the left-hand glass; the table becomes "left-handed." A small initial contingency—a choice—puts the table into one or the other of the two very distinct classes.

Assume now that many people are milling around, waiting to be seated for dinner. Some of these people are indifferent to the handedness of the table they will sit at. Others will only sit at a left-handed table. The remainder will only sit at a right-handed table. The three classes of people making up the banquet are randomly distributed throughout the hall. The *a priori* symmetry is that people of any of these classes are equally likely to be found anywhere. After the invitation to be seated, the symmetry is broken, and a structure is created in which people of the same class are clustered together. Suppose further that tables are waited on only if two neighboring tables are of the same handedness; the rest go hungry. The original crowd of people is now structured into two sets: one hungry, the other satiated. But one cannot know in advance who, if anyone, will go hungry; one has to go through the process, step by step, to find out. Moreover, if the process were to be repeated under identical circumstances, different sets of people would be selected to go hungry.

The model could be extended and made more complex. For example, suppose that collections of three or more neighboring tables of the same handedness are served from large pots of food that are kept sanitary. The remaining people who are served, are served from small pots that harbor dangerous bacteria. The result is a new structure of ill and healthy people, arising from the original simple contingency. (There could, of course, be a series of contingencies; for example, only some of the small pots, randomly distributed, are contaminated.)

Each step in the model is deterministic. It is rigidly connected to the previous step by rigorous rules. The randomness of the input—who initially sits at each table—determines the outcome, which cannot be foreseen. The process of getting from the initial unstructured state to the final structured one, in the deterministic system, is dependent on the path, resulting from a small random element within the model or in the interaction between the model and its environment. It is important that the system be very sensitive to the small random element, that it "bifurcate" into one of the two very different paths as a result of the small disturbance. This sensitivity, which makes contingency possible, can come about only if the mathematical model describing the system of interest is inherently nonlinear—that is, if the changes in the dependent variables are not simply proportional to these variables.

The above paradigm is easily demonstrated in current ideas of biological growth. Instead of the whole future evolution of the entire organism being contained in each single cell, as in the predictive theory, the contingency paradigm assumes that each cell contains a complete set of rules as to how it should respond to each potential environment (internal as well as external). It will behave one way if it finds itself surrounded by liver cells, a different way if surrounded by bone cells, and so on. Which specific environment it finds itself in depends on the "path" it has taken since the "beginning." At each stage along the path there will be random small disturbances as well as deterministic major rules. The sensitivity at any point, which will vary from point to point, will determine whether and which bifurcations occur. In this way biological organisms grow with the similarities and differences commonly observed.

The simple banquet game outlined above could be played by a computer rather than real people, with random-number generators taking the place of individual volitions. The resulting path-dependent numerical structures would be isomorphic to the people-generated

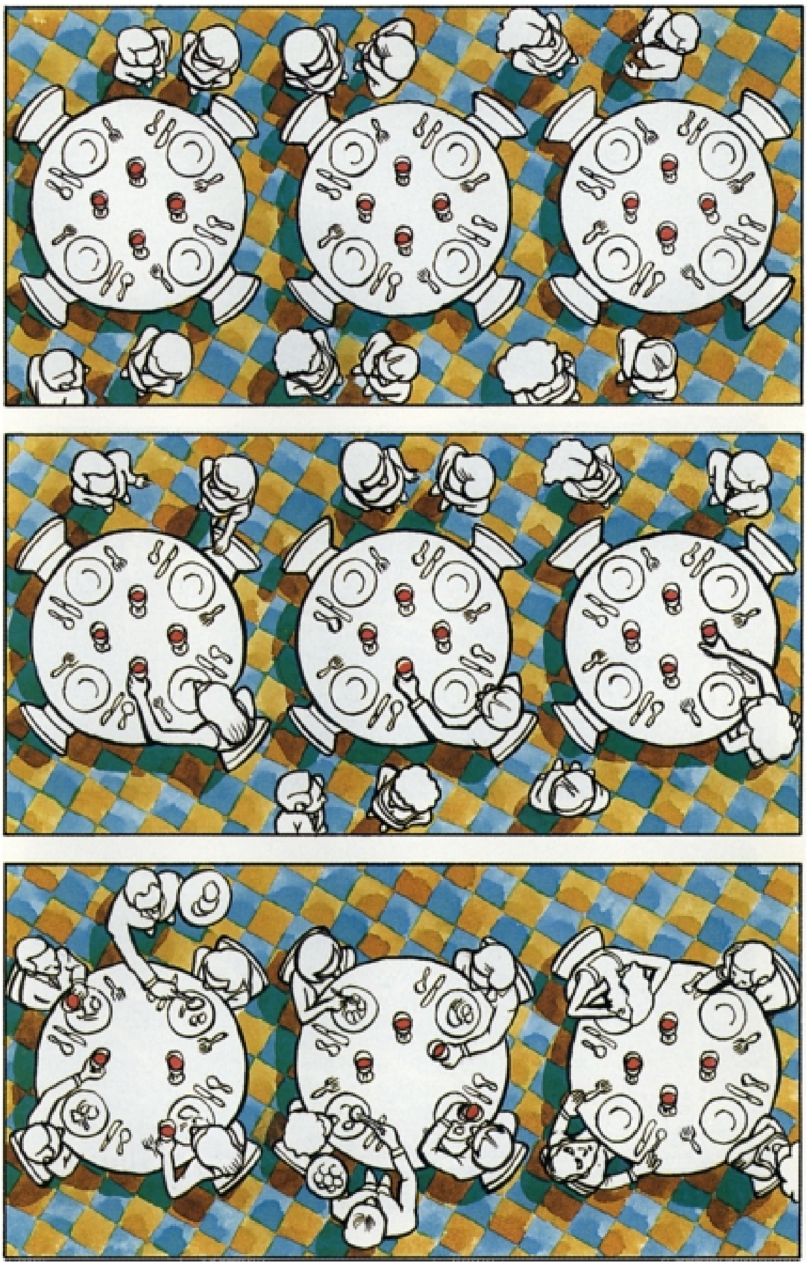

Figure 5. Reactions in human activities are rarely proportional to actions and therefore are rarely predictable. The context and contingencies in a situation influence the outcomes, so that, depending on the context, similar situations may have very different outcomes. For example, assume a dinner party where tables are set so that wine glasses are placed midway between the settings. *A priori*, each glass could "belong" to either of its neighboring settings *(top)*. The guests are milling about, but eventually, some arbitrarily sit down at a table and reach for a wine glass *(center)*. The people at the two tables on the left have chosen the wine glasses to their left. They have broken the symmetry of the table and imposed a handedness on each table. Likewise the person at the rightmost table, by reaching for the glass at the right, has imposed a different handedness on her table. Unbeknownst to the guests, the host has decided that diners will be served only if they are sitting adjacent to a like-handed table *(bottom)*. As a result, the two left-handed tables have been served. The people at the right-handed table go hungry. In this example, the seemingly random act of choosing a wine glass yields disproportionate and unpredictable consequences. Furthermore, the process is completely random. If these events were to be repeated, a very different pattern of fed and starving tables would emerge.

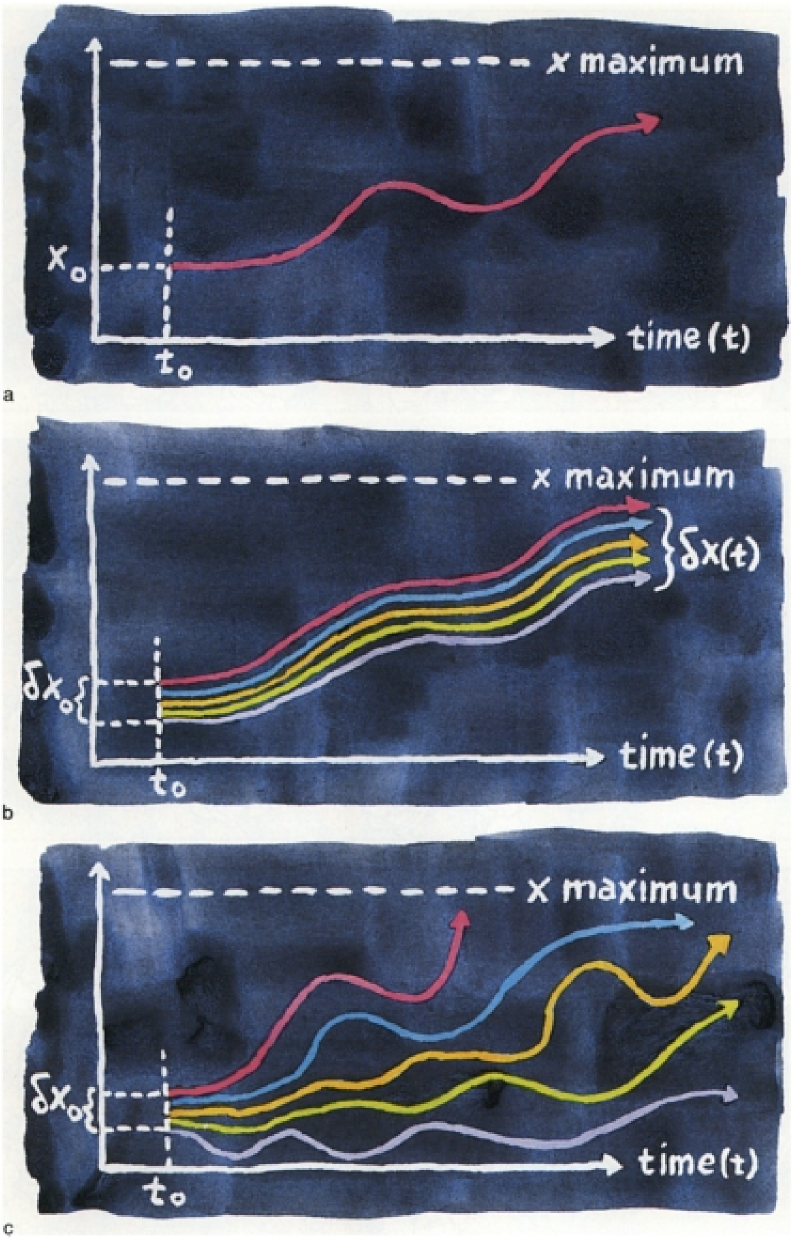

Figure 6. In mathematics, a deterministic law creates a single "orbit," or eventual outcome, given a particular starting point. Shown is a function, *x(t)*, that changes in time, reaching a single solution *(a)* for a given starting value of *x*, x_0. If this same function were reiterated with different starting points (that is, different values for x_0), the endpoints would be different. However, the differences in output would be proportional to the difference in input. If the value of the input changes by one increment, then the value of the output is expected to vary by one increment as well *(b)*. In this case, the future can be predicted to the extent that the present input is known. The accuracy of the output is dependent on the accuracy of the input. If the different outputs diverge greatly from each other *(c)* prediction is impossible. This situation describes chaos.

ones. We cannot program a computer to carry out as many complex deterministic rules as are carried out by real biological cells, but programs have been written to follow the evolution of systems of moderate numbers of moderately complexly interacting computational cells. The resultant behaviors of the computer "organisms" show many interesting parallels to the behavior and development of real living structures. Similar attempts have been made to model the growth of political structures, one of which is sketched below.

Robert Axelrod of the University of Michigan has produced a model of a linear array of "nations," with arbitrary initial distributions of wealth and no "commitments" to one another. At each annual cycle, random nations are chosen to make demands on their neighbors—war or pay ransom. A fairly simple set of rules determines the outcomes of the demands and the consequent changes in national wealth. Also, as a result of making war and transferring ransoms, commitments between the nations begin to develop following rules fairly similar to those of Lee and colleagues: Losers (of war or ransom) become connected to the winners. The enemy of an enemy becomes a friend—that is, they become more committed.

As a nation accumulates increasing commitments from others, its chances of dominating in a war or ransom demand also increase. The computer simulations are run over 1,000 cycles, each run representing one system path through time. At the end of a run, groups of nations that are strongly tied to each other (have large resultant numerical values of mutual commitment) may be considered to be single entities, either alliances or larger nations. Thus a structured international system may arise in which there are fewer independent participants than existed initially. Many runs, all with the same rules and parameters, produce many different paths and therefore many different international structures. Some of these paths look strikingly similar to historical images of the evolution of the real world, implying that Axelrod's model may incorporate some real understanding of important factors in world evolution.

Dissolution of Structure and Chaos

Having formed, all structures, be they physical or social, are vulnerable to dissolution. A good physical illustration of the breakdown of structure is the behavior of a river as it flows from a wide, deep, unobstructed channel into a narrow, constrained, boulder-strewn gorge. The initially slowly and smoothly flowing water is completely predictable and is said to exhibit laminar flow. Knowing the behavior of the fluid at one point and one time allows one to know the detailed behavior of the water at neighboring points at the same time or at the same point at later times. This coherence of behavior in space and time is a structure determined by the physical properties of the fluid as expressed in the Newtonian laws of motion. When the water flows into the twisty, uneven channels of the gorge, it becomes turbulent and is no longer predictable. The same body of water has passed from structured to unstructured behavior—from predictability to chaos (and, of course, later downstream might transform back into structured laminar flow again). Any reasonable mathematical model of fluid behavior must be able to represent these two types of actual fluid behavior and the transitions between them.

The system in this case is nonlinear so that the responses to small changes are not necessarily proportional to the changes themselves. Under these circumstances, small changes in the channel's architecture, may result in tumultuous changes in the fluid's behavior—or they may not. The solutions to equations describing such a situation have regions in which small disturbances remain small and other regions in which the disturbances grow. In the latter regions, two solutions that start off very close to each other (differing only by a small "disturbance") soon bear no resemblance to each other. Prediction in this case becomes impossible.

A situation in which deterministic theory precludes prediction, in which small random disturbances lead to wildly fluctuating incoherent motions, is called "chaos." The concept is just what is needed to describe turbulence. Thus the same theory can represent both laminar flow and turbulence. Although the theory cannot predict what happens in the chaotic regime, it *can* predict the regimes in which turbulence is possible. The theory allows calculation of those parameter values that bound the chaotic region.

At its most useful, then, the theory provides a prediction of unpredictability. So although the theory would not allow one to make the turbulent water calm, it does allow one to predict the parameters that will give rise to turbulence. And this provides the possibility of rationally designing a fluid conduit in such a way as to avoid turbulence.

A corresponding theory of international relations might allow the prediction of the loss of controlled behavior by nations within an international system—a loss we then use to represent the outbreak of war—and thus allow it to be avoided.

Linear Models of War

Many writers on war and peace have introduced the concept of crisis instability. The idea is that large consequences can be wrought in the international system as a result of small disturbances or affronts. A standard example of such a system instability is the outbreak of World War I, following the assassination in Serbia of the Austrian Archduke Francis Ferdinand and his wife. The disparity between disturbance and its consequence is very similar to that presumed in the definition of chaos. It is thus natural to associate the transition from predictability to chaos in a mathematical model with crisis instability and the outbreak of war in the international system being modeled. This association is particularly natural since war has long been associated with the nonmathematical concept of chaos, which bears great similarity to the corresponding mathematical concept. If we wish to make this association, and thus understand the outbreak of war (or of some wars) as the transition to deterministic chaos, we must first believe in the possibility of valid deterministic models for the evolution of international behavior.

But how can nations, each made up of so many seemingly autonomously acting people, behave deterministically? Again, a physical analogy is helpful. A gas is made up of many, many molecules. The overall motions of the myriad molecules are completely random. And yet, when examined from the view of the system as a whole, the gas behaves simply and predictably. It obeys deterministic laws that result from averaging over the many random components of the system. (Similar deterministic averages over many random variables are used by insurance companies to construct mortality and other tables, which form the basis of much common social life.) On this basis, we can expect "laws" to govern the interactions of states. Cer-

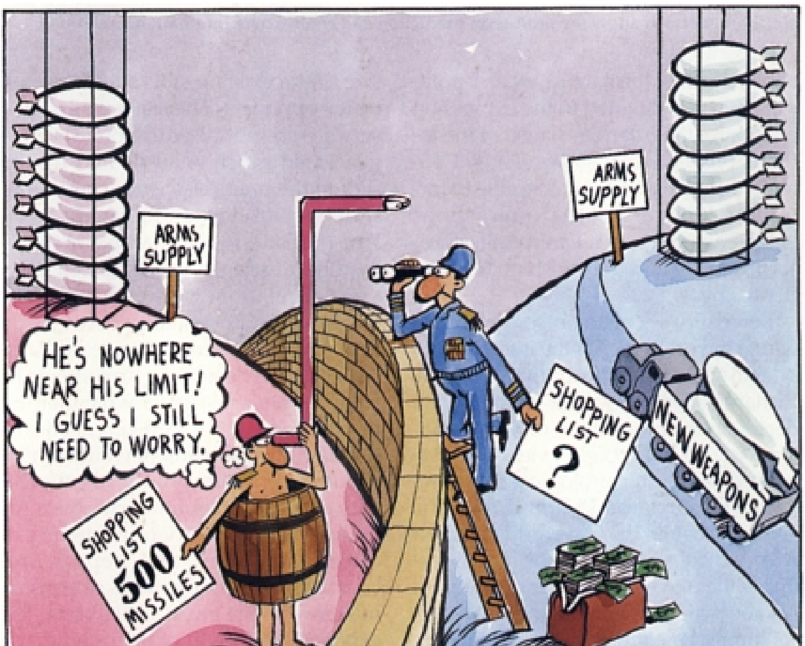

Figure 7. In the author's nonlinear version of the Richardson model, a nation's fear of its opponent's arms stocks diminishes as the size of those stocks approaches the maximum sustainable by that opponent. Hence the arms procurement of one nation is less tightly linked to the size of the competitor's arms stocks than it would be in the Richardson model; procurement grows at a lesser rate with increasing stock size. The result is a nonlinear relation between procurement and stocks that can lead to an unpredictable variation of both, with wild fluctuations over the full range of possibilities. This transition to a chaotic regime represents "crisis instability" in the relationship between the two nations, a contingent relationship not possible in the original linear model. In this cartoon, the red nation, acting out of fear, orders the wildly disproportionate—and unpredictable—number of 500 new missiles.


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Figure 8. Twisted iron bedframe on a World War I battlefield in Villers-Bretonneux, France, stands as a reminder of a calmer time amidst the chaos of war. The first world war erupted following a single assassination in Sarajevo. Today, there is again fighting in that part of the world, but so far its seems unlikely to result in another world war. Only time will tell whether the complexity model will allow for short-term prediction and control over international relations.

tainly, much of formal history and political science is devoted to the expression of apparent regularities (laws) of the interactions between states. Thus, I entered this study hoping to be able to formulate laws of hostile interactions between nations that can manifest crisis instability and the transition from order to chaos.

Lewis Fry Richardson sought to lay war to the properties of the international system rather than to human volition. Richardson was a Quaker who believed deeply in the possibilities of human betterment. And his point of view was buttressed by the technological improvements he saw around him during the years just before the first world war. So it was particularly difficult for him to reconcile the universal disaster of war with his conviction of humankind's struggle for improvement. Therefore he sought to attribute the onset of war to properties of the international system. It certainly seemed to many observers that World War I broke out against the wishes of everybody.

The system Richardson saw at that time was an arms race, and he proceeded to provide a mathematical description of the obvious characteristics of such a race. The rate of arms acquisition of each of a pair of hostile powers (he did not explain the origin of hostility but took it as a given) depended on the size of the existing stocks of arms held by the opponent. The result was a linear set of differential equations that has generated much analysis and discussion in the political-science community since its introduction following World War II. (For a recent example of its use in political science, see Hill, 1992.) One class of relations between the coefficients (of proportionality between the rate increase and the size of arms stocks) led to decreasing armaments for all opponents, no matter what their initial stocks were. Richardson called such a system stable because it would remain peaceful, no matter how hostile the feelings of the nations for each other. Another class of coefficient relations produced solutions that represented constantly increasing stocks of arms, no matter what the initial stocks were. This exponential growth of armaments was claimed to be unstable and would lead to war. Avoiding war would simply require changing the system so the coefficients representing it fell into the appropriate class.

The linear Richardson equations can predict that a system's arms stocks will continually grow either larger or smaller without a change. There is no change allowed in the character of the solutions. What the Richardson equations cannot do is yield a model in which an initially peaceful system suddenly becomes turbulent. In other words, the Richardson equations cannot predict unpredictability. There is nothing in his linear mathematical model that could be interpreted as "crisis instability."

Nonlinear Models of War

But the Richardson approach can be made nonlinear, a task I set for myself. In the Richardson model, a nation increases its arms supplies based on what its opponent had the previous year. It occurred to me that other factors could be included in the decision to increase the number of arms. I made the equation a little more complicated by recognizing that a nation's need to increase its arms supply in proportion to its opponent's supply decreases as the nation fears its opponent less. And a nation fears its opponent less if it suspects that the opponent is near the limit of what it can acquire. I could build these conditions into the Richardson equations mathematically.

Trying to devise the simplest model I could, I set the amount of arms held by one nation at x (expressed as a fraction of the nation's total arms-purchasing capacity). If we set the limit of this nation's ability to purchase or manufacture arms at 1, then $1-x$ expresses how close that nation has come to its saturation point. The factor $1-x$ modifies the Richardson coefficient of proportionality between the size of arms stocks and the rate at which arms are acquired by the opponent. This nation's opponent will acquire arms in proportion to the expression $x(1-x)$. In the meantime, the opponent nation has a total arms supply of y, so the first nation will increase its supply in proportion to $y(1-y)$. It is easy to see that as one nation nears its limit, the expression $1-x$ or $1-y$ approaches zero. This means a nation's arms purchases become increasingly less linked to the purchases of its opponent. If, on the other hand, the new purchase nowhere near approximates the opponent's limit, the coupling coefficient approaches 1, and the nation's acquisition decision will be very tightly coupled to its opponent's existing supply, as in the original Richardson model.

The easiest way to obtain chaos from this is to feed these expressions into a spreadsheet and let the program calculate the outcomes over 100 iterations of the program. I would run the program several times, altering the starting numbers very slightly, say by one-tenth of one percent. If the program was completely predictable, I would expect the outcomes to alter by one-tenth of one percent. However, if the system were successful at generating chaos, the change in outcome would not be expected to be proportional to the changes in inputs. I found that this system could in fact become chaotic.

Since the nonlinearized Richardson equations represent the interactions between competing states, changes in the forms of these equations represent changes in the nature or dominant characteristics of international relations. Thus questions about the outcomes of particular types of organization of the international system can be addressed via stability investigations of the corresponding sets of equations. For example, a question particularly pertinent today, after the demise of a cold-war world dominated by two superpowers, is whether a bipolar world is more or less stable than a multipolar world. A simple approach to this question, leading to an unambiguous answer, is to take a nonlinear Richardson model of a two-nation arms race and bring in a third nation competing with the first two by the use of a coupling parameter that can vary from zero to one. A zero value means that there is no coupling of the third nation to the first two. The system is a bipolar arms race. When the parameter reaches a value of unity, the model represents three equivalent nations, each arms-racing with the other two in a fully tripolar world. With a fixed intermediate value of the coupling parameter, the size of the region of stability of the model (as measured by the allowed ranges of values of the other model parameters that produce predictive classes of solutions to the model equations) is determined. It is found that the stability region shrinks as the coupling parameter is increased. This implies that the system becomes more and more unstable as it departs further and further from pure bipolarity. Other, more traditional, political science analyses have led to the same conclusion. However, any science grows and thrives via the confirmation of its results by many different procedures. Hence this simple mathematical model, invoking chaos as a representation of crisis instability, is a valuable adjunct to conventional political science.

I have modified the model to address other questions of global stability. For example, I have compared the relative stabilities of a world system consisting of democratic states with that of a system of autocracies. The result—democracies yield a more stable world system—has also been obtained by other, less model-oriented approaches. Another question I have modeled concerns the relative stability of balance-of-power worlds versus systems made up of nations, each of which stands alone against the rest. The chaos domain analysis comes down on the side of those who maintain that balance-of-power (nations combining their power in shifting alliances to balance the power of the dominant nation at the moment) is the more stable regime.

The above examples of chaos analysis of the origin of war have used the military capabilities of the competing nations as the dependent variables. In the previous discussion of the model presented by Lee and coworkers of the origin of international structures, the dependent variables are the intents of the nations towards one another. A more realistic model includes both types of variables and their interactions. Intent incites capability, and capability modifies intent. The resulting combined model, including military and economic capability as well as inter-nation relations, is much more complex and contains many more parameters, all of which are difficult to obtain from real-world data. Thus specific conclusions are harder to obtain.

However, some familiar qualitative results can be simply obtained. War will not occur if military-industrial-governmental complexes have no significant influence on the evolution of overall economic-political power. Neither will war break out if changes in military capabilities do not lead to changes in national intent towards other nations.

The similarity between the calculated behavior of complexity-dominated systems and the behavior of sociopolitical systems in the real world gives considerable credence to the idea that the real world is dominated by deterministic rules and that the observed contingency is due to the occasional sensitivity of the real system to minor, but always present, random perturbations. This paradigm is quite different from that which supports the observed contingencies of the world on an underlying stochastic foundation. The choice between the two approaches to sociopolitical reality is not purely academic but has profound practical consequences. Both paradigms rule out the possibility of long-term prediction, but the complexity scheme does allow for short-term prediction and thus offers the possibility of control. If the ruling outlook in a population is that sociopolitical life is based on the stochastic paradigm, there is no point in political activity—in trying to form and direct collective behavior towards predetermined ends. The complexity paradigm makes it reasonable for a population to expect effective government and to believe in the possibility of a collective solution to a collective problem. Public-opinion polls in contemporary America seem to indicate that we are not convinced of the complexity model these days, but the evidence is not yet all in.

Bibliography

Axelrod, R. 1994. A model of the emergence of new political actors. In *Many Agent Simulation and Artificial Life*, ed. E. Hillebrand and J. Stender. Amsterdam: IOS Press, pp. 200–217.

Ekeland, I. 1993. *The Broken Dice*. Chicago: The University of Chicago Press.

Flam, F. 1994. Artificial-life researchers try to create social reality. *Science* 265: 868–869.

Hill, W. A. 1992. Several sequential augmentations of Richardson's arms race model. *Mathematical and Computer Modelling* 16 (No. 8/9): 201–212.

Lee, S. C., R. G. Muncaster and D. A. Zinnes. 1991. The friend of my enemy is my enemy: Modeling triadic international relationships. *Synthese* 100 (No. 1): 333–358.

Nagel, E. 1961. *The Structure of Science*. New York: Harcourt, Brace, and World.

Ott, E. 1993. *Chaos in Dynamical Systems*. Cambridge, England: Cambridge University Press.

Prigogine, I., and I. Stengers. 1984. *Order Out of Chaos*. New York: Bantam.

Richardson, L. F. 1960. *Arms and Insecurity*. Pittsburgh: Boxwood Press.

Richardson, L. F. 1960. *Statistics of Deadly Quarrels*. Pittsburgh: Boxwood Press.

Saperstein, A. M. 1984. Chaos—A model for the outbreak of war. *Nature* 309: 303–305.

Saperstein, A. M. 1991. The long peace—result of a bi-polar competitive world? *Journal of Conflict Resolution* 35 (No. 1): 68–79.

Saperstein, A. M. 1992. Are democracies more or less prone to war? A dynamical model approach. *Mathematical and Computer Modelling* 16 (No. 8/9): 213–221.

Saperstein, A. M. 1992. Alliance building vs independent action: A non-linear modelling approach to comparative international stability. *Journal of Conflict Resolution* 36 (No. 3): 518–545.

Saperstein, A. M. 1994. Mathematical modelling of the effects of 'capability' and 'intent' on the stability of a competitive international system. *Synthese* 100 (No. 3): 359–378.

Schuster, H. G. 1988. *Deterministic Chaos — An Introduction*. Weinman, FRG: VCH Verlagsgesellschaft mbH.

Tuchman, B. 1962. *The Guns of August*. New York: Dell.

von Clausiwitz, C. 1832. *On War* (English translation, 1968; A. Rapaport, ed.). New York: Penguin.

Waldrop, M. M. 1992. *Complexity*. New York: Simon and Schuster.