2007.05.03. csütörtök X. Gyakorlat (12. hét)

$f(z) = u(x,y) + i\,v(x,y)$

diffható az fv. $\Leftrightarrow$ C-R teljesül $\boxed{\begin{aligned} u'_x &= v'_y \\ u'_y &= -v'_x \end{aligned}}$

reguláris: előző óra

$\Updownarrow$

harmonikus: $\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

$\Delta v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$

u és v harmonikus fv pár

konstanstól eltekintve meghatározzák egymást

ANALITIKUS

Harmonikus társkeresés

① $u(x,y) = x^2 - y^2$

$u'_x = 2x \qquad u'_y = -2y$

$u''_{xx} = 2 \qquad u''_{yy} = -2$

$u''_{xx} + u''_{yy} = 0 \Rightarrow$ u harmonikus $\Rightarrow$ harmonikus társ kereshető

v(x,y) harmonikus társ, ha

$v'_x = -u'_y = 2y \qquad \Rightarrow v(x,y) = \int 2y\,dx = 2xy + c(y)$

$v'_y = u'_x = 2x$

$v'_y = 2x + \frac{d}{dy}c = 2x$

$\frac{d}{dy}c = 0$

$c(y) = c$

$\Rightarrow v(x,y) = 2xy + c \Rightarrow \underline{f(z) = (x^2 - y^2) + i\,2xy}$ $(c=0)$ ANALITIKUS ✓

② $V(x,y) = \cos x + \sin y$

V

③ $v(x,y) = e^{x}(x\cos y - y\sin y)$

$V'_x = e^x(x\cos y - y\sin y) + e^x \cos y$

$V''_{xx} = e^x(x\cos y - y\sin y) + 2e^x \cos y$

$V'_y = -e^x x \sin y - \sin y - y\cos y$

$V''_{yy} = -e^x x\cos y - \cos y - \cos y + y\sin y$

④ $v(x,y) = y + \operatorname{arctg}\frac{y}{x}$

$$V'_x = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \left(-\frac{y}{x^2}\right) = \frac{-y}{x^2+y^2}$$

$$V''_{xx} = \frac{2xy}{(x^2+y^2)^2}$$

$$V'_y = 1 + \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x} = 1 + \frac{1}{x+\frac{y^2}{x}} \qquad \frac{x}{x^2+y^2}$$

$$V''_{yy} = \frac{-1}{\left(x+\frac{y^2}{x}\right)^2} \cdot 2y = -\frac{2xy}{(x^2+y^2)^2}$$

$$\int \frac{f'}{f} = \ln|f|$$

$V''_{xx} + V''_{yy} = 0 \Rightarrow$ u(x,y) harmonikus pár

$$\int f' \cdot f^{\alpha} = \frac{f^{\alpha+1}}{\alpha+1} \qquad \alpha \neq -1$$

$$u'_x = v'_y = 1 + \frac{\frac{1}{x}}{1+\left(\frac{y}{x}\right)^2}$$

$$u'_y = -v'_x = \frac{y}{(x^2+y^2)^2} \Rightarrow u(x,y) = \int \frac{y}{(x^2+y^2)^2}\,dy = \frac{1}{2}\int \frac{2y}{(x^2+y^2)^2}\,dy =$$

$$= \qquad = \frac{1}{2}\ln(x^2+y^2) + C(x)$$

$$\cancel{u'_x = \frac{-(x^2+y^2) + 2x \cdot x}{(x^2+y^2)^2} + c'(x) = \frac{x^2-y^2}{}}$$

$\Rightarrow$

$$u'_x = \frac{x}{x^2+y^2} + C'(x) = \frac{x}{x^2+y^2} + 1$$

$$C'(x) = 1$$

$$C(x) = x + C$$

$$\boxed{V(x,y) = \ln\sqrt{x^2+y^2} + x + C}$$