ZH $\boxed{3}$

$\dot{x}_1 = x_1 + 2x_2 \qquad x_1(0) = 1$

$\dot{x}_2 = 2x_1 + x \qquad x_2(0) = 2$

$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad \underline{\underline{A}} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x}$

$\det \begin{pmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{pmatrix} = (1-\lambda)^2 - 4 = 0$

$(1-\lambda+2)(1-\lambda-2) = 0$

$(3-\lambda)(-1-\lambda) = 0$

$\lambda_1 = 3 \qquad \lambda_2 = -1$

$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$\begin{matrix} v_1 \\ v_2 \end{matrix}$

$\underline{\underline{A}}\,\underline{v} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} v_1 + 2v_2 \\ 2v_1 + v_2 \end{pmatrix}$

$\lambda_1 = 3$

$v_1 + 2v_2 = 3v_1 \quad \Rightarrow v_2 = v_1 = 1$

$2v_1 + v_2 = 3v_2$

$\Rightarrow \underline{v_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda_2 = -1$

$v_1 + 2v_2 = -v_1 \quad \Rightarrow v_2 = -v_1 = 1 \qquad \underline{v_2} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$2v_1 + 1v_2 = -v_2$

$$\underline{x} = c_1 e^{3t}\begin{pmatrix}1\\1\end{pmatrix} + c_2 e^{-t}\begin{pmatrix}-1\\1\end{pmatrix}$$

$$x_1 = c_1 e^{3t} - c_2 e^{-t} \qquad 1 = c_1 - c_2 \qquad c_1 = \frac{3}{2}$$

$$x_2 = c_1 e^{3t} - c_2 e^{-t} \qquad 2 = c_1 + c_2 \qquad c_2 = \frac{1}{2}$$

$\boxed{\text{ZH } \boxed{5}}$

Skalárpot. létez-e

$$\underline{v}(\underline{r}) = \ln |\underline{r}|^2 \cdot \underline{r} = \qquad \underline{r} = (x, y, z)$$

$$= \left(x \ln(x^2+y^2+z^2),\ y \ln(x^2+y^2+z^2),\ z \ln|x^2+y^2+z^2| \right)$$

$$\operatorname{rot} \underline{v}(\underline{r}) = \nabla \times \underline{v} = \det \begin{pmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x\ln(..) & y\ln(..) & z\ln(..) \end{pmatrix} =$$

$$= \underline{i}\underbrace{\left(\frac{2yz}{x^2+y^2+z^2} - \frac{2zy}{x^2+y^2+z^2} \right)}_{0 \ - \ 0} - \underline{j}\underbrace{\left(\frac{2xz}{x^2+y^2+z^2} - \frac{2zx}{x^2+y^2+z^2} \right)}_{0 \ - \ 0} + \underline{k}\left(0 - 0 \right)$$

$$\underline{k}:\ \frac{2xy}{x^2+y^2} - \frac{2yx}{}$$

$$= 0 \Rightarrow \exists \text{ pot fv}$$

$\boxed{ZH\ 4}$

$$v(r) = (y^2 - x^2)\underline{i} + 2yz\underline{j} - x^2\underline{k}$$

$L:\ x = t,\ y = t^2,\ z = t^3 \qquad 0 \le t \le 1$

$$\underline{r}(t) = (t,\ t^2,\ t^3)$$
$$\dot{\underline{r}}(t) = (1,\ 2,\ 3t^2)$$

$$v(r(t)) = (t^4 - t^2)\underline{i} + 2t^5\underline{j} - t^2\underline{k}$$

$$\int_L \underline{v}(\underline{r})d\underline{r} = \int_0^1 \underline{v}(\underline{r}(t))\,\dot{\underline{r}}(t)\,dt = \int_0^1 (t^4 - t^2 + 4t^6 - 3t^4)\,dt =$$

$$= \left[-2\frac{t^5}{5} - \frac{t^3}{3} + \frac{4t^7}{7}\right]_0^1 = -\frac{2}{5} - \frac{1}{3} + \frac{4}{7}$$

$\boxed{1}$ $y' - \frac{2}{x}y = x^2 + 1 \qquad$ LIN 2 => 1

H: $y' - \frac{2}{x}y = 0$

$$\int \frac{dy}{y} = \int \frac{2}{x}dx$$

$$\ln|y| = 2\ln|x| + \ln|c|$$

$$\boxed{y_H = cx^2}$$

IH: $y_p = c(x)\cdot x^2$

$y_p' = c'x^2 + 2xc$

$c'x^2 + \cancel{2xc} - \cancel{\frac{2}{x}cx^2} = x^2 + 1$

$$c' = 1 + \frac{1}{x^2}$$

$$c = \int \left(1 + \frac{1}{x^2}\right)dx = x - \frac{1}{x}$$

$y_p = x^3 - x$

$y_{ált} = cx^2 + x^3 - x$

$\boxed{2}$

$$\underbrace{(3x^2+6xy^2)}_{p}dx+\underbrace{(6x^2y+4y^3)}_{q}dy=0$$

$$\left.\begin{aligned}\frac{\partial p}{\partial y}&=12xy\\ \frac{\partial q}{\partial x}&=12xy\end{aligned}\right\}\ \text{egzakt}$$

$F(x,y)=C$

$$\frac{\partial F}{\partial x}=p=3x^2+6xy^2$$

$$\frac{\partial F}{\partial y}=q=6x^2y+4y^3$$

$$F=\int(3x^2+6xy^2)\,dx=x^3+3x^2y^2+\varphi(y)$$

$$\frac{\partial F}{\partial y}= \quad \cancel{6x^2y}+\frac{d}{dy}c=\cancel{6x^2y}+4y^3$$

$$\varphi(y)=\int 4y^3\,dy=y^4$$

$$\underline{\underline{F(x,y)=x^3+3x^2y^2+y^4=c}}$$

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- Integrális defin
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