2007. 02. 14. Szerda

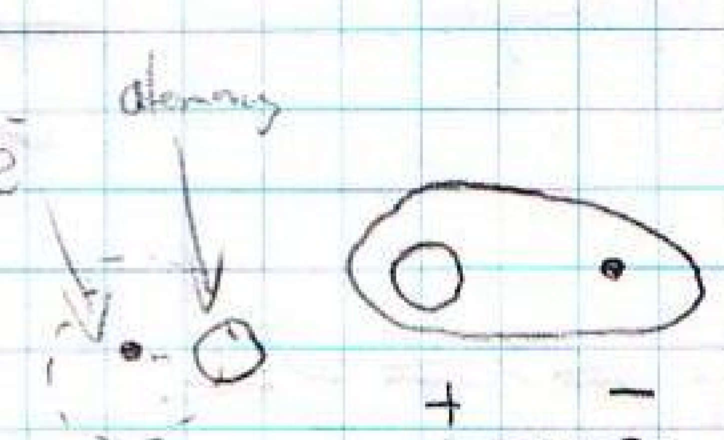
II. Előadás (1. hét)

EMT alaptörvényei

Maxwell egyenletek	Integrális	Differenciális	Szinuszos változás
I. ált. gerj. törv.	$\oint_l \bar{H} d\bar{l} = \int_a \left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right) d\bar{a}$ $l \rightarrow a$	$rot\,\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$	$\bar{H}(\bar{r},t) = Re\{\bar{H}(\bar{r})\, e^{j\omega t}\}$ $rot\,\bar{H}(\bar{r}) = \bar{J}(\bar{r}) + j\omega \bar{D}(\bar{r})$
II. Faraday	$\oint_l \bar{E} d\bar{l} = -\int_a \frac{\partial \bar{B}}{\partial t} d\bar{a}$ $l \rightarrow a$	$rot\,\bar{E} = -\frac{\partial \bar{B}}{\partial t}$ (örvényesség)	$rot\,\bar{E}(\bar{r}) = -j\omega \bar{B}(\bar{r})$
III. Mágn. tér forrásmentes	$\oint_a \bar{B} d\bar{a} = 0$	$div\,\bar{B} = 0$	$div\,\bar{B}(\bar{r}) = 0$
IV. Elektrost. Gauss t.	$\oint_a \bar{D} d\bar{a} = \int_V \rho\, dv$ $a \rightarrow V$	$div\,\bar{D} = \rho$ (forrásosság)	$div\,\bar{D}(\bar{r}) = \rho(\bar{r})$

Anyag és EMT kölcsönhatása:

• Szigetelő anyagok – Mikroszkópikus:



dipólus nyomaték

$\bar{p} = Q\bar{l} \sim \alpha \bar{E}$

– Makroszkópikus:

$\bar{p}_i = Q_i\, \bar{l}_i \qquad i = 1, \dots, N$

$\bar{p}_i \sim \bar{p}_j \sim \bar{p}$

$$\bar{P} = \lim_{dv \to 0} \frac{1}{dv} \sum_{i=1}^{N} \bar{p}_i = \varepsilon_0 \mathcal{K} \bar{E}$$

szuszceptibilitás

POLARIZÁCIÓ VEKTORA

$$\bar{D} = \varepsilon_0 \bar{E} + \bar{P} = \varepsilon_0 \underbrace{(1 + \mathcal{K})}_{\varepsilon_r} \bar{E}$$

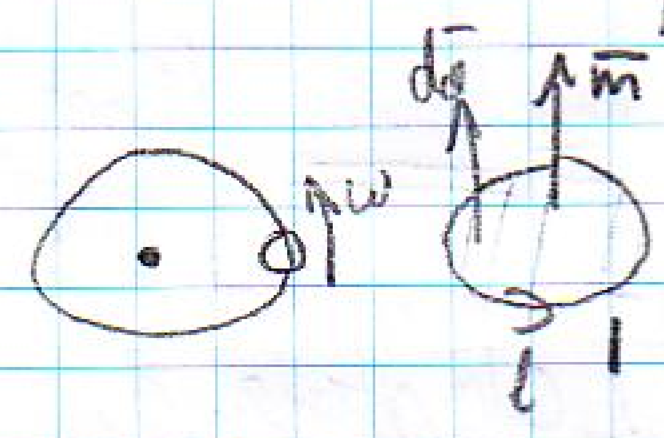
ε_r (rel. permittivitás)

$\mathcal{K} > 1$

$\varepsilon_{r\,víz} \cong 80$

• Mágneses anyagok:

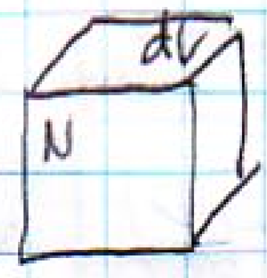
- mikroszkópikus modell



mágn. dipólus

mágn. dipólus-nyomaték: $\overline{m} = I\,d\overline{a} \sim \alpha \overline{H}$

- makroszkópikus



$\overline{m}_i \quad i=1,\dots,N$

$\overline{m}_i \sim \overline{m}_j \sim \overline{m}$

$$\overline{M} = \lim_{dV\to 0} \frac{1}{dV} \sum_{i=1}^{N} \overline{m}_i \sim K\overline{H} \left[\frac{A}{m}\right]$$

MÁGNESEZETTSÉG VEKTORA

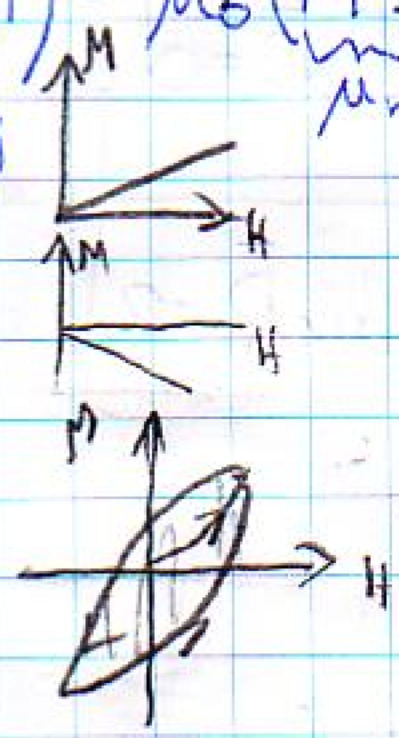
MÁGNESES SZUSZCEPTIBILITÁS

$$\overline{B} = \mu_0(\overline{H}+\overline{M}) = \mu_0(\underbrace{1+K}_{\mu_r})\overline{H}$$

paramágneses $K>0$

diamágneses $K<0$

ferromágneses anyagok

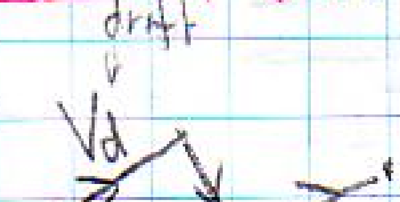


$\mu_r \gg 1$

• Fémes (vezető) anyagok

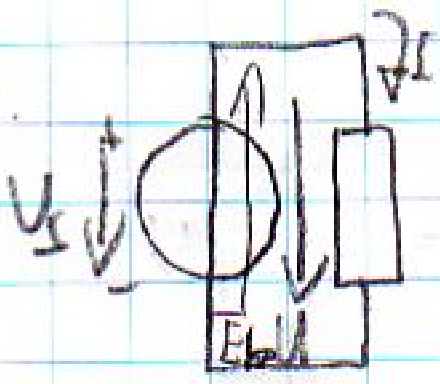
drift

$\overline{v}_d = \mu_e \overline{E}$ elekt. mozgékonyság



$\boxed{\overline{J} = \rho \overline{v}_d = \sigma \overline{E}}$ $\quad \sigma = \left[\frac{S}{m}\right]$ VEZETŐ KÉPESSÉG

Diff. Ohm t.



$\boxed{\overline{J} = \sigma(\overline{E} + \overline{E}_b)}$ DIFFERENCIÁLIS OHM-TÖRVÉNY

V Maxwell

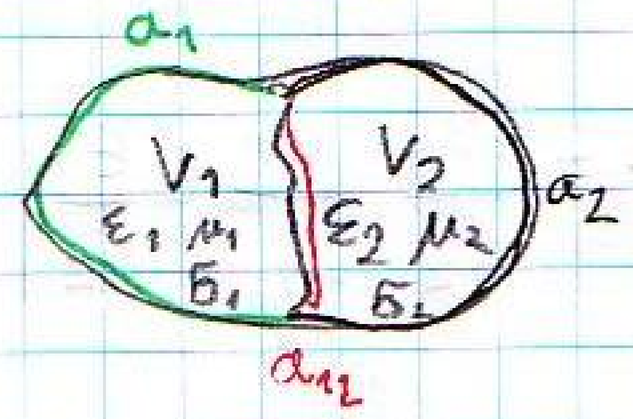
szigetelő anyagok

$$\overline{D} = \varepsilon\overline{E} = \varepsilon_0\varepsilon_r\overline{E} = \varepsilon_0\overline{E} + \overline{P}$$

$$\overline{B} = \mu\overline{H} = \mu_0\mu_r\overline{H} = \mu_0(\overline{H}+\overline{M})$$

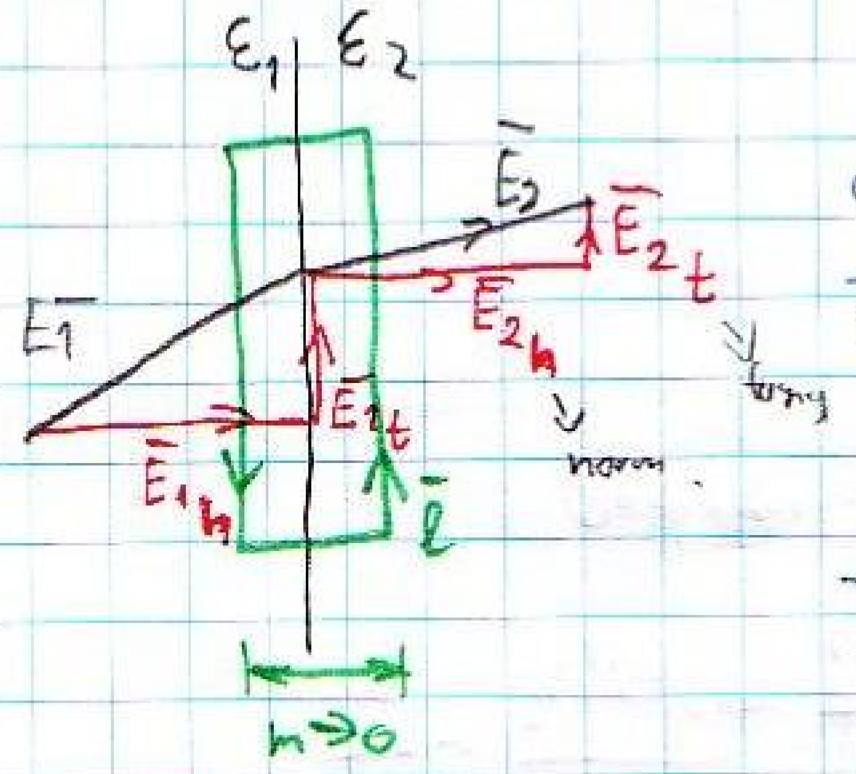
$$\overline{J} = \sigma(\overline{E} + \overline{E}_b)$$

Határfeltétel



a_1, a_2 : peremfeltételek
a_{12} : folytonossági

} Rendes Határfeltétel

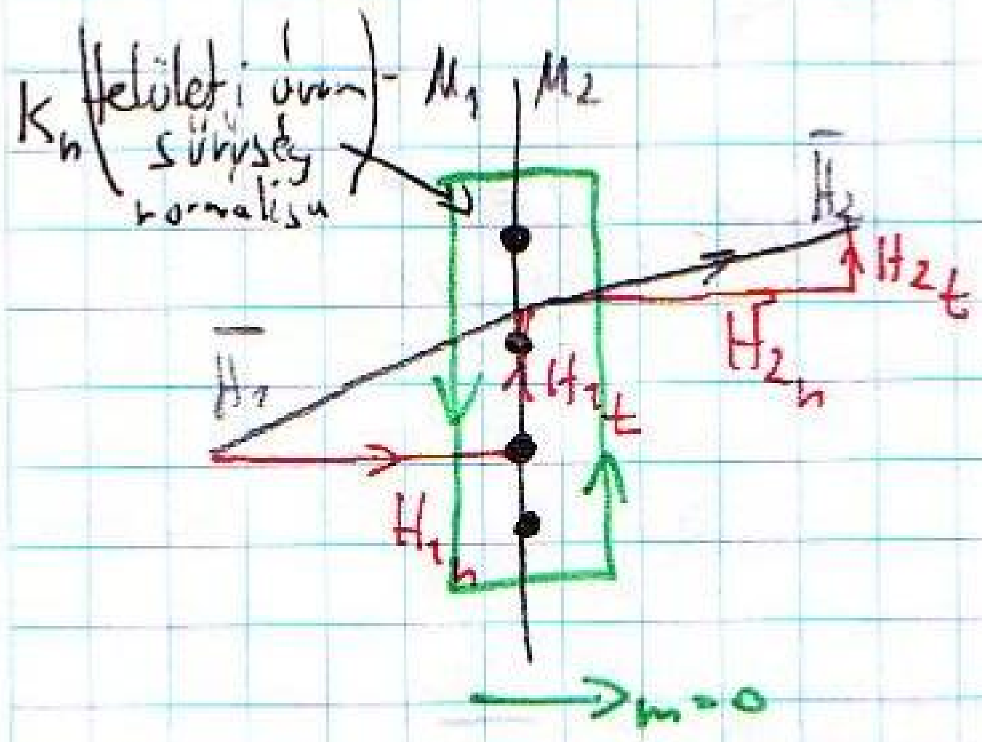


$$\oint_l \bar E d\bar l = -\int_a \frac{\partial \bar B}{\partial t} d\bar a$$

$l \longrightarrow a$

$$-E_{1_t} l + E_{2_t} l = 0$$

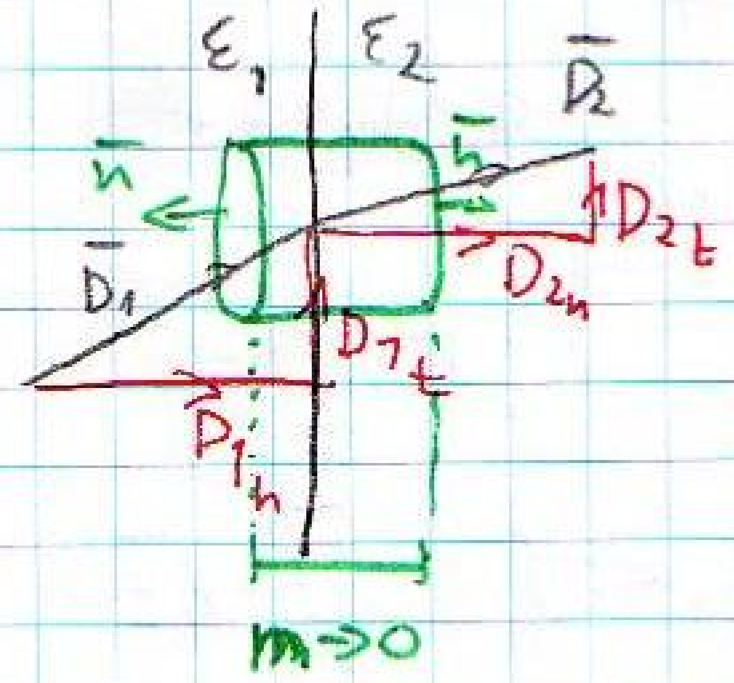
$$\boxed{E_{1_t} = E_{2_t}}$$



$$\oint_l \bar H d\bar l = \int_a \left(\bar J + \frac{\partial \bar D}{\partial t}\right) d\bar a$$

$l \longrightarrow a$

$$-H_{1_t} l + H_{2_t} l = K_n l \qquad \boxed{H_{2_t} - H_{1_t} = K_n}$$

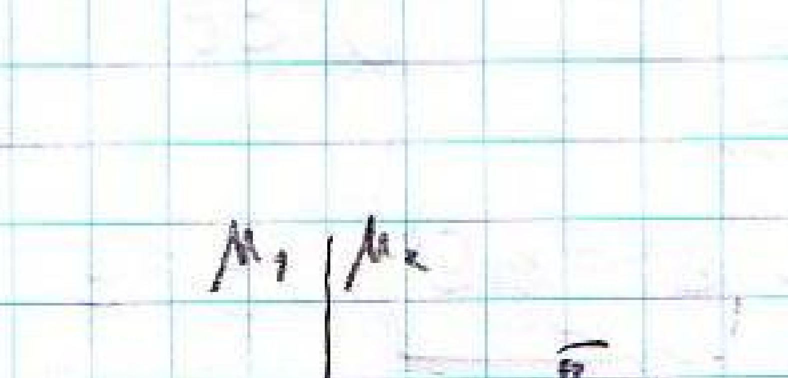


$$\oint_a \bar D d\bar a = \int_V \rho dV$$

$a \longrightarrow V$

$$-D_{1_n} a + D_{2_n} a = \sigma a$$

$$\boxed{D_{2_n} - D_{1_n} = \sigma}$$ ha $\sigma = 0 \Rightarrow D_{1_n} = D_{2_n}$

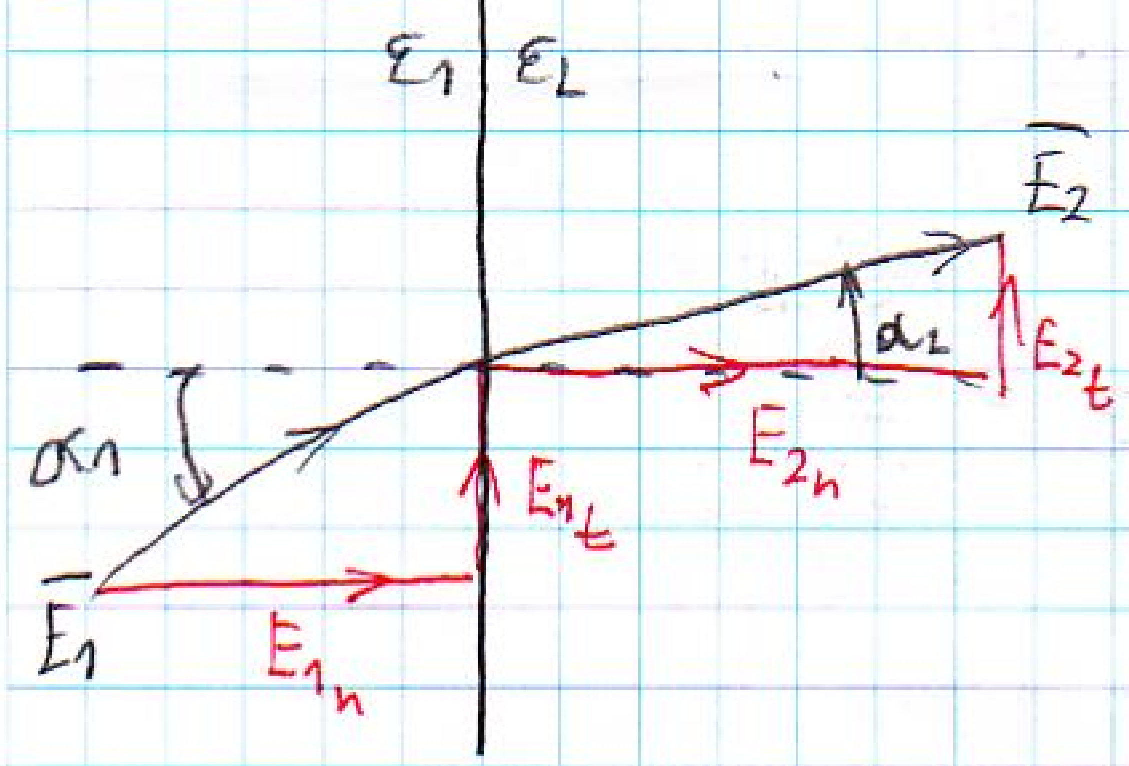


$$\oint_a \bar B d\bar a = 0$$

$$-B_{1_n} a + B_{2_n} a = 0$$

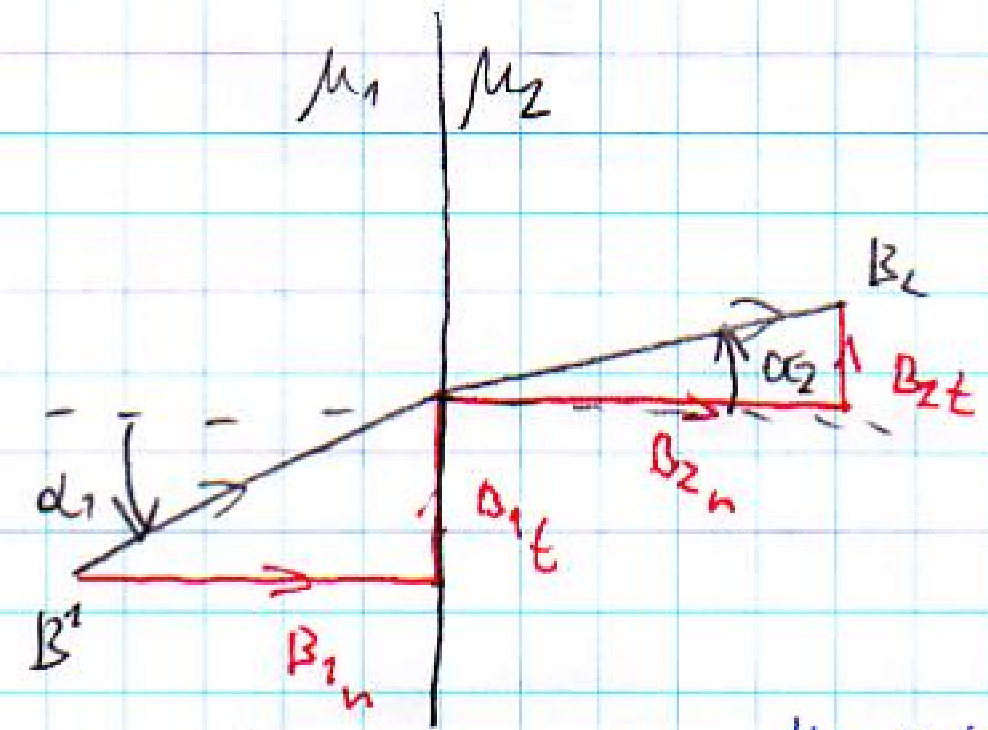
$$\boxed{B_{1_n} = B_{2_n}}$$

Törési törvény



előző old. ha $\sigma=0$

$$\frac{tg\,\alpha_1}{tg\,\alpha_2}=\frac{\cancel{E_{1t}}}{E_{1n}}\cdot\frac{E_{2n}}{\cancel{E_{2t}}}=\frac{D_{2n}}{\varepsilon_2}\cdot\frac{\varepsilon_1}{D_{1n}}=\frac{\varepsilon_1}{\varepsilon_2}$$

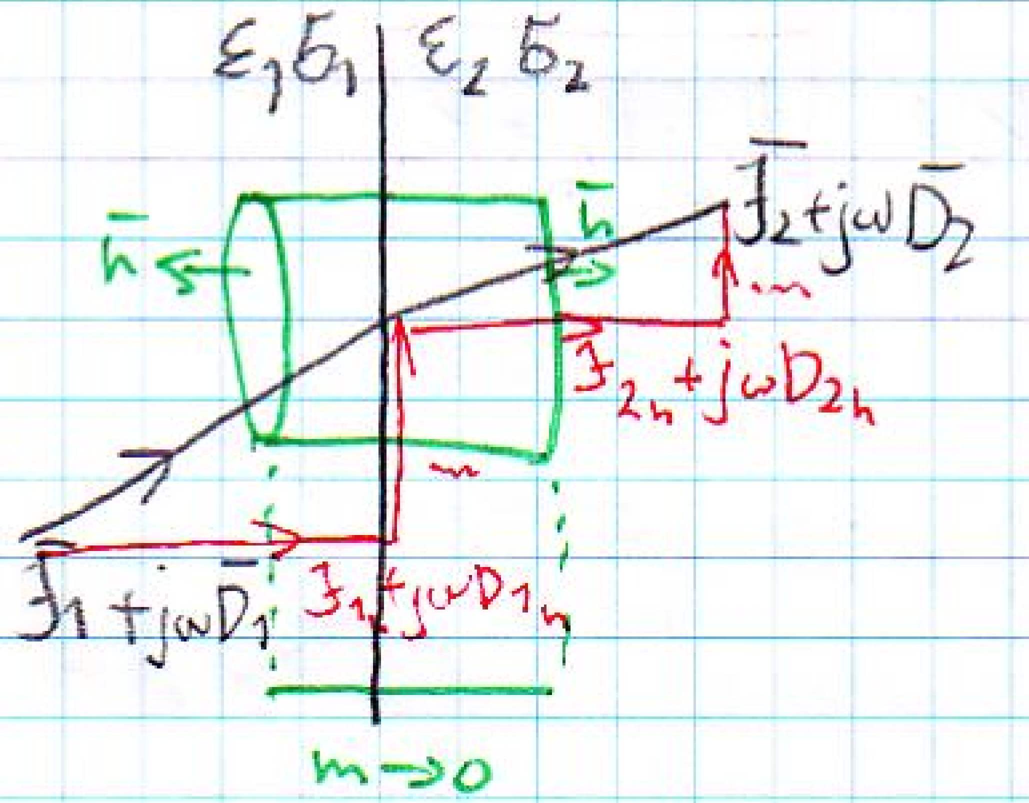


előző ha $K=0$

$$\frac{tg\,\alpha_1}{tg\,\alpha_2}=\frac{B_{1t}}{\cancel{B_{1n}}}\cdot\frac{\cancel{B_{2n}}}{B_{2t}}=\frac{\mu_1 \cancel{H_{1t}}}{\mu_2 \cancel{H_{2t}}}=\frac{\mu_1}{\mu_2}$$

$\mu_1 \sim \mu_0$ $\qquad$ $\mu_2\, tg\,\alpha_1 = \mu_1\, tg\,\alpha_2$ $\qquad$ $\mu_r \gg 1$

$\infty \approx \mu_2 \gg \mu_0$ (ferro)



$$\oint_a \left(\bar{J}+\frac{\partial\bar{D}}{\partial t}\right)d\bar{a}=0 \quad (\text{div})\ \text{rot}\,\bar{H}=\bar{J}+\frac{\partial\bar{D}}{\partial t}$$

$$0=\text{div}\left(\bar{J}+\frac{\partial\bar{D}}{\partial t}\right)$$

$$\oint_a \left(\bar{J}+j\omega\bar{D}\right)d\bar{a}=0$$

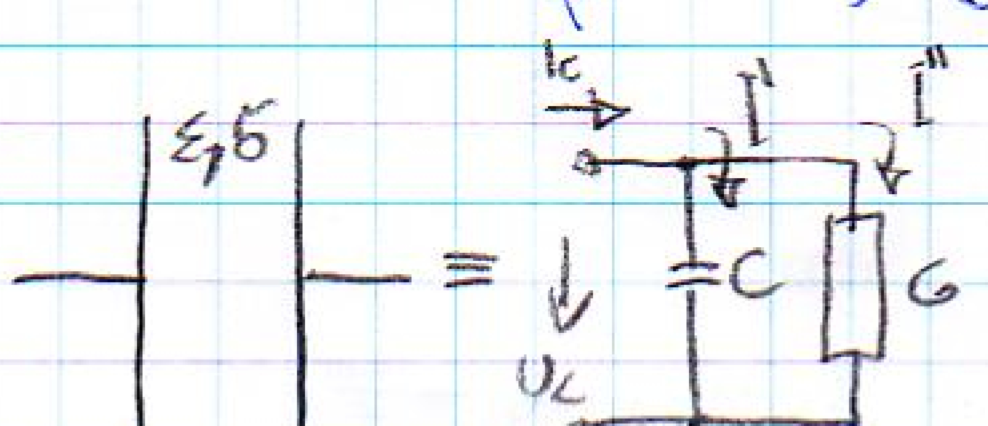
$$-(J_{1n}+j\omega D_{1n})a+(J_{2n}+j\omega D_{2n})a=0 \qquad \bar{J}=\sigma\bar{E}$$

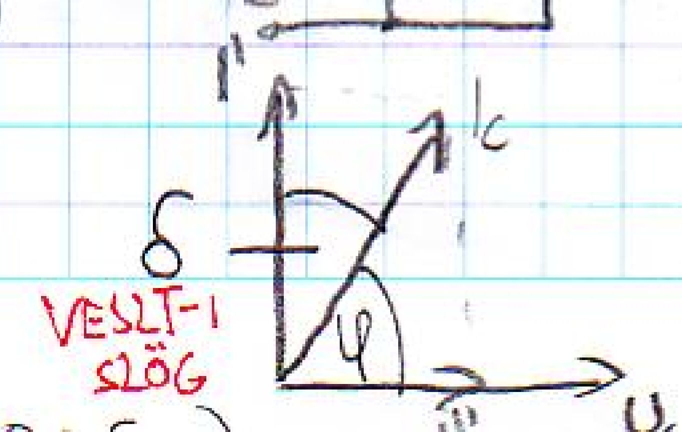
$$\bar{D}=\varepsilon\bar{E}$$

$$+(\sigma_1+j\omega\varepsilon_1)E_{1n}=(\sigma_2+j\omega\varepsilon_2)E_{2n}$$

$$(j\omega\varepsilon+\sigma)=j\omega\varepsilon\left(1+\frac{\sigma}{j\omega\varepsilon}\right)=j\omega\underbrace{(\varepsilon'-j\varepsilon'')}_{\varepsilon_{\text{komplex p.}}\text{(permi)}} \qquad \varepsilon'=\varepsilon$$

$$\varepsilon''=\frac{\sigma}{\omega\varepsilon}$$



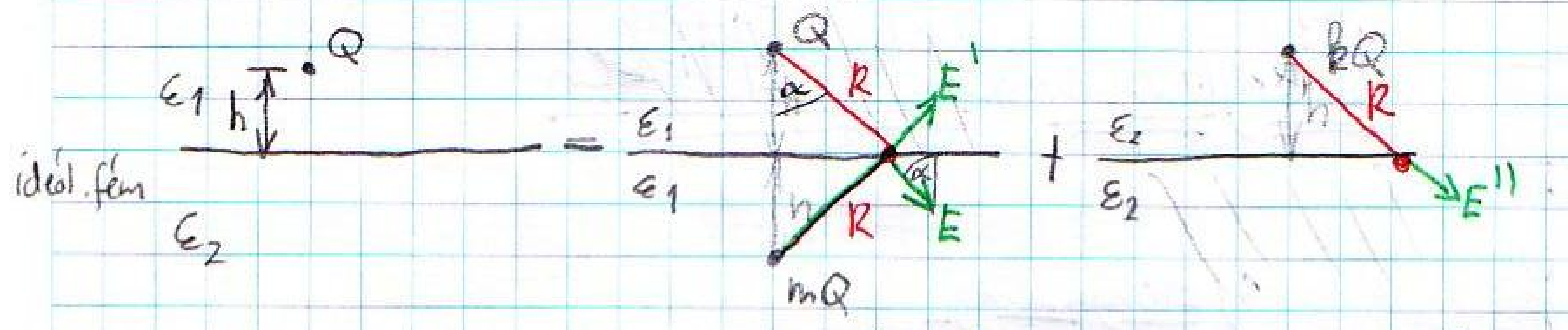


$$tg\,\delta=\frac{I''}{I'}=\frac{U_c G}{\frac{U_c}{1/j\omega C}}=\frac{G}{\omega C}\approx\frac{\sigma}{\omega\varepsilon}$$

$$\boxed{\varepsilon_{\text{kompl.p.}}=\varepsilon(1-j\,tg\,\delta)}$$

$U_c \Rightarrow I' \Rightarrow I'' \Rightarrow I_c$

Folyt.-i feltételek $\Rightarrow$ Dielektromos tükrözés! (vill tükrözés)



$$E_{1t}=E_{2t}: \quad \frac{Q}{4\pi\varepsilon_1}\frac{1}{R^2}\sin\alpha + \frac{mQ\sin\alpha}{4\pi\varepsilon_1 R^2} = \frac{kQ}{4\pi\varepsilon_2}\frac{1}{R^2}\sin\alpha \rightarrow$$

$$\Rightarrow \frac{1}{\varepsilon_1}(1+m)=\frac{k}{\varepsilon_2}$$

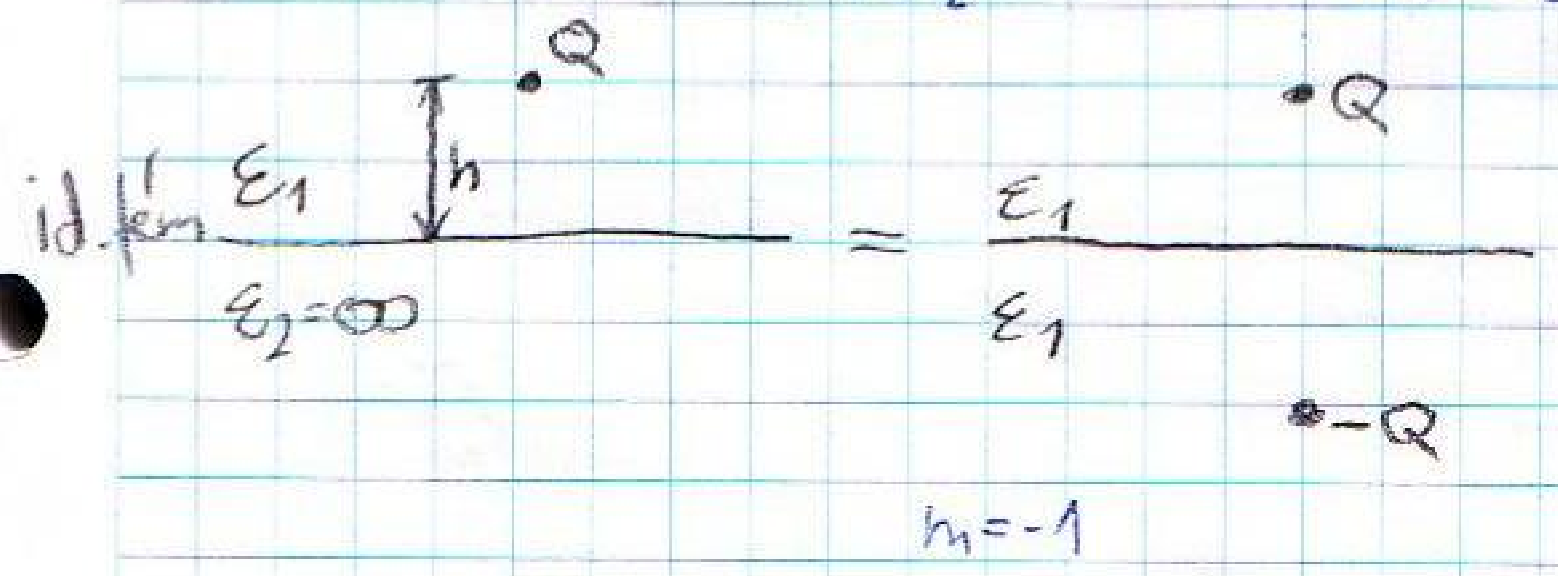
$$D_{1n}=D_{2n}: \quad \frac{Q}{4\pi R^2}\cos\alpha - \frac{mQ}{4\pi R^2}\cos\alpha = \frac{kQ}{4\pi R^2}\cos\alpha \rightarrow$$

$$\rightarrow 1-m=k$$

$$\Rightarrow$$

$$\varepsilon_2(1+m)=\varepsilon_1(1-m)$$

$$\Rightarrow \quad m=\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2} \qquad k=\frac{2\varepsilon_2}{\varepsilon_1+\varepsilon_2}=1-m$$



$m=-1$