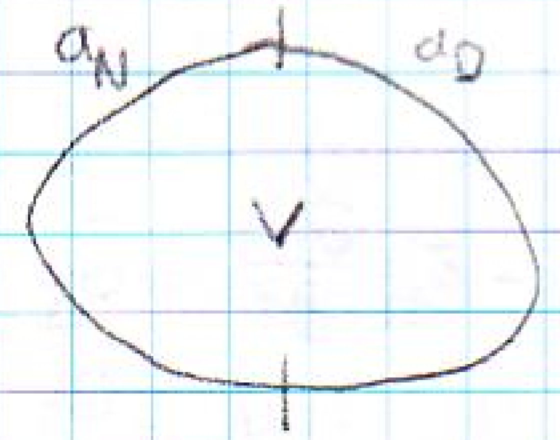
2007.03.20. kedd IX. Előadás (6. hét)

Stacionárius mágneses tér

$\mathrm{rot}\,\bar{H} = \bar{J}$

$\mathrm{div}\,\bar{B} = 0$

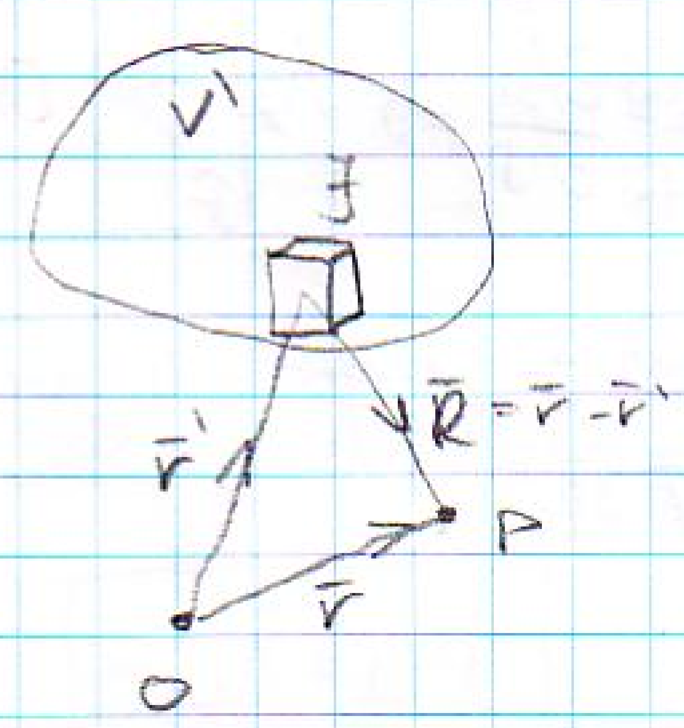
$\bar{B} = \mu \bar{H}$



$\bar{B} = \mathrm{rot}\,\bar{A}$

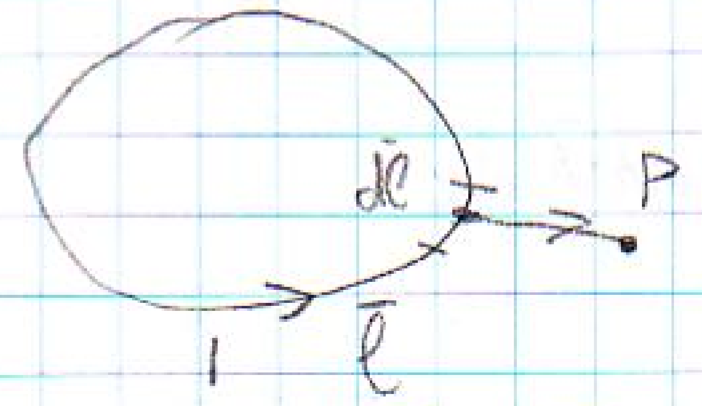
$\mathrm{div}\,\bar{A} = 0$ (Coulomb felt.)

$-\Delta \bar{A} = \mu \bar{J}$



$$\bar{A}(\bar{r}) = \frac{\mu}{4\pi} \int_{V'} \frac{\bar{J}(\bar{r}')}{|\bar{R}|} dV'$$

vonalas vez.



$$A(\bar{r}) = \frac{\mu I}{4\pi} \oint_l \frac{d\bar{l}}{|\bar{R}|}$$

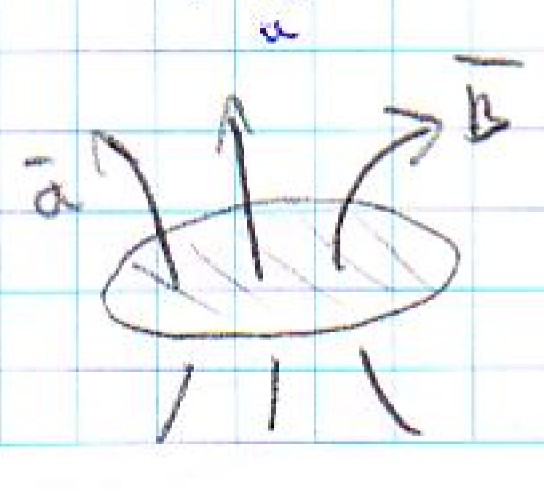
a_D $\quad \bar{A}\Big|_{a_D} = \bar{A}_D \quad \bar{A} = \text{áll} \Rightarrow \bar{B}$

a_N $\quad \Longrightarrow \quad \frac{\partial A}{\partial n}\Big|_{a_N} = \text{adott}$

Következmény:

1) fluxus

$$\Phi = \int_a \bar{B} d\bar{a} = \int_a \mathrm{rot}\,\bar{A} d\bar{a} \overset{\text{Stokes}}{=} \oint_l \bar{A} d\bar{l}$$



2) Energia:

$$W_m = \frac{1}{2}\int_V \bar{H}\bar{B}\,dv = \frac{1}{2}\int_V \bar{H}\,\mathrm{rot}\,\bar{A}\,dv = \frac{1}{2}\int_V \Big[\bar{A}\underbrace{\mathrm{rot}\,\bar{H}}_{\bar{J}} - \underbrace{\mathrm{div}(\bar{H}\times\bar{A})}_{G.O.}\Big]\,dv$$

$$\mathrm{div}(\bar{H}\times\bar{A}) = \bar{A}\,\mathrm{rot}\,\bar{H} - \bar{H}\,\mathrm{rot}\,\bar{A}$$

$$W_m = \frac{1}{2}\int_V \bar{A}\bar{J}\,dv - \frac{1}{2}\oint_a (\bar{H}\times\bar{A})\,d\bar{a}$$

a) nyitott a tér

$$a \Rightarrow \infty \qquad \lim_{r\to\infty} \bar{H}(\bar{r}) \approx \frac{1}{r}$$

$$\lim_{r\to\infty} \bar{A}(\bar{r}) \approx \frac{1}{r^2} \qquad \lim_{r\to\infty} \oint_a (\bar{H}\times\bar{A})\,r^2\,d\bar{a} \approx 0$$

$$W_m = \frac{1}{2}\int_V \bar{J}\bar{A}\,dv$$

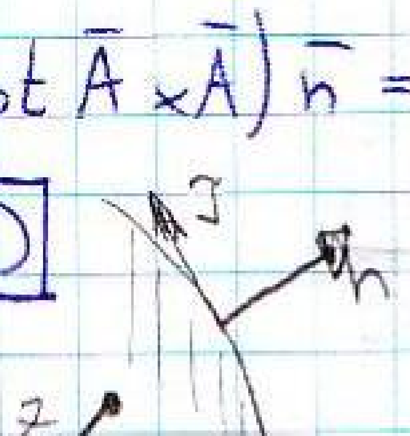
b) zárt a tér

$$a = a_D \cup a_N$$

$$W_m = \frac{1}{2}\int_V \bar{J}\bar{A}\,dv - \frac{1}{2}\oint_{\substack{a \\ a_D \cup a_N}} \left(\frac{\mathrm{rot}\,\bar{A}}{\mu}\times\bar{A}\right)\bar{n}\,da$$

$$(\mathrm{rot}\,\bar{A}\times\bar{A})\,\bar{n} = \mathrm{rot}\,\bar{A}\cdot\overbrace{(\bar{A}\times\bar{n})}^{A_J} = \overbrace{(\bar{n}\times\mathrm{rot}\,\bar{A})}^{H_J}\cdot\bar{A}$$

$\boxed{2D}$



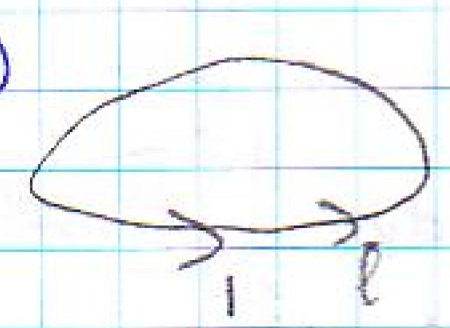
$$\bar{A} = A_z\bar{e}_z$$

$$\bar{B} = \bar{B}_J + \bar{B}_n$$

$$W_m = \frac{1}{2}\int_V \bar{J}\bar{A}\,dv - \frac{1}{2}\int_{a_D} \mathrm{rot}\,\bar{A}\cdot\underbrace{(\bar{A}\times\bar{n})}_{A_D = A_J}\,da - \frac{1}{2}\int_{a_N} \underbrace{(\bar{n}\times\mathrm{rot}\,\bar{A})}_{H_J}\bar{A}\,da$$

Le Color

3) Biot - Savart törvény: (vonalas vezetőre)

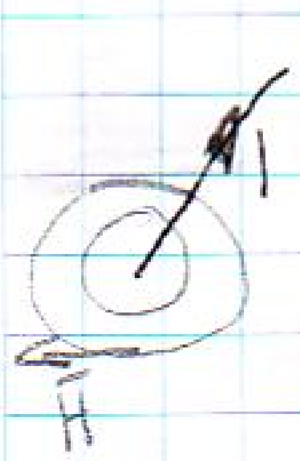


$$\bar{H} = \frac{\bar{B}}{\mu} = \frac{\mathrm{rot}\,\bar{A}}{\mu} = \mathrm{rot}\left(\frac{1}{4\pi}\oint_l \frac{d\bar{l}}{r}\right) = \frac{1}{4\pi}\oint_l \mathrm{rot}\,\frac{d\bar{l}}{r} \ominus$$

$$\mathrm{rot}(a\,\bar{u}) = \mathrm{grad}\, a \times \bar{u} + a \cdot \mathrm{rot}\,\bar{u} \qquad a = \frac{1}{r}, \quad \bar{u} = d\bar{l}$$

$$\ominus \frac{1}{4\pi}\oint_l \left(\mathrm{grad}\,\frac{1}{r} \times d\bar{l} + \frac{1}{r}\,\underbrace{\mathrm{rot}\, d\bar{l}}_{0}\right) = \frac{1}{4\pi}\oint_l -\frac{\bar{r}_0}{r^2} \times d\bar{l} = \frac{1}{4\pi}\oint_l \frac{d\bar{l} \times \bar{r}_0}{r^2} = \frac{1}{4\pi}\oint_l \frac{d\bar{l} \times \bar{r}}{r^3}$$

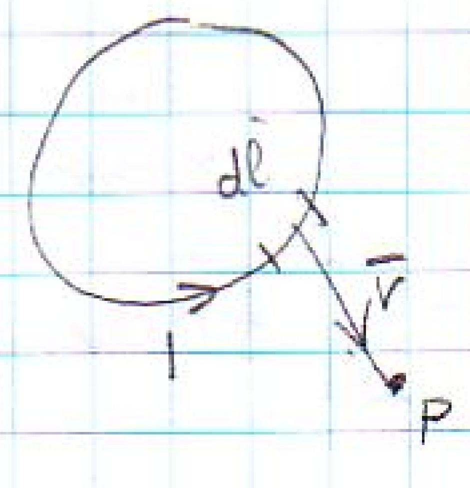
Megj.! gerj-i törv. vs. Bios -S. törv



$$\oint_l \bar{H}\, d\bar{l} = \sum I$$

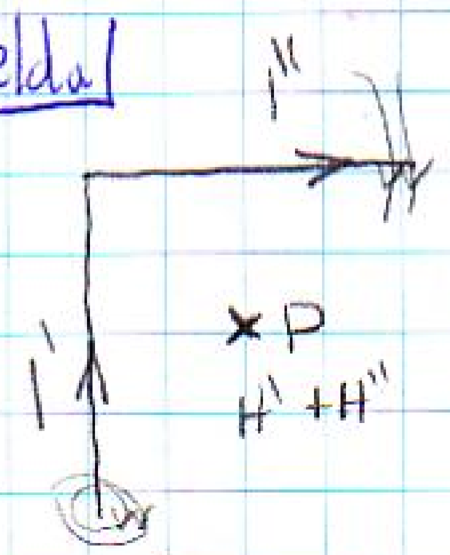
$\bar{H} \parallel d\bar{l}$
mágn erővonal

a vezető ismert, de az erővonalkép NEM

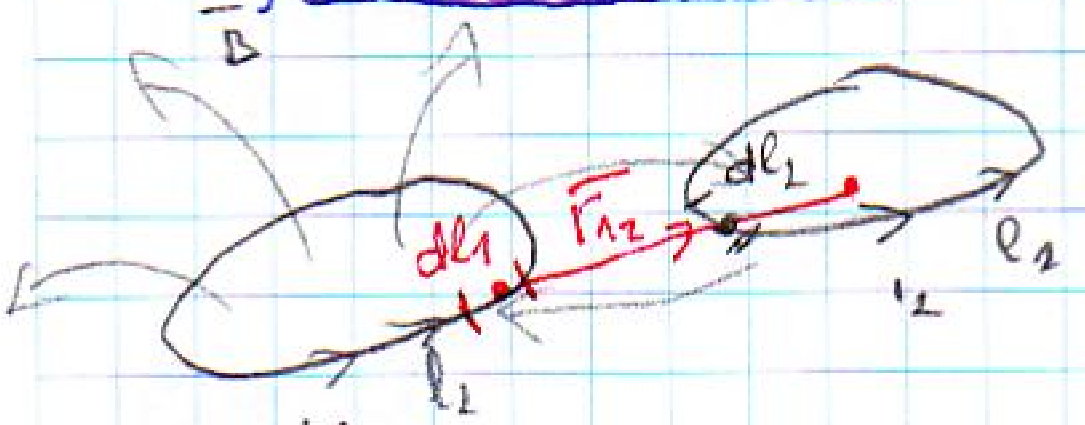


példa!

Csak B-S



4) kölcsönös indukció



$$I_1 \Rightarrow \bar{A}_1 = \frac{I_1 \mu}{4\pi}\oint_{l_1} \frac{d\bar{l}_1}{r_{12}}$$

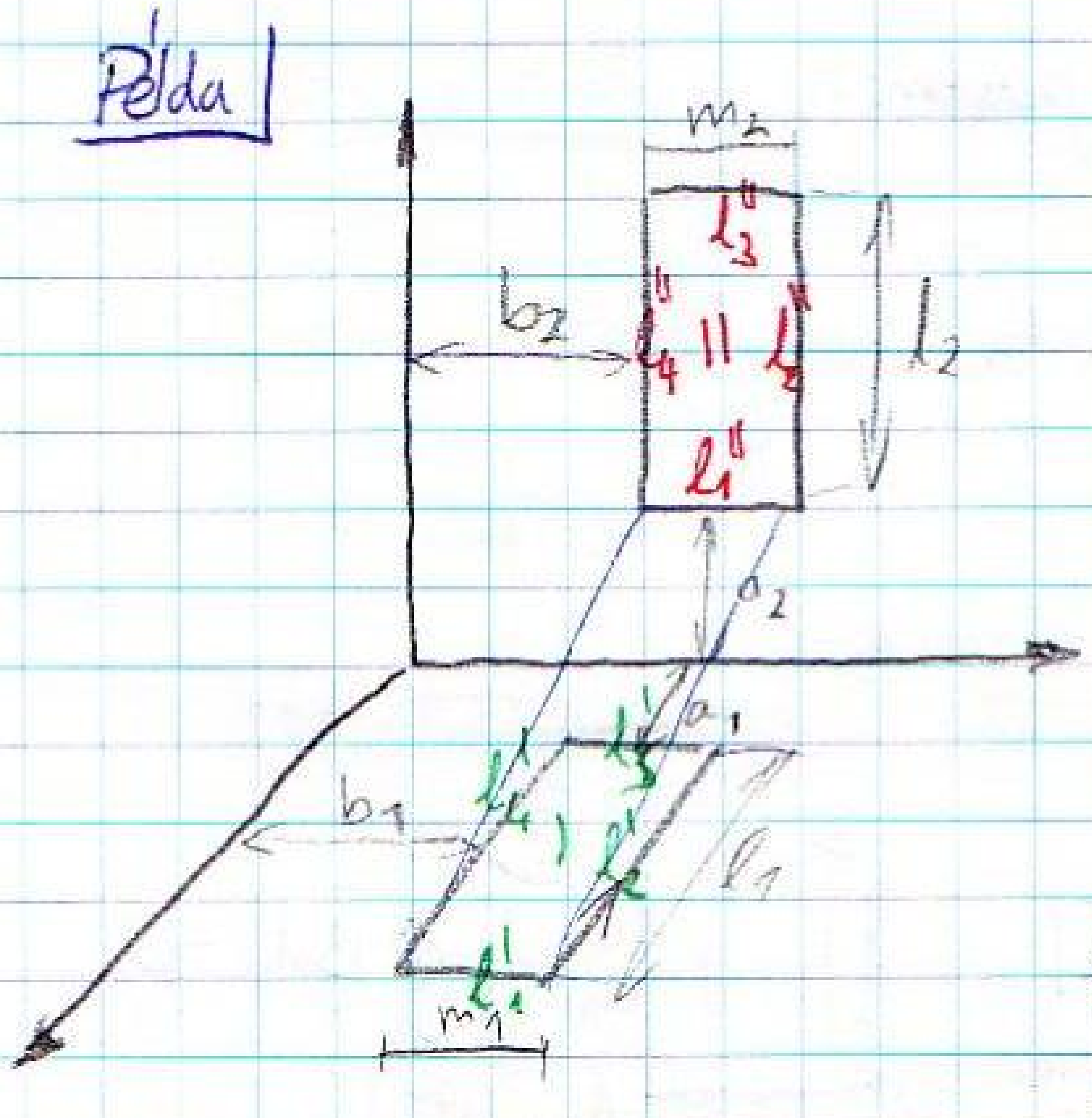
$$\Phi_{12} = \oint_{l_1} \bar{A}_1\, d\bar{l}_2 = \frac{\mu I_1}{4\pi}\oint_{l_2}\oint_{l_1} \frac{d\bar{l}_1 d\bar{l}_2}{r_{12}}$$

$$L_{12} = \frac{\Phi_{12}}{I_1}\bigg|_{I_2=0}$$

$$\boxed{L_{12} = \frac{\mu}{4\pi}\oint_{l_1}\oint_{l_2} \frac{d\bar{l}_2 \cdot d\bar{l}_1}{r_{12}}}$$

NEUMANN FORMULA

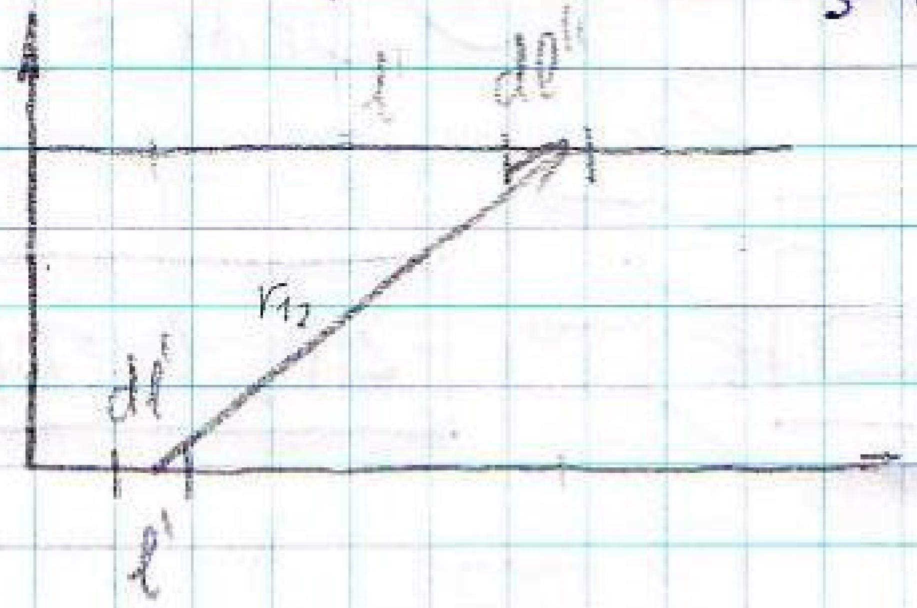
Példa



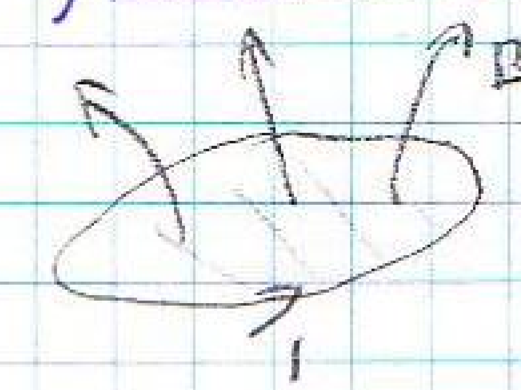
$L_{12} = ?$

$$INT = I \iint_{l_1 l_2} \frac{d\bar{l}_1 \, d\bar{l}_2}{r_{12}}$$

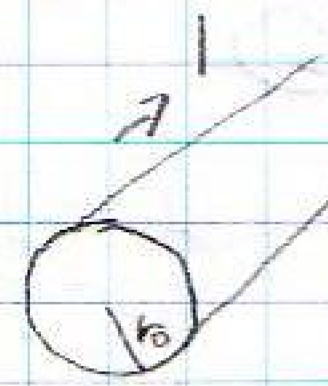
$$L_{12} = \frac{\mu}{4\pi}\left(I_{l_1' l_1''} + I_{l_1' l_3''} + I_{l_3' l_1''} + I_{l_3' l_3''} \right)$$

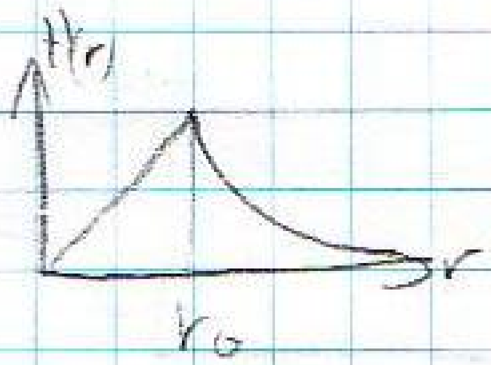


5) Öninduktciós együttható:



$$\frac{\Phi}{I} = L_{ön}\ (\text{külső})$$

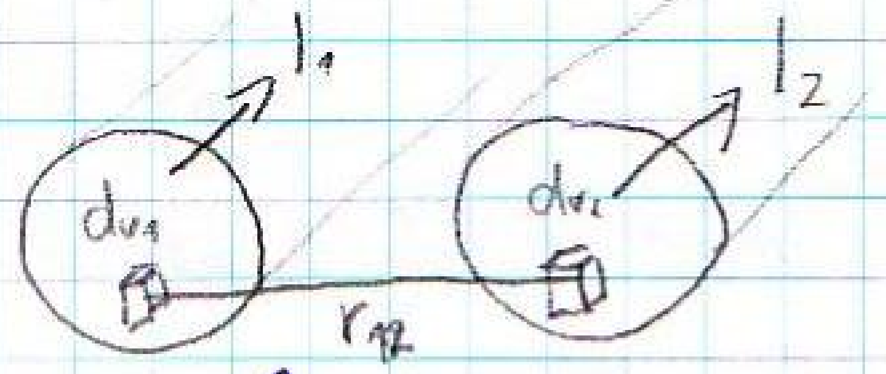




$$W_m = \frac{1}{2} \int_{V_{belső}} \bar{H}\bar{B}\, dv = \frac{1}{2} L_{ön_b} I^2$$

külső belső

$$L_{ön} = L_{ön_k} + L_{ön_b}$$



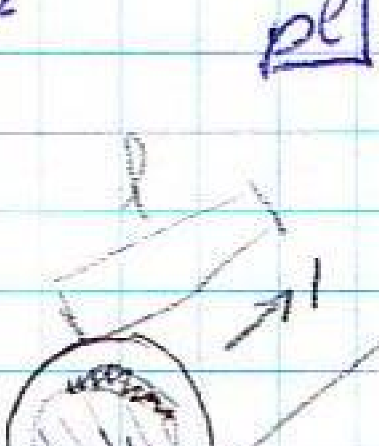
$$W_{m_2} = \int_{V_2} \bar{J}_2 \bar{A}_2 \, dv_2 = \int_{V_2} \frac{\mu}{4\pi} \int_{V_1} \frac{\bar{J}_1 \bar{J}_2}{r_{12}} dv_1 dv_2$$

$$\bar{A}_2 = \frac{\mu}{4\pi} \int_{V_1} \frac{\bar{J}_1}{r_{12}} dv_1$$

$$L_{b_2} = \frac{2}{I_1^2} W_{m_2}$$

$$\bar{I}_1 = -\bar{I}_2 = I$$

pl



$$\mu = \text{áll} \quad W_m = \frac{1}{2} \int H^2 \mu \, dv = \frac{\mu}{2} \int_0^{r_0} \left(\frac{I r}{2\pi r_0^2} \right)^2 2r\pi l \, dr =$$

$$dv = 2r\pi \, dr \cdot l$$

$$= \frac{\mu}{2} \frac{I^2 l}{2\pi r_0^4} \int_0^{r_0} r^3 dr =$$

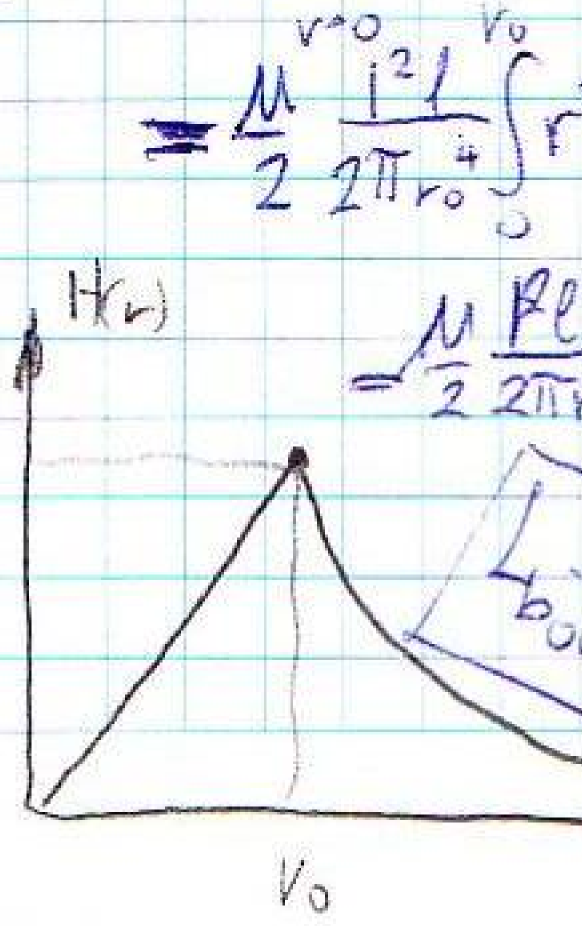
$$= \frac{\mu}{2} \frac{I^2 l}{2\pi r_0^4} \frac{r_0^4}{4} = \frac{1}{2} L_{b_{ön}} I^2$$

$L_{b_{ön}} = ?$

$$\oint \bar{H} d\bar{l} = \Sigma I$$

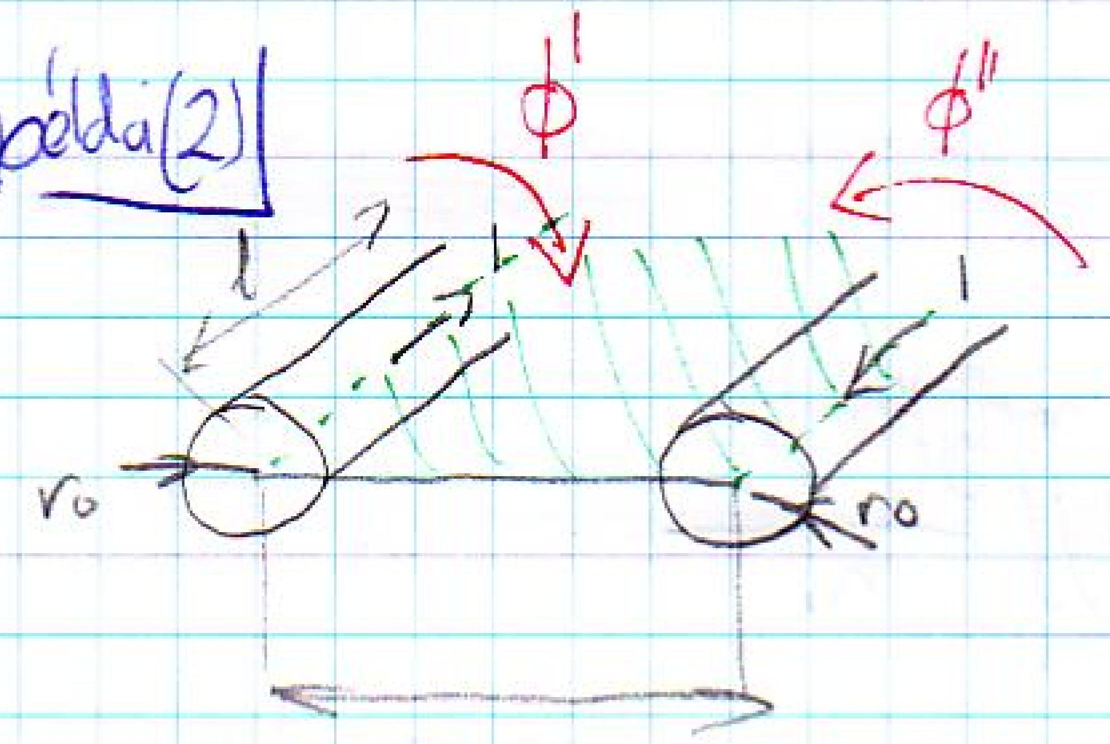
$$H 2r\pi = \frac{I}{r_0^2 \pi} r^2 \pi$$

$$H = \frac{I}{2\pi r_0^2} r$$

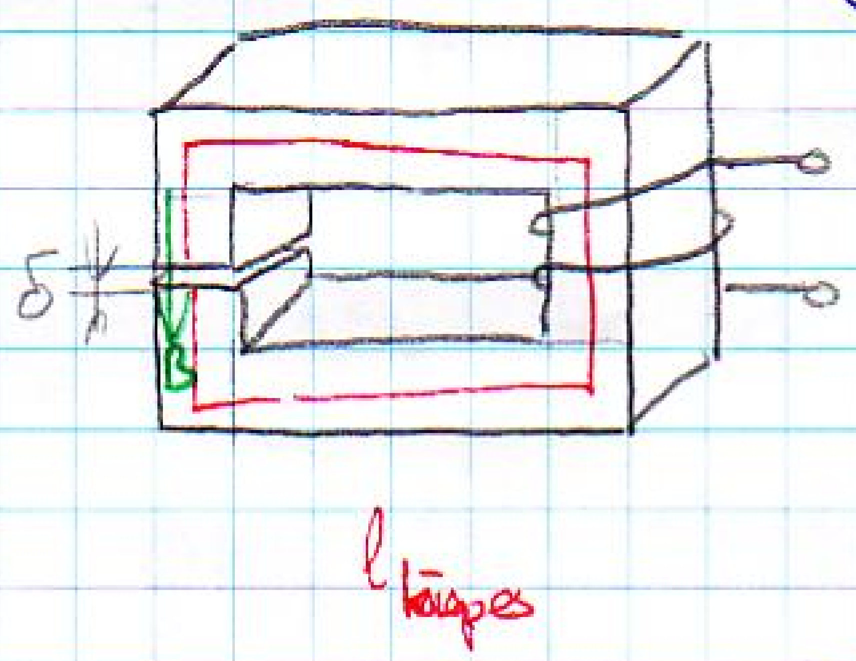


$$\boxed{L_{b_{ön}} = \frac{\mu l}{8\pi}}$$

példa(2)

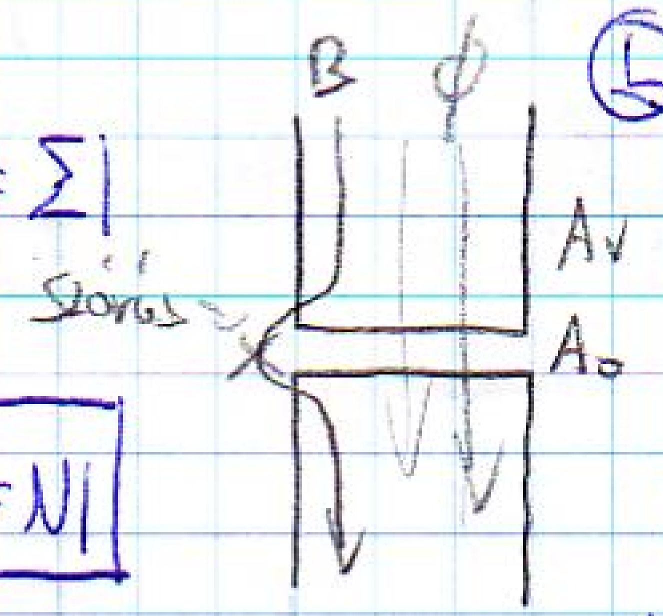


6) Mágneses körök:



ⓐ $\oint_l \bar{H} d\bar{l} = \Sigma I$

$$\boxed{\Sigma H_k \cdot l_k = NI}$$



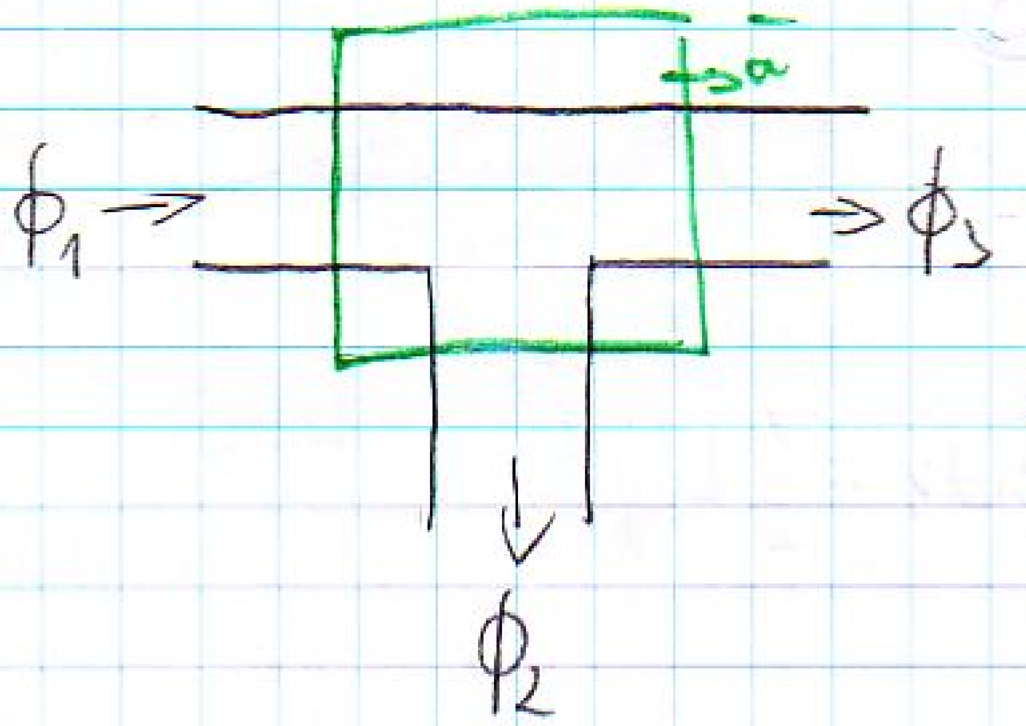
ⓑ $\oint_a \bar{B} d\bar{a} = 0$

$-B_V A_V + A_0 B_0 = 0$

szórás nincs $A_V = A_0$

$\Downarrow$

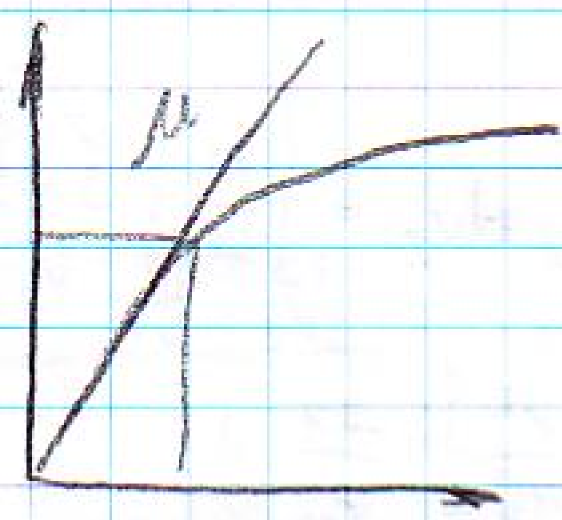
$B_V = B_0$



$\phi_1 = \phi_2 + \phi_3$

$B_1 A_1 = B_2 A_2 + B_3 A_3$

ⓒ



C1) $\mu = \mu_0 \mu_r$

C2) karakterisztika

Mágneses ellenállás (k: közepes)

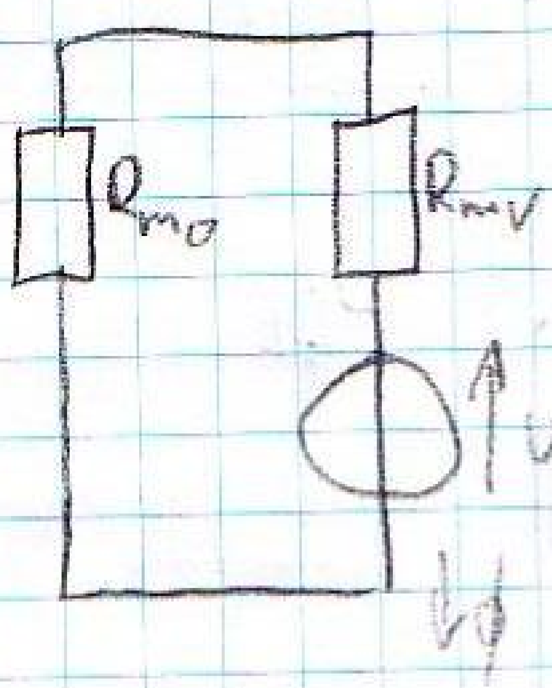
$$\Phi_k = B_k \cdot a_k = \mu H_k \cdot a_k = \mu \frac{NI}{l_k} a_k = \frac{\mu a_k}{l_k} \cdot U_m = \frac{U_m}{R_m}$$

$I \Longleftrightarrow \Phi$

$U \Longleftrightarrow U_m$

$R \Longleftrightarrow R_m$

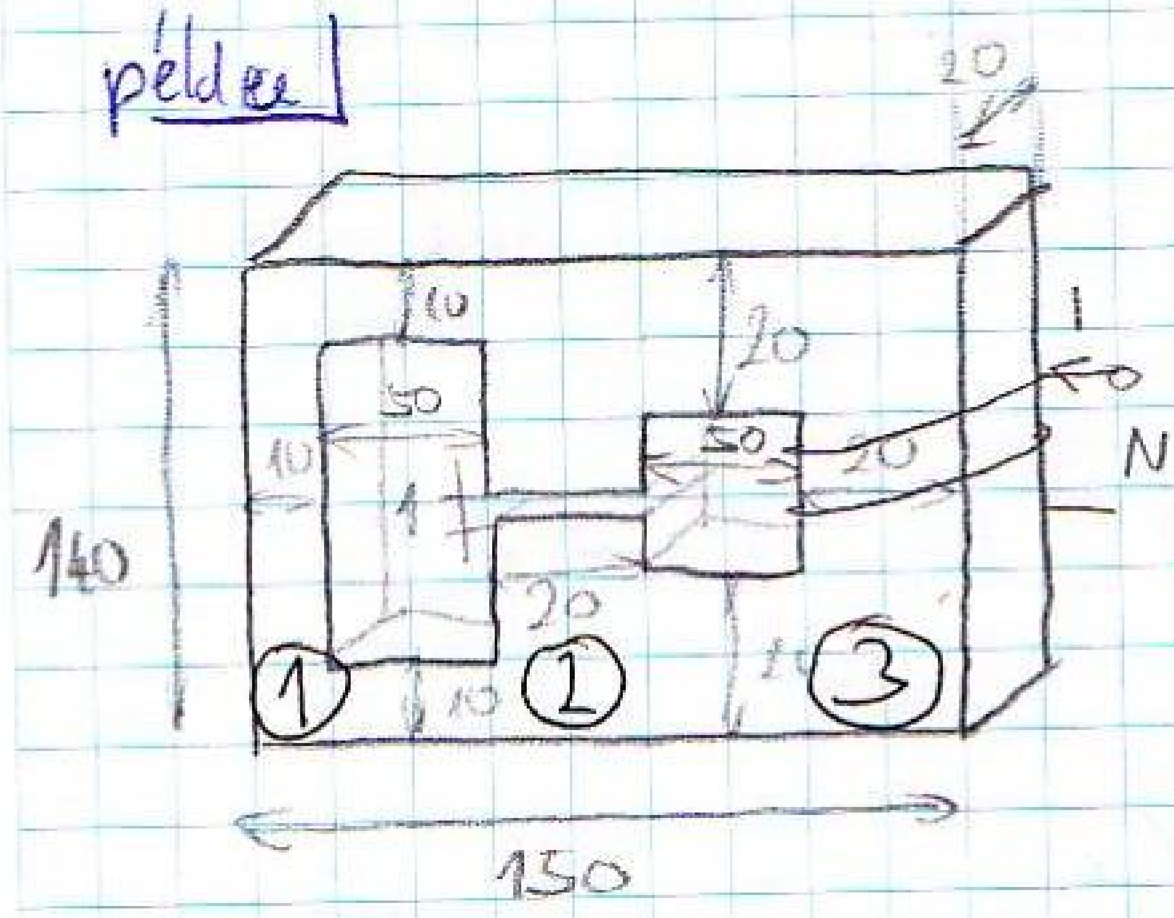
$$\Rightarrow \boxed{R_m = \frac{l_k}{\mu a_k}}$$

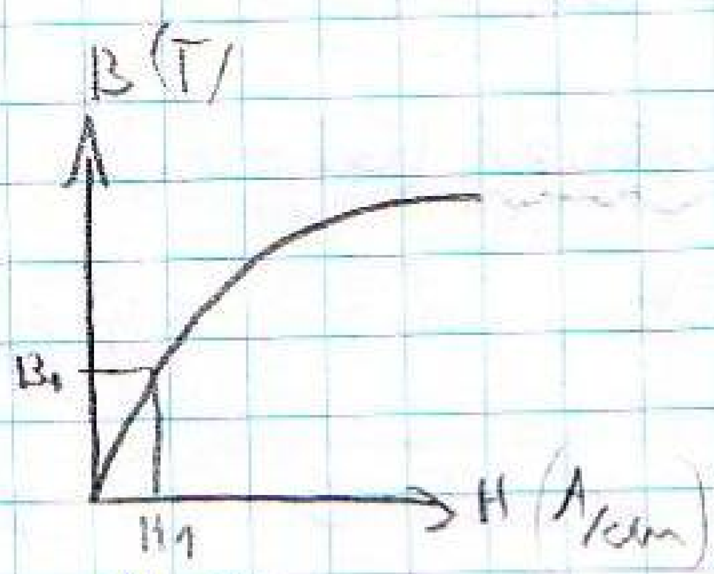


$$\phi_0 = \frac{U_m}{R_{m0} + R_{mV}}$$

légrés vas

példa





$NI = ?$

$B_0 = 0{,}8\,T$

$\phi_1 = \phi_0$

$A_1 = A_0 \Rightarrow B_0 = B_1 = 0{,}8\,T \overset{\text{leolv.}}{\Rightarrow} H_1 = 2{,}7\ A/cm$

$$H_0 = \frac{B_0}{\mu_0} = \frac{0{,}8}{4\pi \cdot 10^{-7}} = 0{,}637 \cdot 10^6\ \frac{A}{m} = 6370\ \frac{A}{cm}$$

$H_1 l_1 + H_0 l_0 - H_2 l_2 = 0 \Rightarrow H_2 \Rightarrow B_2$

$\phi_3 = \phi_1 + \phi_2 \Rightarrow \phi_3 \Rightarrow B_3 \Rightarrow H_3$

$H_2 l_2 + H_3 l_2 = NI \Rightarrow$ ✓