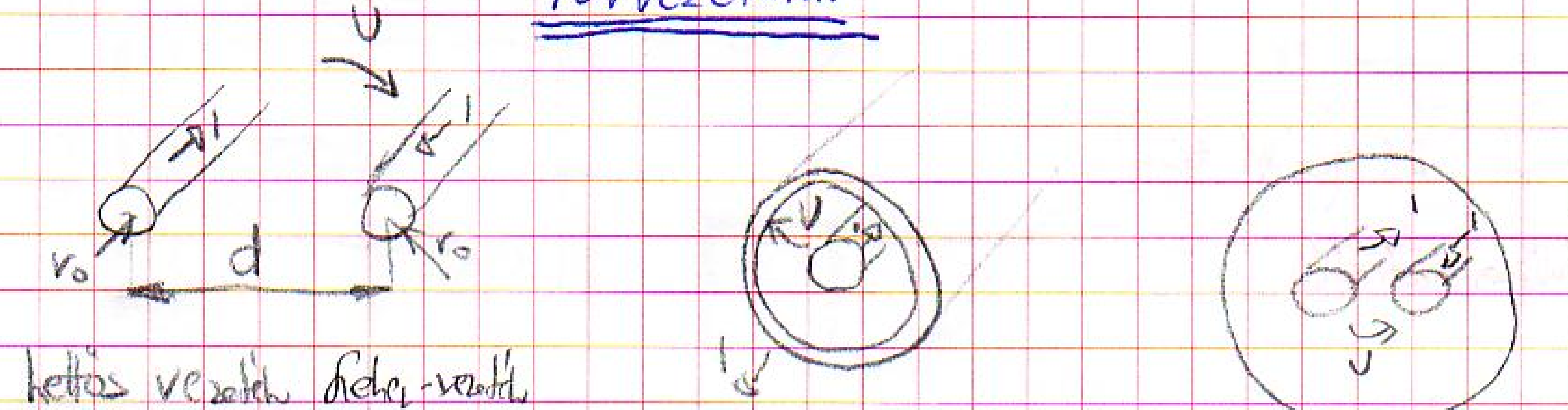
2007.03.28. szerda   XI. Előadás (7. hét)

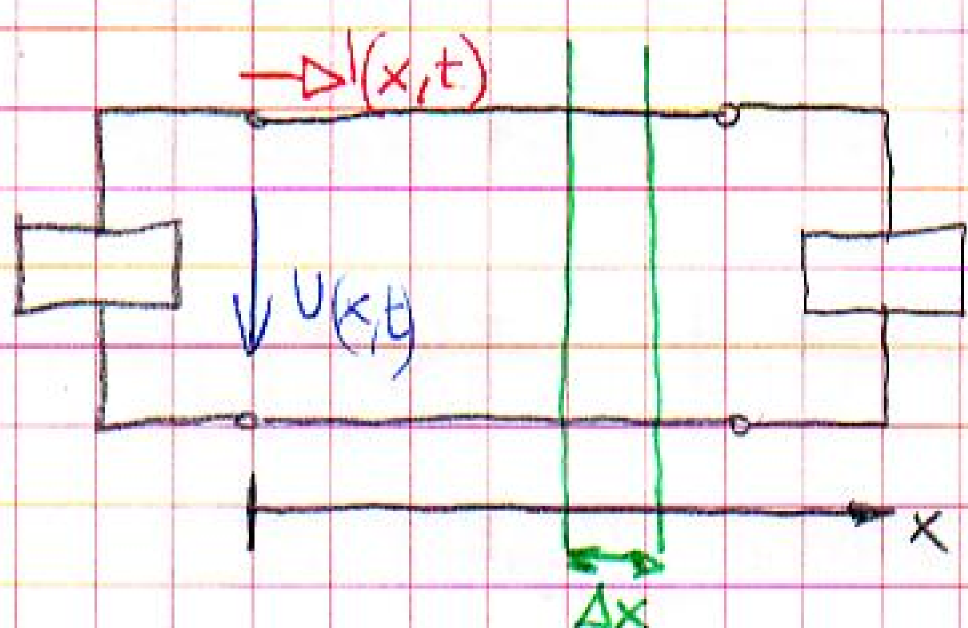
Távvezetékek



kettős vezeték   koax-vezeték

$\Phi = 0$
árnyékolt vez.

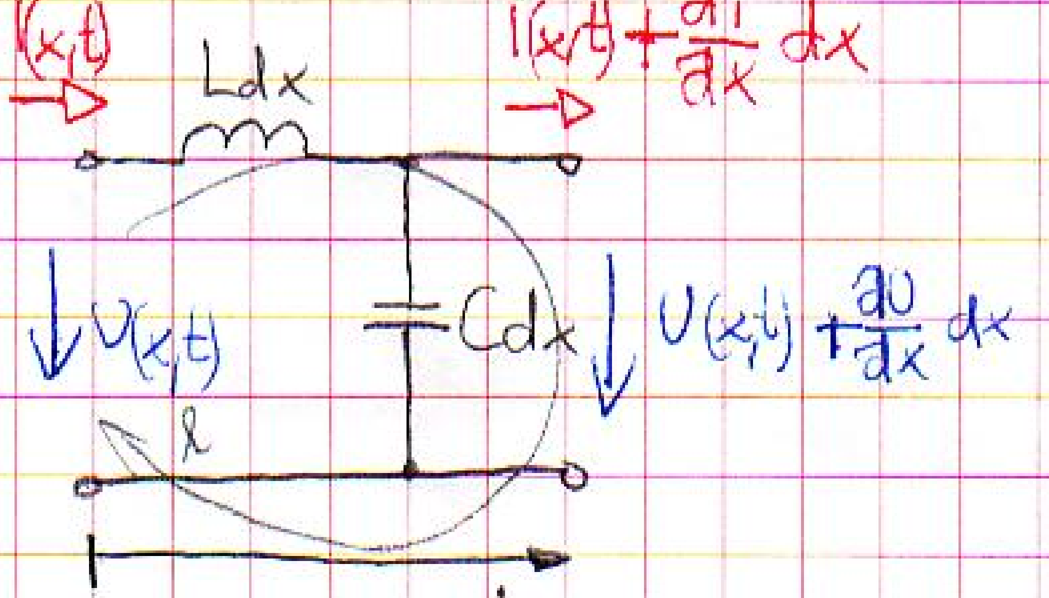
szalagvezeték   koplanáris távvez



dx

$$\oint_l \bar{E}\,d\bar{l} = -\frac{\partial}{\partial t}\int_a \bar{E}\,d\bar{a}$$

$$\oint_l \bar{H}\,d\bar{l} = \int_a \underbrace{\left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right)d\bar{a}}_{\text{folytonossági egyenlet}}$$



$$q = C\,dx\,u(x,t)$$

$$-\frac{dq}{dt} = -C\,dx\,\frac{\partial u(x,t)}{\partial t} = \cancel{i(x,t)} + \frac{\partial i}{\partial x}dx - \cancel{i(x,t)}$$

$$\cancel{u(x,t)} + \frac{\partial u}{\partial x}dx - \cancel{u(x,t)} = -\frac{\partial \phi}{\partial t} = -\frac{\partial}{\partial t} L\,dx\, i(x,t) = -L\,dx\,\frac{di}{dt}$$

$$\boxed{-\frac{\partial u(x,t)}{\partial x} = L\,\frac{\partial i(x,t)}{\partial t}} \qquad \boxed{-\frac{\partial i(x,t)}{\partial x} = C\,\frac{\partial u(x,t)}{\partial t}}$$

$$\boxed{-\frac{\partial^2 u}{dx^2} = L\frac{\partial}{\partial t}\frac{di}{dx} = -LC\frac{\partial^2 u}{\partial t^2}} \qquad \boxed{\frac{\partial^2 i(x,t)}{\partial x^2} = LC\frac{\partial^2 i(x,t)}{\partial t^2}}$$

1D hullámegyenlet megoldása $\quad f\left(t \mp \frac{x}{v}\right)$ haladó hullám $\pm x$

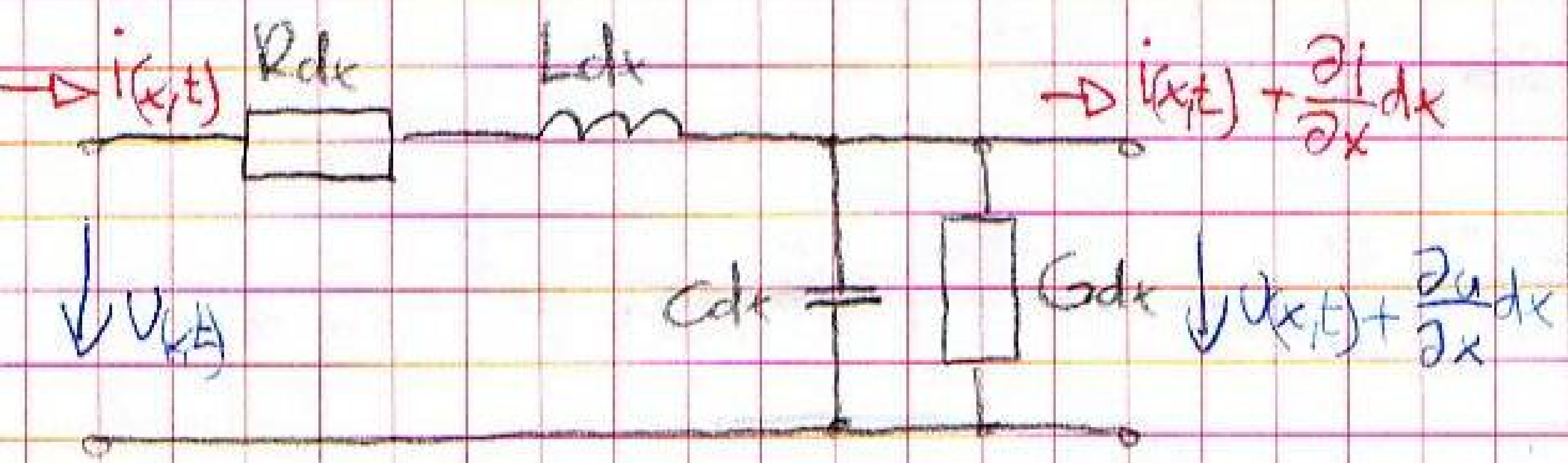
$$t \mp \frac{x}{v} = \alpha$$

$$\left.\begin{aligned} \frac{\partial f(\alpha)}{\partial x} &= \frac{\partial f}{\partial \alpha}\frac{\partial \alpha}{\partial x} = \frac{\partial f}{\partial \alpha}\left(\mp\frac{1}{v}\right) \\ \frac{\partial f(\alpha)}{\partial t} &= \frac{\partial f}{\partial \alpha}\cdot\frac{\partial \alpha}{\partial t} = \frac{\partial f(\alpha)}{\partial \alpha} \end{aligned}\right\} \qquad \begin{aligned} \frac{\partial f}{\partial x} &= \mp\frac{1}{v}\frac{\partial f}{\partial t} \\ \frac{\partial^2 f}{\partial x^2} &= +\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \end{aligned}$$

$$\frac{1}{v^2} = LC = \mu\varepsilon \qquad v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\varepsilon_0\mu_0}}\cdot\frac{1}{\sqrt{\varepsilon_r\mu_r}} = \frac{c_0}{\sqrt{\varepsilon_r\mu_r}}$$

hullám ~~impedancia~~ $Z_0 = \frac{U^+}{I^+} = \sqrt{\frac{L}{C}}$

Veszteséges távvezeték



$$-\frac{\partial u(x,t)}{\partial x} = R\,i(x,t) + L\frac{d\,i(x,t)}{\partial t}$$

$$-\frac{\partial i(x,t)}{\partial x} = G\cdot u(x,t) + C\frac{d\,u(x,t)}{\partial t}$$

$$-\frac{\partial^2 u}{\partial x^2} = -R\left(Gu + C\frac{\partial u}{\partial t}\right) - L\left(G\frac{\partial u}{\partial t} + C\frac{\partial^2 u}{\partial t^2}\right)$$

$$\frac{\partial^2 u}{dx^2} = LC\frac{\partial^2 u}{\partial t^2} + (RC + LG)\frac{\partial u}{\partial t} + RGu$$

Ha $R=0,\ G=0 \qquad \frac{\partial^2 u}{\partial x^2} = LC\frac{\partial^2 u}{\partial t^2}$

Ha $\frac{R}{L} = \frac{G}{C}$ torzításmentes hullámterjedés (THOMSON-FÉLE)

$$U^+(x,t) = U_0^+ e^{-\alpha x}\, f\left(t - \frac{x}{v}\right) \quad \xrightarrow{+x}$$

$$U^-(x,t) = U_0^- e^{\alpha x}\, f\left(t + \frac{x}{v}\right) \quad \xleftarrow{-x}$$

$$-\frac{\partial u}{\partial x} = Ri + L\frac{\partial i}{\partial t}$$

$$-\frac{\partial i}{\partial x} = Gu + C\frac{\partial u}{\partial t}$$

Szinuszos gerj.

$$i(x,t) = \mathrm{Re}\{ \bar{I}(x) e^{j\omega t} \}$$
$$u(x,t) = \mathrm{Re}\{ \bar{U}(x) e^{j\omega t} \}$$

$$-\frac{\partial \bar{U}(x)}{\partial x} = R\bar{I}(x) + j\omega L \bar{I}(x) = (R + j\omega L)\bar{I}(x) = Z_s(j\omega)\bar{I}(x)$$

$$-\frac{\partial \bar{I}(x)}{\partial x} = G\bar{U}(x) + j\omega C \bar{U}(x) = (G + j\omega C)\bar{U}(x) = Y_p(j\omega)\bar{U}(x)$$

$$\frac{\partial^2 \bar{U}(x)}{\partial x^2} = Z_s Y_p \bar{U}(x) = \gamma^2 \bar{U}(x)$$

$$\bar{U}(x) = U_0^+ e^{-\gamma x} + U_0^- e^{+\gamma x}$$

$$\bar{I}(x) = -\frac{\partial \bar{U}(x)}{\partial x}\frac{1}{Z_s} = \frac{1}{Z_s}\left(\gamma U_0^+ e^{-\gamma x} - \gamma U_0^- e^{+\gamma x}\right) = \frac{\gamma}{Z_s}\left(U_0^+ e^{-\gamma x} - U_0^- e^{+\gamma x}\right)$$

$$\frac{1}{Z_0} = \frac{\gamma}{Z_s} = \frac{\sqrt{Z_s Y_p}}{Z_s} = \sqrt{\frac{Y_p}{Z_s}} \Rightarrow Z_0 = \sqrt{\frac{Z_s}{Y_p}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$ HULLÁMIMPEDANCIA

$$I_0^+ e^{-\gamma x} + I_0^- e^{+\gamma x} = \frac{U_0^+}{Z_0} e^{-\gamma x} - \frac{U_0^-}{Z_0} e^{+\gamma x}$$

$$I_0^+ = \frac{U_0^+}{Z_0} \qquad I_0^- = -\frac{U_0^-}{Z_0} \qquad \Rightarrow \boxed{Z_0 \triangleq \frac{U_0^+}{I_0^+} = -\frac{U_0^-}{I_0^-}}$$

A megoldás értelmezése

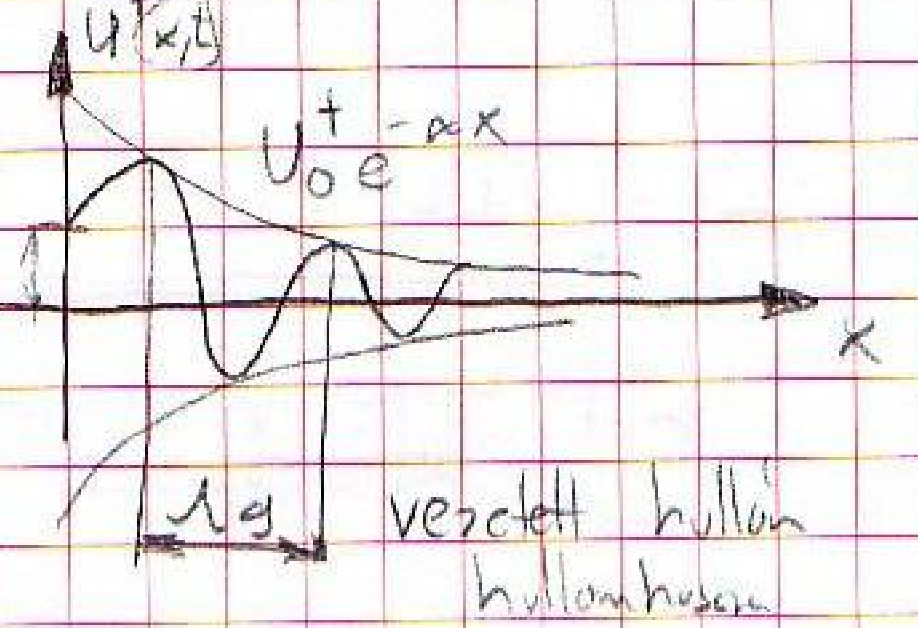
$$\gamma = \sqrt{Z_s Y_p} = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$ TERJEDÉSI EGYÜTTHATÓ

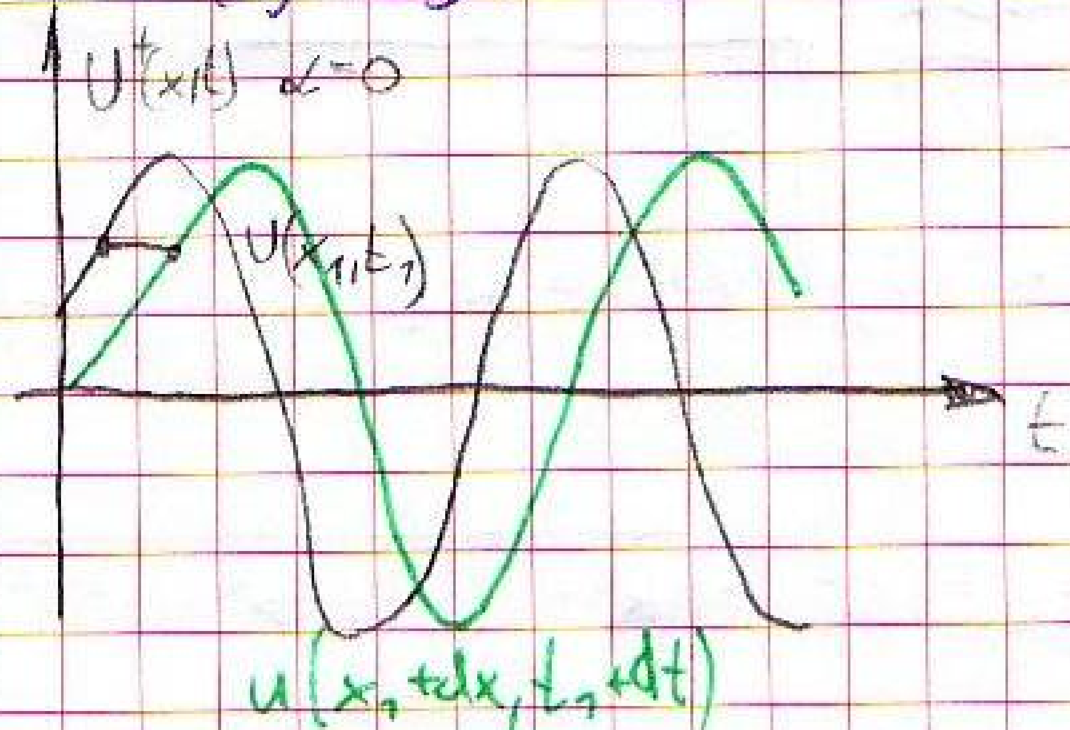
$$U^+(x) = U_0^+ e^{-\gamma x} = U_0^+ e^{-(\alpha + j\beta)x} = U_0^+ e^{-\alpha x} e^{-j\beta x}$$

$$u^+(x,t) = \mathrm{Re}\{ U_0^+ e^{-\alpha x} e^{-j\beta x} e^{j\omega t} \} = \mathrm{Re}\{ U_0^+ e^{-\alpha x} e^{j(\omega t - \beta x)} \}$$

csillapítási tényező — fázis tényező

$$u^+(x,t) = U_0^+ e^{-\alpha x} \cos(\omega t - \beta x) \qquad U^+(x) = U_0^+ e^{-\alpha x} e^{j\omega\left(t - \frac{x}{v}\right)} \qquad v_f = \frac{\omega}{\beta}$$





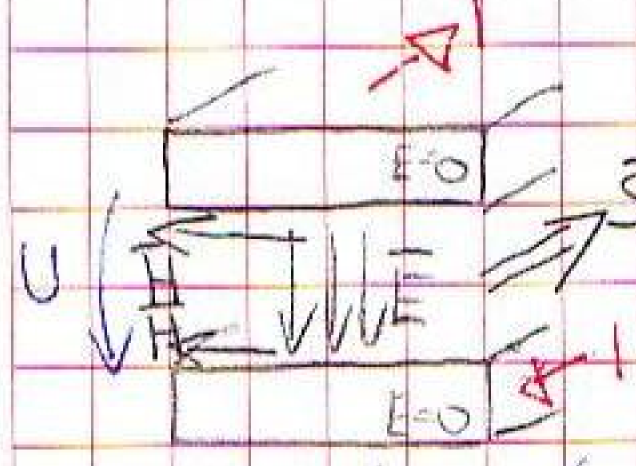
$$U_0^+ e^{-j(\omega t_1 - \beta x_1)} = U_0^+ e^{-j\left(\omega(t_1+dt) - (x_1+dx)\beta\right)}$$

$$-(\omega t_1 - \beta x_1) = -\left[\omega(t_1+dt) - (x_1+dx)\beta\right] \Rightarrow \omega dt - \beta dx = 0$$

$$\frac{dx}{dt} = \frac{\omega}{\beta} = v_f$$

(ideális távvez.) $\gamma = \alpha + j\beta$

$$\begin{matrix} R=0 \\ G=0 \end{matrix} \Rightarrow \gamma = \sqrt{j\omega L \cdot j\omega C} = j\omega\sqrt{LC} = j\beta$$



$\bar{S} = \bar{E} \times \bar{H}$ (szigetelőben!!!)

$$U_0^+ e^{-j(\omega t - \beta(x + d))} = U_0^+ e^{-j(\omega t - \beta x \pm 2\pi)}$$

$$\beta \Lambda_g = 2\pi \Rightarrow \Lambda_g = \frac{2\pi}{\beta} \neq \lambda_{\text{szabadtéri hullám}}$$

$$U(x,t) = U_0^+ e^{-\alpha x} e^{j\omega\left(t - \frac{x}{v}\right)} + U_0^- e^{+\alpha x} e^{j\omega\left(t + \frac{x}{v}\right)}$$

$$I(x,t) = \frac{U_0^+}{Z_0} e^{-\alpha x} e^{j\omega\left(t - \frac{x}{v}\right)} + \frac{U_0^-}{Z_0} e^{+\alpha x} e^{j\omega\left(t + \frac{x}{v}\right)}$$

$$u(x,t) = U_0^+ e^{-\alpha x} \cos(\omega t - \beta x) + U_0^- e^{+\alpha x} \cos(\omega t + \beta x)$$

$$i(x,t) = \frac{U_0^+}{Z_0} e^{-\alpha x} \cos(\omega t - \beta x) + \frac{U_0^-}{Z_0} e^{+\alpha x} \cos(\omega t + \beta x)$$

$\alpha = ?$, $\beta = ?$

$$\gamma = \sqrt{Z_s Y_p} = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta$$

$$\gamma^2 = \alpha^2 + 2j\beta - \beta^2 = (R+j\omega L)(G+j\omega C) = RG - \omega^2 LC + j\omega(RC+GL)$$

$$\left.\begin{array}{ll} Re:\ \alpha^2 - \beta^2 = RG - \omega^2 LC & - \\ |\gamma| = \alpha^2 + \beta^2 = \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)} & + \end{array}\right\} \beta = \pm\frac{1}{\sqrt{2}}\sqrt{\omega^2 LC - RG + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}$$

$$\left.\begin{array}{l} + \\ + \end{array}\right\} \alpha = \pm\frac{1}{\sqrt{2}}\sqrt{RG - \omega^2 LC + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}$$

ha $\omega \to \infty \Rightarrow \alpha \to 0$

Fázis sebesség, csoport sebesség:

ω_1, ω_2 $\quad \beta_1, \beta_2$ $\qquad \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$$u(x,t) = U_0\left[\cos(\underbrace{\omega_1 t - \beta_1 x}_{\alpha}) + \cos(\underbrace{\omega_2 t - \beta_2 x}_{\beta})\right]$$

$$= 2U_0 \cos\frac{(\omega_1+\omega_2)t - (\beta_1+\beta_2)x}{2}\cos\frac{(\omega_1-\omega_2)t - (\beta_1-\beta_2)x}{2}$$