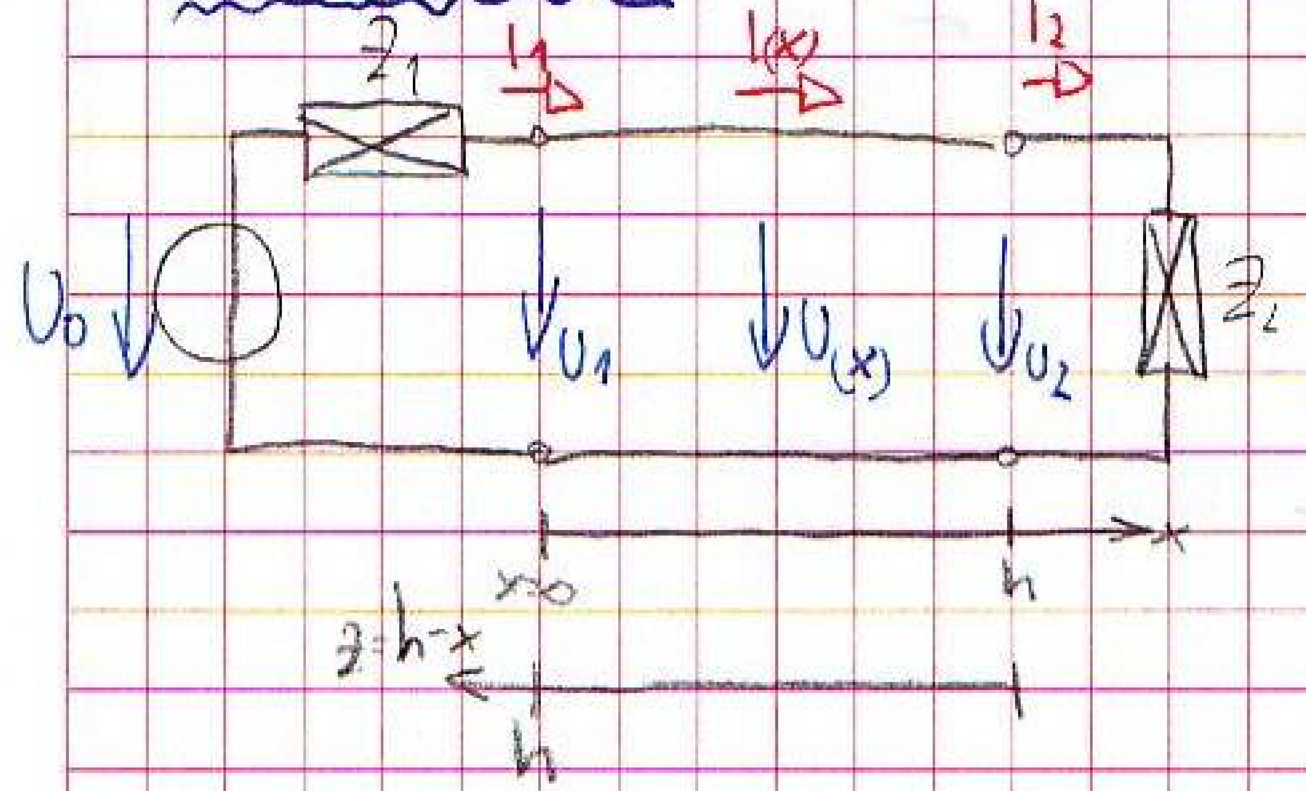
2007. 04. 03. kedd   XII. Előadás (8. hét)

Távvezetékek (folyt)

Lezárt távvezeték



$$U(x) = U_1^+ e^{-\gamma x} + U_1^- e^{+\gamma x}$$
$$I(x) = \frac{U_1^+}{Z_0} e^{-\gamma x} - \frac{U_1^-}{Z_0} e^{+\gamma x}$$
$$Z(x) = \frac{U(x)}{I(x)} = Z_0 \frac{U_1^+ e^{-\gamma x} + U_1^- e^{+\gamma x}}{U_1^+ e^{-\gamma x} - U_1^- e^{+\gamma x}}$$

$x=h$ $\quad U(x=h) = U_2 = U_1^+ e^{-\gamma h} + U_1^- e^{+\gamma h} = U_2^+ + U_2^-$ $\quad U_2^+ = U_1^+ e^{-\gamma h}, \; U_2^- = U_1^- e^{+\gamma h}$

$U(x) = U_2^+ e^{+\gamma(h-x)} + U_2^- e^{-\gamma(h-x)}$, $\quad U(z) = U_2^+ e^{+\gamma z} + U_2^- e^{-\gamma z}$

$I(x=h) = I_2 = \frac{U_1^+}{Z_0} e^{-\gamma h} - \frac{U_1^-}{Z_0} e^{+\gamma h} = \frac{U_2^+}{Z_0} e^{\gamma(h-x)} - \frac{U_2^-}{Z_0} e^{-\gamma(h-x)}$, $\quad I(z) = \frac{U_2^+}{Z_0} e^{+\gamma z} - \frac{U_2^-}{Z_0} e^{-\gamma z}$

$$Z(z) = \frac{U(z)}{I(z)} = Z_0 \frac{U_2^+ e^{\gamma z} + U_2^- e^{-\gamma z}}{U_2^+ e^{\gamma z} - U_2^- e^{-\gamma z}}$$

$x=h$ $\quad Z_2 = Z(h) = Z_0 \frac{U_1^+ e^{-\gamma h} + U_1^- e^{+\gamma h}}{U_1^+ e^{-\gamma h} - U_1^- e^{+\gamma h}}$

$z=0$ $\quad Z(z=0) = Z_2 = \frac{U_2^+ + U_2^-}{U_2^+ - U_2^-} Z_0 = Z_0 \frac{1 + \frac{U_2^-}{U_1^+}}{1 - \frac{U_2^-}{U_2^+}}$

(definíciós)

$\frac{U_2^-}{U_2^+} \triangleq r_2$ reflexiós tényező a lezáráson

$$Z_2 = Z_0 \frac{1+r_2}{1-r_2} \Rightarrow r_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0}$$

(Smith)

$$Z(z) = Z_0 \frac{1 + \frac{U_2^-}{U_2^+} e^{-2\gamma z}}{1 - \frac{U_2^-}{U_2^+} e^{-2\gamma z}}$$

$$r(z) \triangleq \frac{U_2^-}{U_2^+} e^{-2\gamma z} = r_2 e^{-2\gamma z}$$

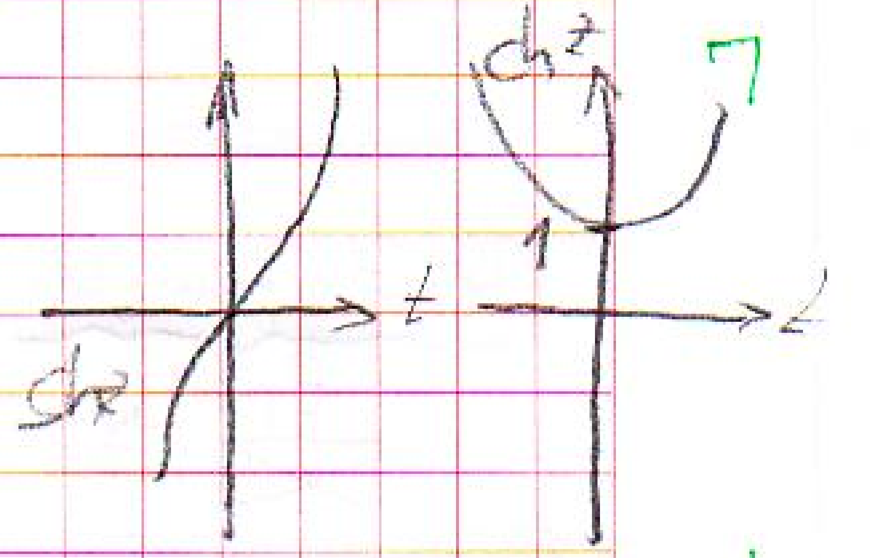
$$U(z) = U_2^+\left(e^{\gamma z} + r_2 e^{-\gamma z}\right) = U_2^+\left((1+r_2)\,ch\gamma z + (1-r_2)\,sh\gamma z\right)$$

$$I(z) = \frac{U_2^+}{Z_0}\left(e^{\gamma z} - r_2 e^{-\gamma z}\right) = \frac{U_2^+}{Z_0}\left((1-r_2)\,ch\gamma z + (1+r_2)\,sh\gamma z\right)$$

matematika

$$e^{\gamma z} = ch\gamma z + sh\gamma z = \frac{e^{\gamma z}+e^{-\gamma z}}{2} + \frac{e^{\gamma z}-e^{-\gamma z}}{2}$$

$$e^{-\gamma z} = ch\gamma z + sh\gamma z$$



$\boxed{U_2^+ = ?}$ $U(z=0) = U_2$ adott

$$U_2 = U_2^+(1+r_2) \Rightarrow U_2^+ = \frac{U_2}{1+r_2}$$

$$U(z) = U_2\left(ch\gamma z + \underbrace{\frac{1-r_2}{1+r_2}}_{Z_0/Z_2} sh\gamma z\right)$$

$$I(z) = \frac{U_2}{Z_0}\left(sh\gamma z + \underbrace{\frac{1-r_2}{1+r_2}}_{Z_0/Z_2} ch\gamma z\right)$$

$$U(z) = U_2\, ch\gamma z + \overset{U_2/Z_2}{I_2}\, Z_0\, sh\gamma z$$

$$I(z) = \frac{U_2}{Z_0} sh\gamma z + I_2\, ch\gamma z$$

lánc karakterisztika

$z = h$

$$\left.\begin{aligned} U(z=h) = U_1 &= U_2\, ch\gamma h + I_2 Z_0\, sh\gamma h \\ I(z=h) = I_1 &= \frac{U_2}{Z_0} sh\gamma h + I_2\, ch\gamma h \end{aligned}\right\} \begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \underline{\underline{A}} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$$

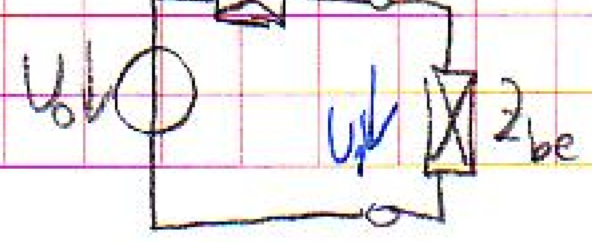
$$\underline{\underline{A}} = \begin{bmatrix} ch\gamma h & Z_0\, sh\gamma z \\ \frac{sh\gamma h}{Z_0} & ch\gamma z \end{bmatrix}$$

$\det \underline{\underline{A}} = ch^2\gamma h - sh^2\gamma h = 1 \Rightarrow$ reciprok!

$A_{11} = A_{22}$ $\Rightarrow$ szimmetrikus!

$$Z_{1be} = Z_{be} = \frac{U_1}{I_1} = Z_0\,\frac{Z_2\, ch\gamma h + Z_0\, sh\gamma h}{Z_2\, sh\gamma h + Z_0\, ch\gamma h}$$

$U_2 = I_2 \cdot Z_2$



$$I_1 = \frac{U_0}{Z_1 + Z_{be}} \qquad U_1 = U_0\,\frac{Z_{be}}{Z_1 + Z_{be}}$$

$$\operatorname{ch}\gamma h = \operatorname{ch}(\alpha + j\beta)h = \operatorname{ch}\alpha h \operatorname{ch}\beta h + j \operatorname{sh}\alpha h \cdot \operatorname{sh}\beta h$$

$$\operatorname{sh}\gamma h = \operatorname{sh}(\alpha + j\beta)h = \operatorname{sh}\alpha h \cos\beta h + j \operatorname{ch}\alpha h \sin\beta h$$

Speciális lezárás:

a) $z_2 = z_0$, $r_2 = \frac{z_2 - z_0}{z_2 + z_0} \Rightarrow \frac{U_2^-}{U_2^+} = r_2 = 0 \quad U_2^+ \neq 0 \quad U_2^- \equiv 0$ szimulál $h \Rightarrow \infty$

illesztés

b) $z_2 = 0$ (rz), $r_2 = -1$ $\quad z_{be_{rz}} = z_0 \operatorname{th} \gamma z$

c) $z_2 = \infty$ (szak), $r_2 = +1$ $\quad z_{be_{sz}} = \frac{z_0}{\operatorname{th}\gamma z}$

$$(\ )\cdot(\ ) = z_0^2$$

$$\Rightarrow z_0 = \sqrt{z_{be_{rz}} \cdot z_{be_{sz}}}$$

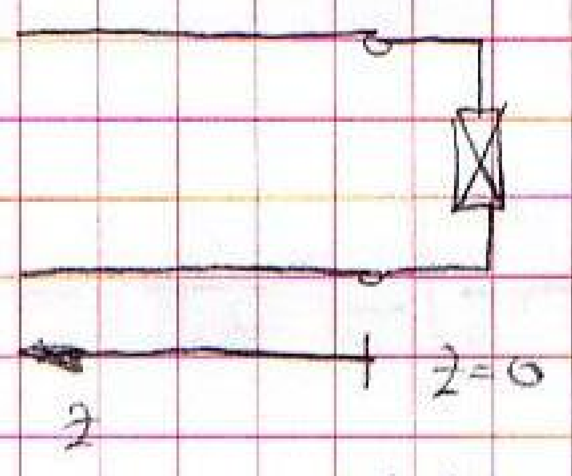
$$\operatorname{th}\gamma z = \sqrt{\frac{z_{be_{rz}}}{z_{be_{sz}}}} = \frac{e^{\gamma z} - e^{-\gamma z}}{e^{\gamma z} + e^{-\gamma z}} = \frac{1 - e^{-2\gamma z}}{1 + e^{-2\gamma z}}$$

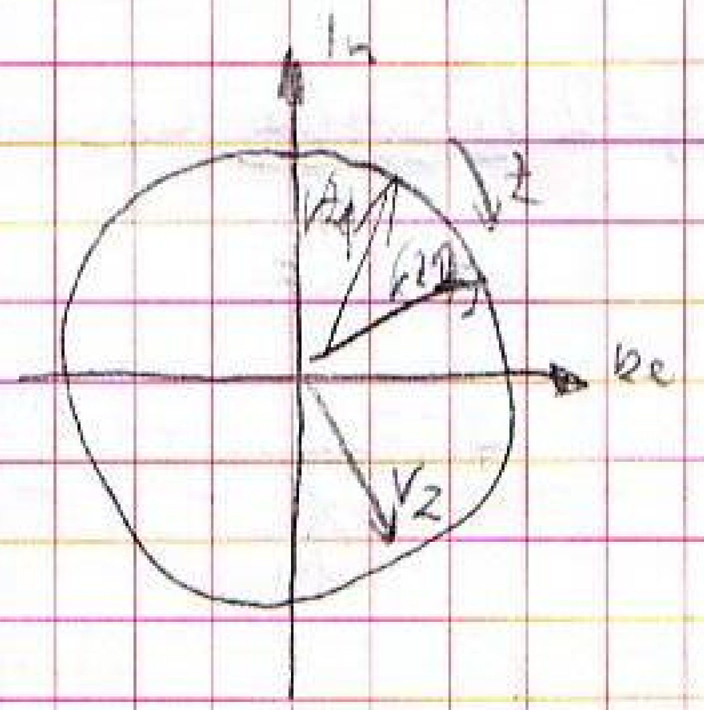
ideális a távvezeték $\quad \alpha = 0$

$$\gamma = j\beta = j\frac{2\pi}{\Lambda_g} \qquad r(z) = r_2 e^{-j2\beta z}$$

$$2\beta z = 2\pi$$

$$z = \frac{\pi}{\beta} = \frac{\pi}{\frac{2\pi}{\Lambda_g}} = \frac{\Lambda_g}{2}$$



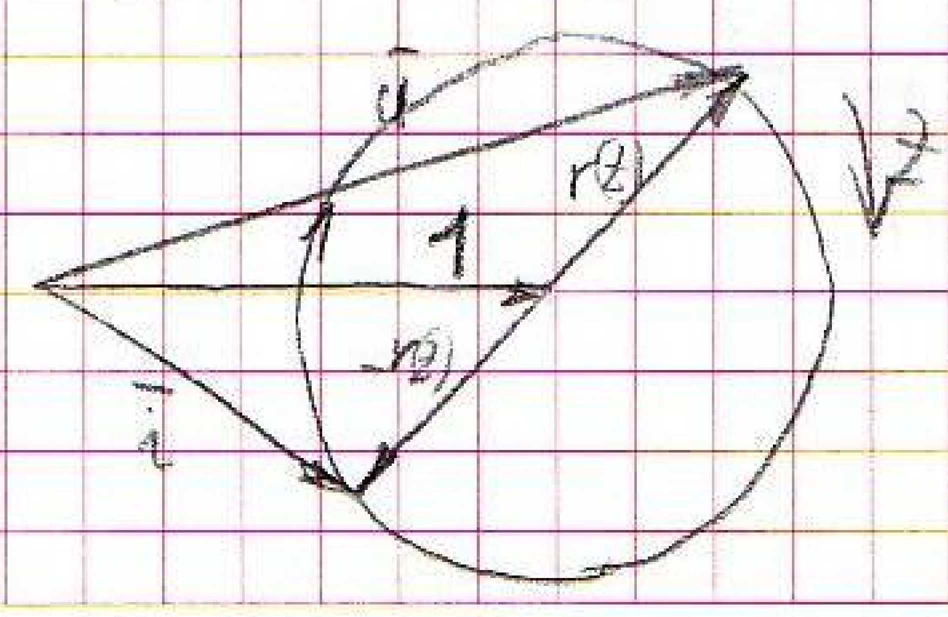


$$\left.\begin{aligned} U(z) &= U_2^+\left(e^{j\beta z} + r_2 e^{-j\beta z}\right) \\ I(z) &= \frac{U_2^+}{z_0}\left(e^{j\beta z} - r_2 e^{-j\beta z}\right) \end{aligned}\right\} \quad z(z) = z_0 \frac{z_2 \cos\beta z + j z_0 \sin\beta z}{z_2 \sin\beta z + z_0 \cos\beta z} = z_0 \frac{z_2 + j z_0 \operatorname{tg}\beta z}{z_0 + j z_2 \operatorname{tg}\beta z}$$

Normalizált $\bar{U}(z)$, $\bar{I}(z)$:

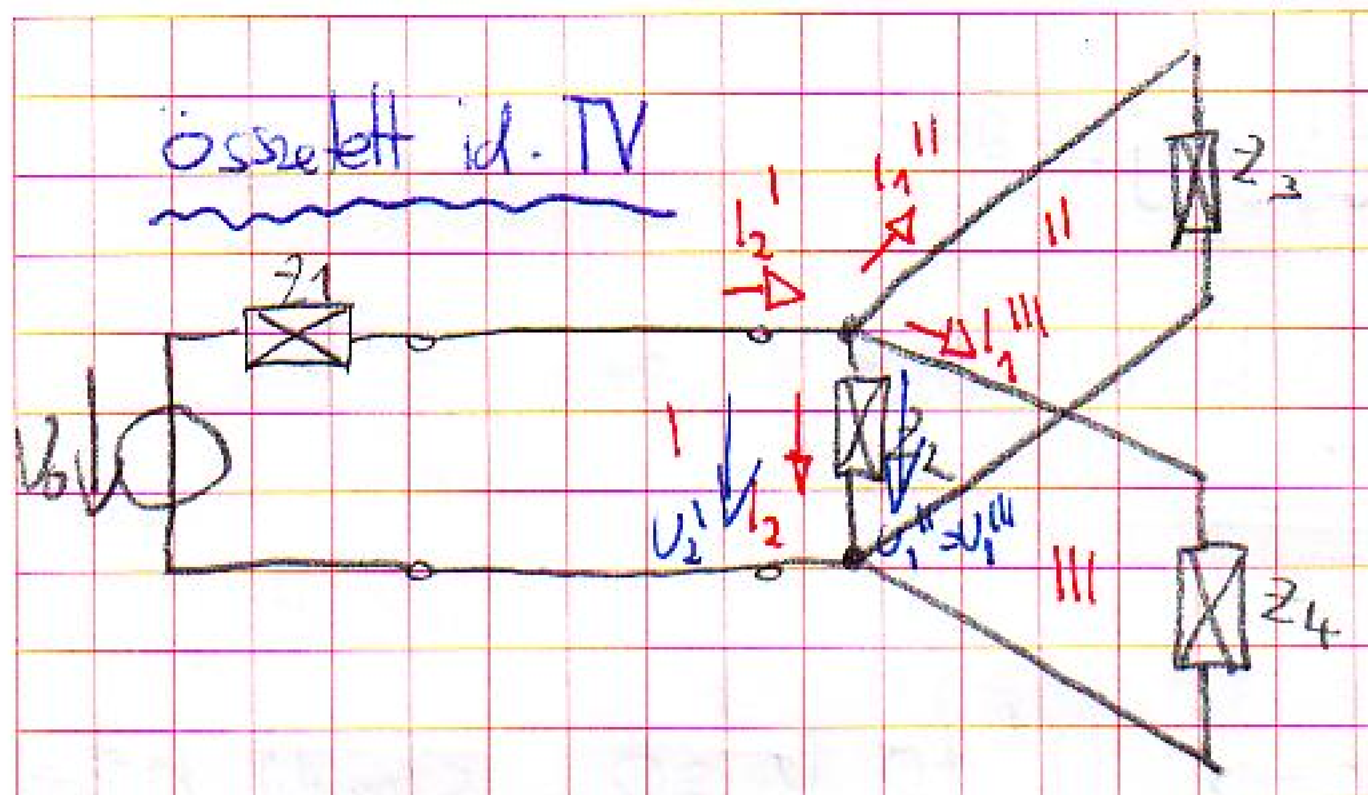
$$\bar{u}(z) = \frac{U(z)}{U_2^+ e^{j\beta z}} = 1 + \underbrace{r_2 e^{-j2\beta z}}_{r(z)} = 1 + r(z)$$

$$\bar{i}(z) = \frac{I(z)}{\frac{U_2^+}{z_0} e^{j\beta z}} = 1 - r(z)$$



SMITH-diagram

Összetett id. TV



id. TV spec. lezárások:

a) $Z_2=Z_0$ $\quad r_2=0$ illesztett, haladó hullám $\quad U(z)=U_2^+e^{j\beta z}$; $u(z,t)=U_2^+\cos(\omega t-\beta t)=$
$=U_2^+\cos\left(t-\frac{z}{v}\right)$ $\quad \frac{\omega}{v}=\beta$

b) $Z_2=0$ $\quad r_2=\frac{Z_2-Z_0}{Z_2+Z_0}=-1$ $\quad Z_{be_{r_2}}=jZ_0\,tg\beta z$ reaktáns

$$U(z)=U_2^+\left(\frac{e^{j\beta z}-1e^{-j\beta z}}{2j}\right)2j=2jU_2^+\sin\beta z$$

$$u(z,t)=2U_2^+\underbrace{\sin\beta z}\cos\left(\omega t+\frac{\pi}{2}\right)$$

$$\beta z=\pm k\pi$$

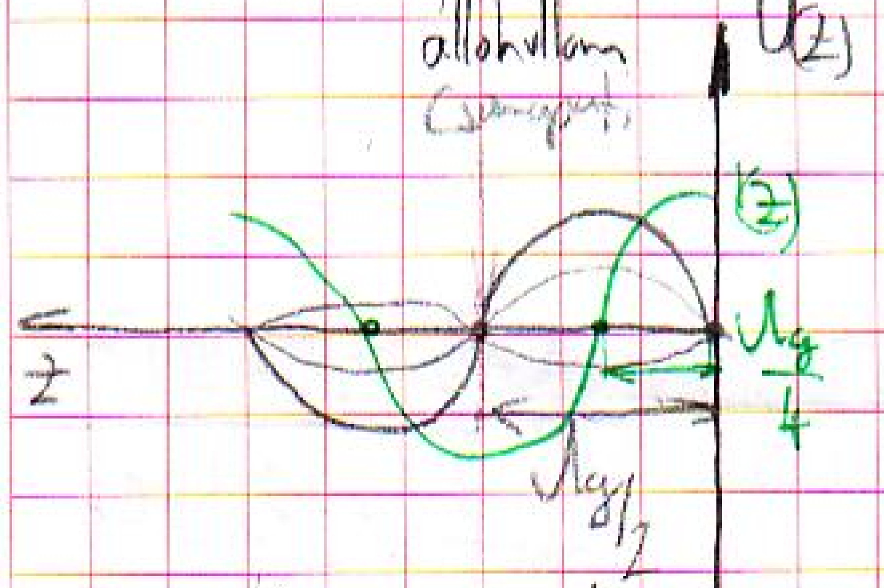
$$z=\pm\frac{\pi k}{\frac{2\pi}{\lambda_g}}=\pm k\frac{\lambda_g}{2}$$

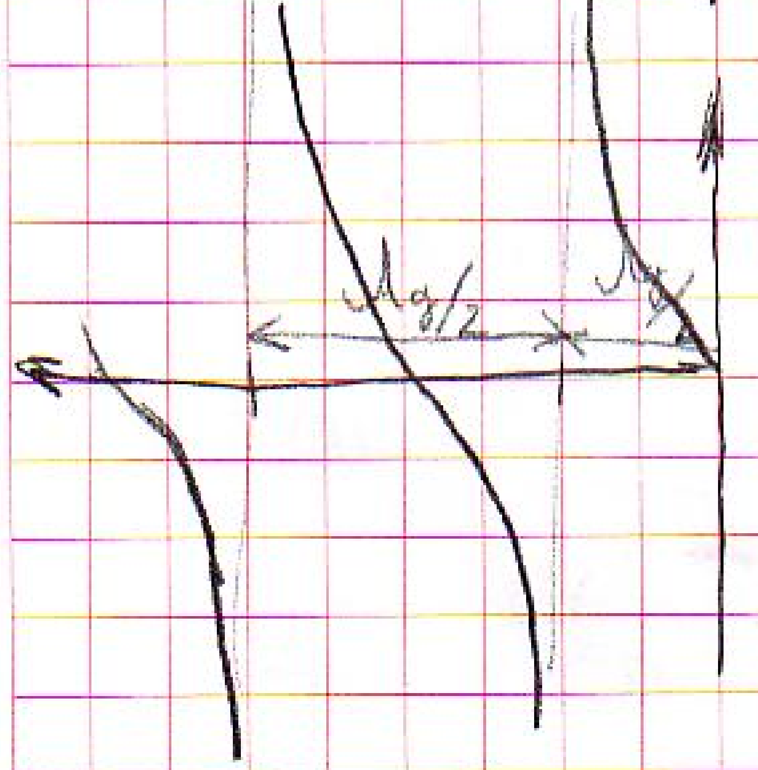
$$I(z)=\frac{U_2^+}{Z_0}\left(\frac{e^{j\beta z}+1e^{-j\beta z}}{2}\right)2=\frac{2U_2^+}{Z_0}\cos\beta z$$

$$i(z,t)=2\frac{U_2^+}{Z_0}\underline{\cos\beta z}\cos(\omega t)$$

$$\beta z=\pm(2k+1)\frac{\pi}{2}$$

$$z=\pm\frac{(2k+1)\frac{\pi}{2}}{\frac{2\pi}{\lambda_g}}=\pm(2k+1)\frac{\lambda_g}{4}$$





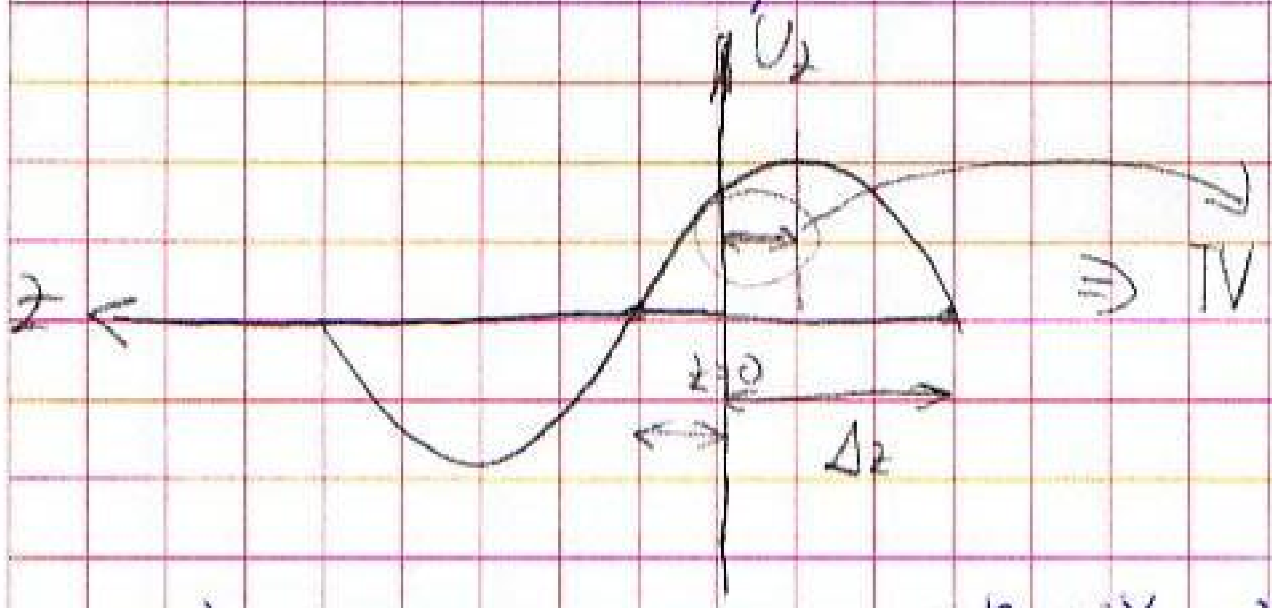
c) $Z_2=\infty$, $r_2=+1$, $Z_{be}(z)=\dfrac{Z_0}{j\,tg\,\beta z}=-jZ_0\,ctg\,\beta z$

d) Reaktancia $Z_2=jX_2$ $\quad r_2=\dfrac{jX_2-Z_0}{jX_2+Z_0}=-\dfrac{Z_0-jX_2}{Z_0+jX_2}=1\cdot e^{j\varphi}$

$$U(z)=U_2^+\left(e^{j\beta z}+e^{j\varphi}e^{-j\beta z}\right)=U_2^+e^{j\frac{\varphi}{2}}\left(\frac{e^{j\left(\beta z-\frac{\varphi}{2}\right)}+e^{-j\left(\beta z-\frac{\varphi}{2}\right)}}{2}\right)2=$$

$$=2U_2^+e^{j\frac{\varphi}{2}}\cos\left(\beta z-\frac{\varphi}{2}\right)=2U_2^+e^{j\frac{\varphi}{2}}\cos\beta\left(z-\frac{\varphi}{2\beta}\right)$$

$$\beta\left(z-\frac{\varphi}{2\beta}\right)=\pm(2k+1)\frac{\pi}{2}\qquad z=\pm(2k+1)\frac{\pi}{2}\frac{1}{\beta}+\frac{\varphi}{2\beta}$$ Csp van a TV végén



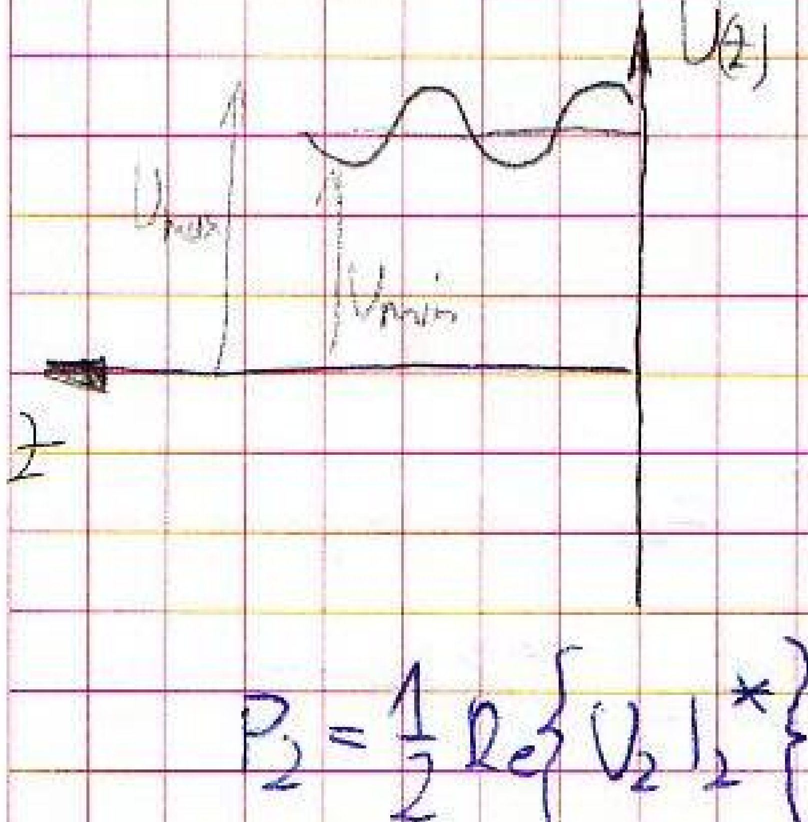
⇒ TV hosszabbnak tűnik

e) $Z_2=R_2+jX_2$, $r_2=\dfrac{R_2+jX_2-Z_0}{R_2+jX_2+Z_0}=|r_2|e^{j\varphi}$

$$|r_2|\leq 1$$

$$U(z)=U_2^+\left(e^{j\beta z}+|r_2|e^{j\varphi}e^{-j\beta z}\right)\pm U_2^+|r_2|e^{j\beta z}=$$

$$=U_2^+\left[\underbrace{e^{j\beta z}\left(1-|r_2|\right)}_{\text{haladó hullám}}+\underbrace{|r_2|e^{j\frac{\varphi}{2}}\left(\frac{e^{j\left(\beta z-\frac{\varphi}{2}\right)}+e^{-j\left(\beta z-\frac{\varphi}{2}\right)}}{2}\right)2}_{\text{állóhullám}}\right]$$



$U_{max}=|U_2^+|\left(1+|r_2|\right)$

$U_{min}=|U_2^+|\left(1-|r_2|\right)$

$\dfrac{U_{max}}{U_{min}}=VSWR$ (voltage standing wave rate)

ÁLLÓHULLÁM ARÁNY

$$VSWR=\frac{1+|r_2|}{1-|r_2|}$$

$$P_2=\frac{1}{2}Re\left\{U_2I_2^*\right\}$$

Le Color