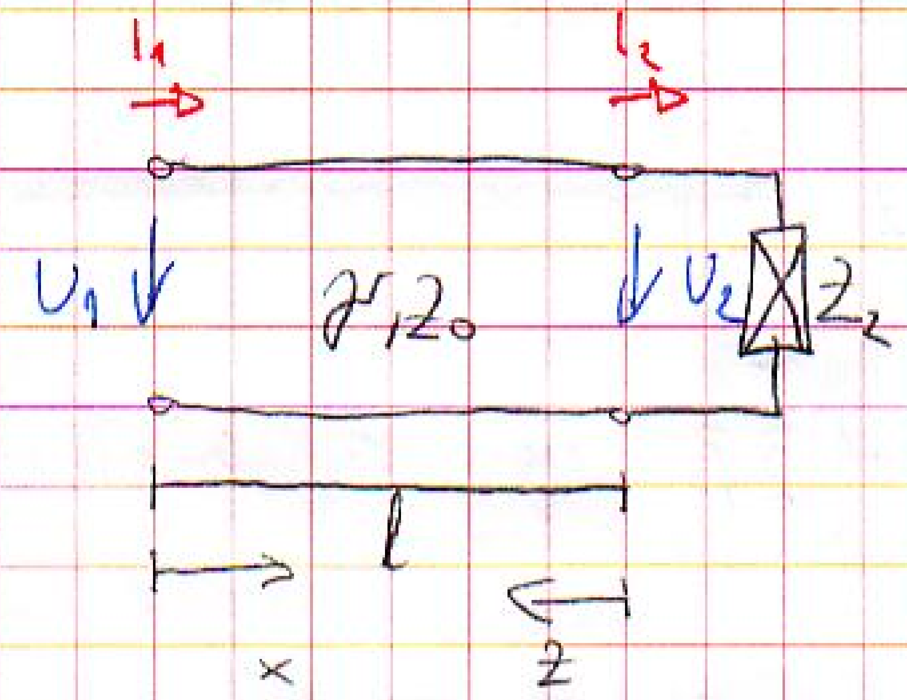
2007. 04. 10. kedd XIII. Előadás (9. hét)

Távvezeték (folyt.)

Távvezeték, mint kétkapu:



$$U_1 = U_2 \operatorname{ch}\gamma l + I_2 Z_0 \operatorname{sh}\gamma l$$

$$I_1 = \frac{U_2}{Z_0}\operatorname{sh}\gamma l + I_2 \operatorname{ch}\gamma l$$

$$Z_{1be} = Z_0 \frac{Z_2 \operatorname{ch}\gamma l + Z_0 \operatorname{sh}\gamma l}{Z_0 \operatorname{ch}\gamma l + Z_2 \operatorname{sh}\gamma l}$$

$\underline{P} = \underline{\underline{A}}\,\underline{S}$, $\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \operatorname{ch}\gamma l & Z_0 \operatorname{sh}\gamma l \\ \frac{\operatorname{sh}\gamma l}{Z_0} & \operatorname{ch}\gamma l \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$

$\det(A) = \operatorname{ch}^2\gamma l - \operatorname{sh}^2\gamma l = 1$
$\Rightarrow$ reciprok

$A_{11} = A_{22} \Rightarrow$ szimmetrikus

$\underline{\underline{Z}}$, $\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \underline{\underline{Z}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$U_2 = \frac{Z_0}{\operatorname{sh}\gamma l}\left(I_1 - I_2 \operatorname{ch}\gamma l\right)$$

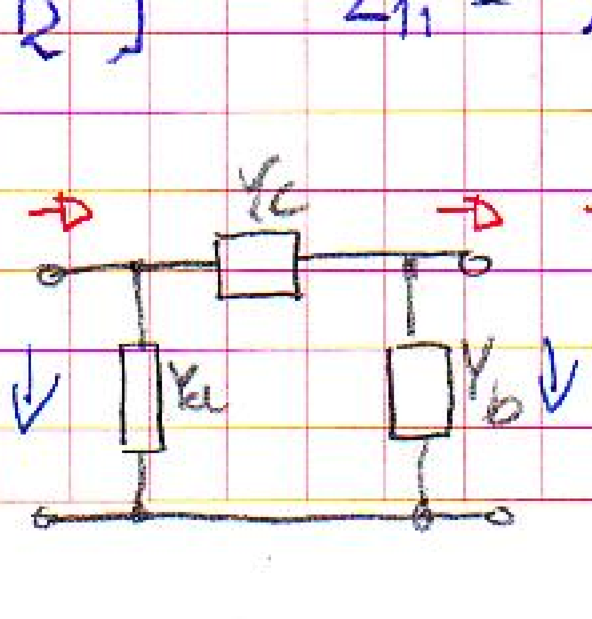
$$U_1 = \frac{Z_0}{\operatorname{sh}\gamma l}\left(I_1 \operatorname{ch}\gamma l - I_2 \operatorname{ch}^2\gamma l\right) + I_2 \frac{Z_0 \operatorname{sh}^2\gamma l}{\operatorname{sh}\gamma l}$$

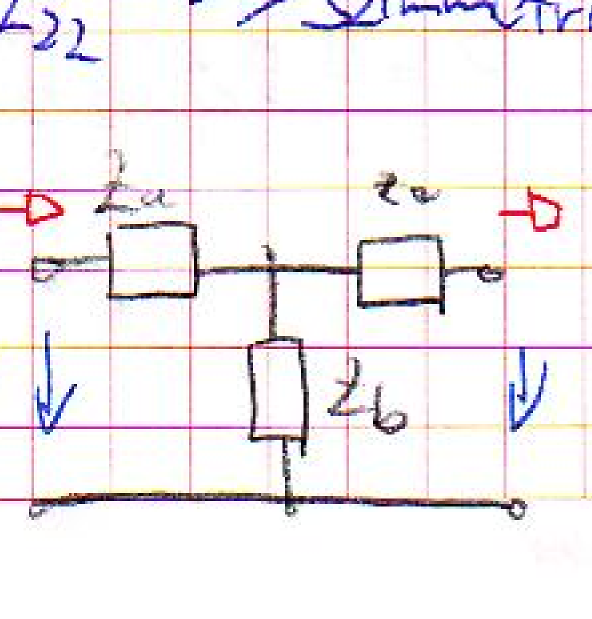
$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \frac{Z_0}{\operatorname{sh}\gamma l} \begin{bmatrix} \operatorname{ch}\gamma l & -1 \\ 1 & -\operatorname{ch}\gamma l \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$Z_{12} = -Z_{21} \Rightarrow$ reciprok

$Z_{11} = -Z_{22} \Rightarrow$ szimmetrikus

$\Rightarrow \exists$ T, Π - helyettesítő kép

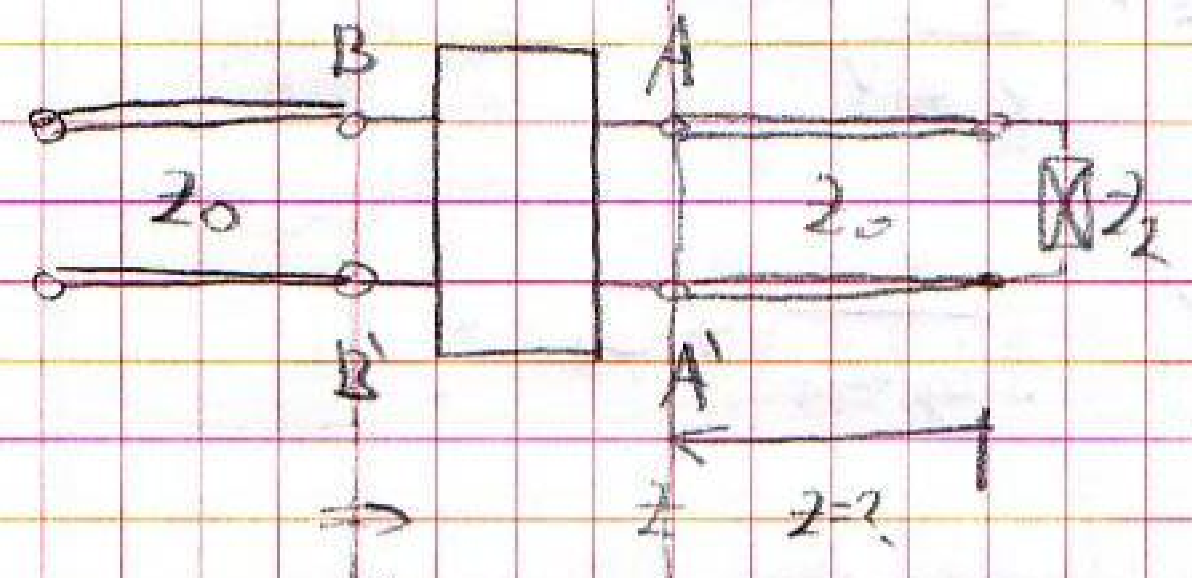




ideális távvezeték $\alpha = 0$ $\gamma = j\beta$

$$\left.\begin{aligned} U_1 &= U_2 \cos\beta l + I_2 Z_0 j \sin\beta l \\ I_1 &= \frac{U_2}{Z_0}\sin\beta l + I_2 \cos\beta l \end{aligned}\right\} \quad Z_{I\,be} = Z_0 \frac{Z_2 + jZ_0 tg\beta l}{Z_0 + jZ_2 tg\beta l}$$

távvezetékek illesztés:



$Z_{be}^{B} = Z_0$

$r(z) = r_2 e^{-j2\beta z}$

$$r(z) = 0 \quad VSWR = \frac{1+|r(z)|}{1-|r(z)|} = 1$$

$$U(z) = U_2^+\left(e^{\gamma z} + r(z)e^{-\gamma z}\right)$$

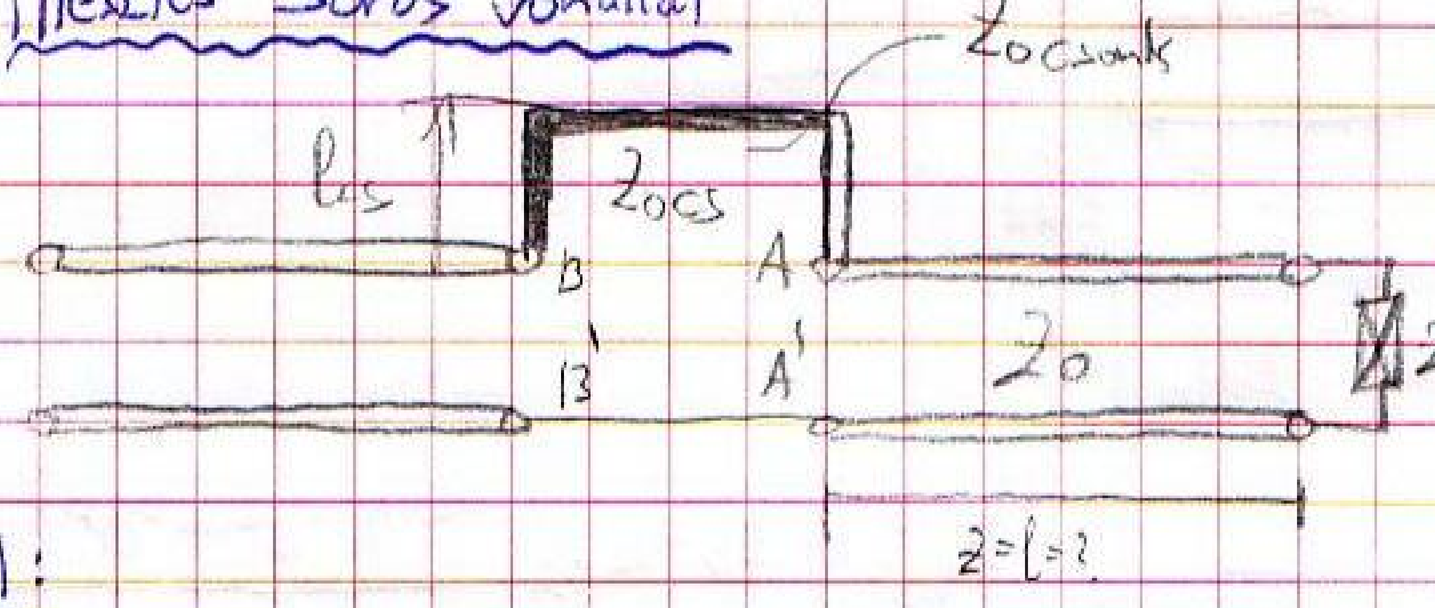
$$I(z) = \frac{U_2^+}{Z_0}\left(e^{\gamma z} - r(z)e^{-\gamma z}\right)$$

$Z_{be}^{B}(z) = Z_0$

$\Rightarrow z = ?$

$$Z_{be}^{A}(z) = Z_0 \frac{Z_2 + jZ_0 tg\beta z}{Z_0 + Z_2 j tg\beta z}$$

a) illesztés soros vonallal



$$Z_{cs}^{be} = Z_{0cs}\, j\, tg\beta l_{cs} = jX_{cs}$$

$$Z_{be}^{AA'} = Z_0 \frac{Z_2 + jZ_0 tg\beta l}{Z_0 + jZ_2 tg\beta l} = R_{AA'} + jX_{AA'}$$

feltétel:

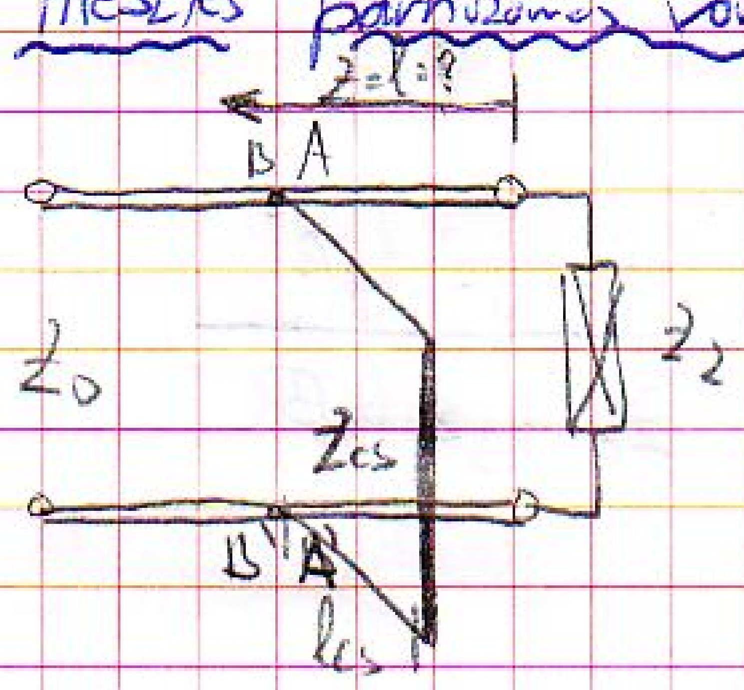
Im: $X_{cs} + X_{AA'} = 0$

Re: $R_{AA'} = 0 \Rightarrow z = l$ meghatározható

$$Z_{be}^{BB'} = Z_0 = Z_{cs}^{be} + Z_{AA'}^{be} =$$

$$= jX_{cs} + R_{AA'} + jX_{AA'} = Z_0$$

b) illesztés párhuzamos vonallal:



$$Y_{BB'} = Y_0 = \frac{1}{Z_0} = Y_{cs}^{be} + Y_{AA'}^{be}$$

$$Y_{be}^{AA'} = \frac{1}{Z_{be}} = \frac{1}{Z_0} \frac{Z_0 + jZ_2 \operatorname{tg}\beta l}{Z_2 + jZ_0 \operatorname{tg}\beta l} \cdot \frac{Z_2 Z_0}{Z_2 Z_0} =$$

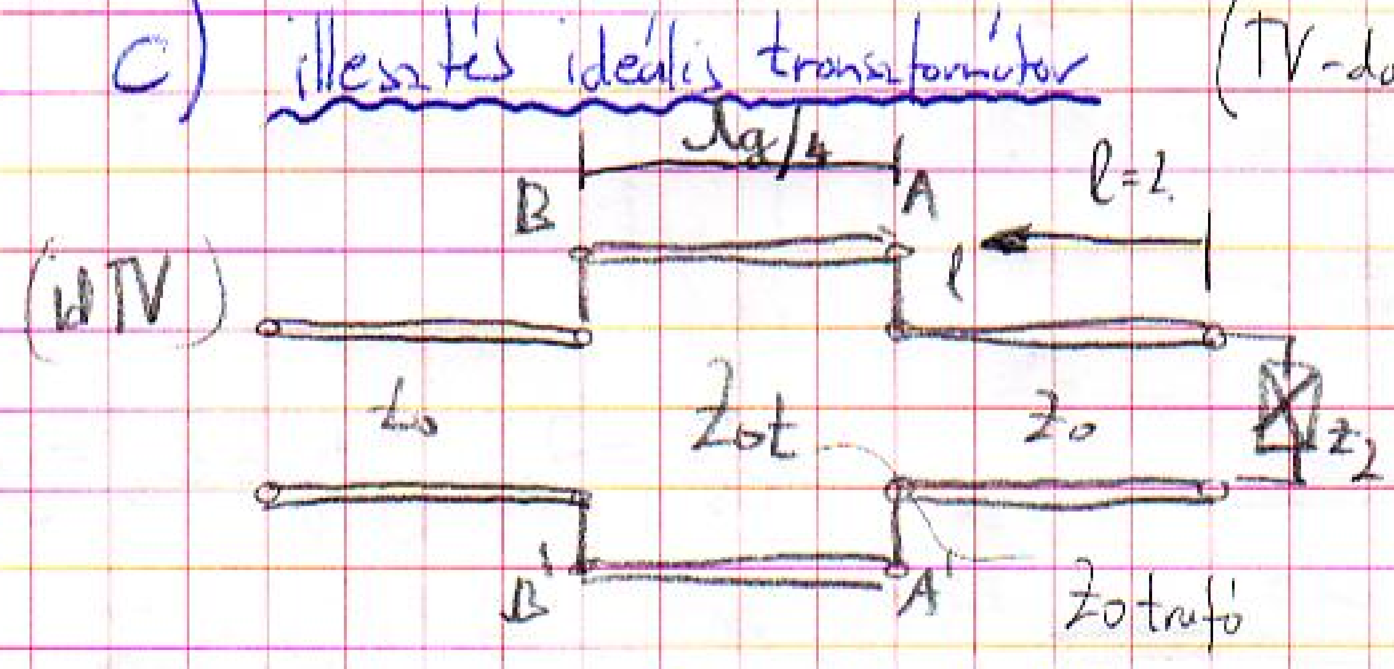
$$= Y_0 \frac{Y_2 + jY_0 \operatorname{tg}\beta l}{Y_0 + jY_2 \operatorname{tg}\beta l} = \underset{\uparrow \text{ konduktancia}}{G_{AA'}} + j\underset{\uparrow \text{ szuszceptancia}}{B_{AA'}}$$

$$Y_{cs}^{be} = -j\frac{Y_0}{\operatorname{tg}\beta l_{cs}} \qquad Y_{be}^{BB'} = Y_0 = -j\frac{Y_0}{\operatorname{tg}\beta l_{cs}} + G_{AA'} + jB_{AA'}$$

Im: $\frac{-Y_0}{\operatorname{tg}\beta l_{cs}} + B_{AA'} = 0 \implies l_{cs}$ meghat.-ható

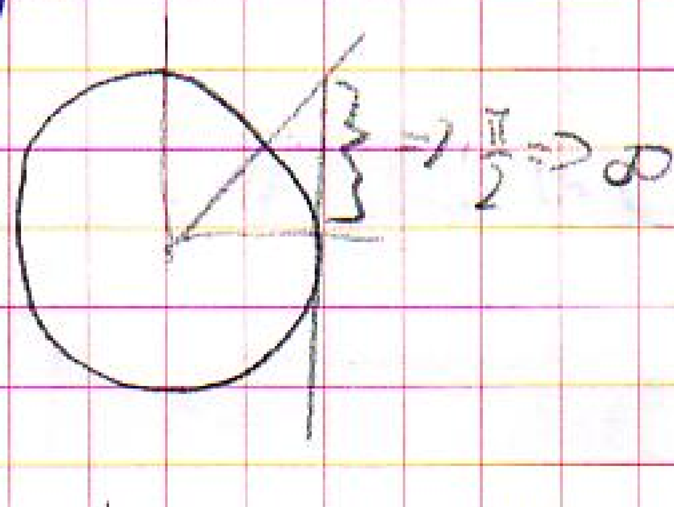
Re: $Y_0 = G_{AA'} \implies l$ meghat. ható

c) illesztés ideális transzformátor (TV-doboz)



$$Z_{AA'}^{be} = Z_0 \frac{Z_2 + jZ_0 \operatorname{tg}\beta l}{Z_0 + jZ_2 \operatorname{tg}\beta l} \quad ; \quad Z_{BB'}^{be} = Z_0 = Z_{0t} \frac{Z_{AA'}^{be} + jZ_{0t} \operatorname{tg}\beta \frac{\lambda_g}{4}}{Z_{0t} + jZ_{AA'}^{be} \operatorname{tg}\beta \frac{\lambda_g}{4}} = \frac{Z_{0t}^2}{Z_{AA'}^{be}} = Z_0$$

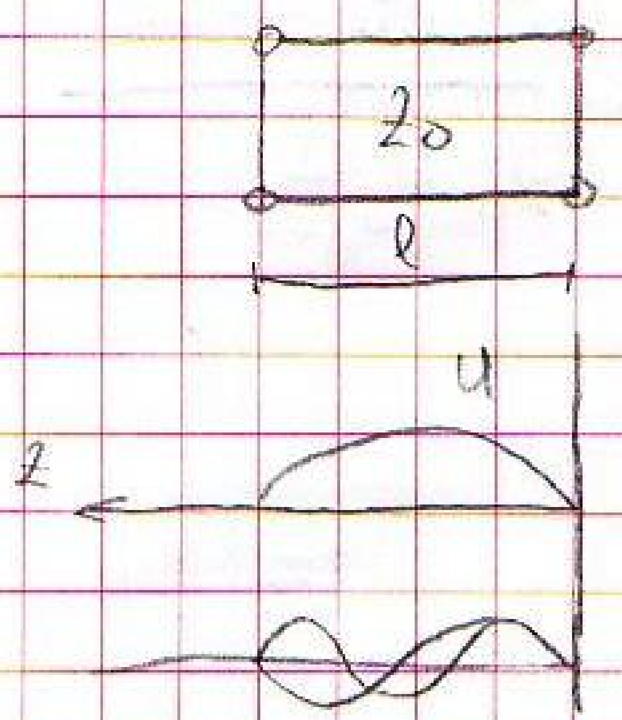
$$\beta \frac{\lambda_g}{4} = \frac{2\pi}{\lambda_g} \frac{\lambda_g}{4} = \frac{\pi}{2}$$



$$Z_{0t} = \sqrt{Z_{AA'}^{be} \cdot \underset{\substack{\uparrow \\ \text{(valós)}}}{Z_{BB'}^{be}}} \quad \text{(valós)} \implies Z_{AA'}^{be} \text{: valósnak kell lennie}$$

ideális távvezeték, mint rezgőkör

a) rövidzár

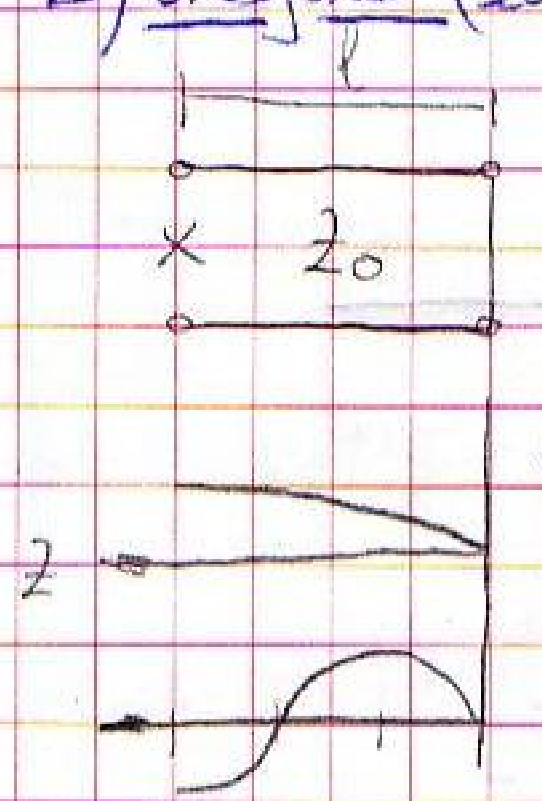


$$\frac{\lambda_g}{2} n = \ell$$

$$\lambda_g = \frac{2\ell}{n} \qquad n = 1, 2 \ldots$$

$$f = \frac{v}{\lambda_g} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \cdot \frac{n}{2\ell}$$ Távvezeték saját frekvencia

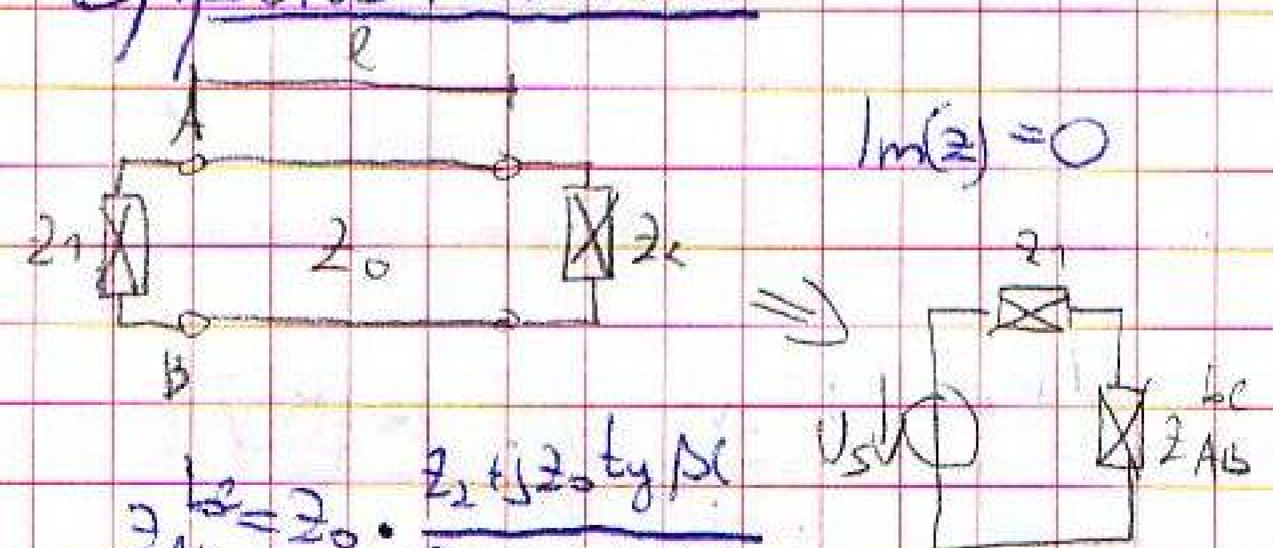
b) üresjárás (szakadás)



$$\frac{\lambda_g}{4}(2n+1) = \ell$$

$$\lambda_g = \frac{4\ell}{2n+1} \qquad n = 0, 1, 2,$$

c/1 soros rezonancia



$$Im(Z) = 0$$

$$Z_{AB}^{be} = Z_0 \cdot \frac{Z_2 + jZ_0 \, tg\, \beta\ell}{Z_0 + jZ_2 \, tg\, \beta\ell}$$

$$Im\left\{Z_1 + Z_{AB}^{be}\right\} = 0 \Rightarrow \ell = \ldots \quad (f \text{ adott})$$

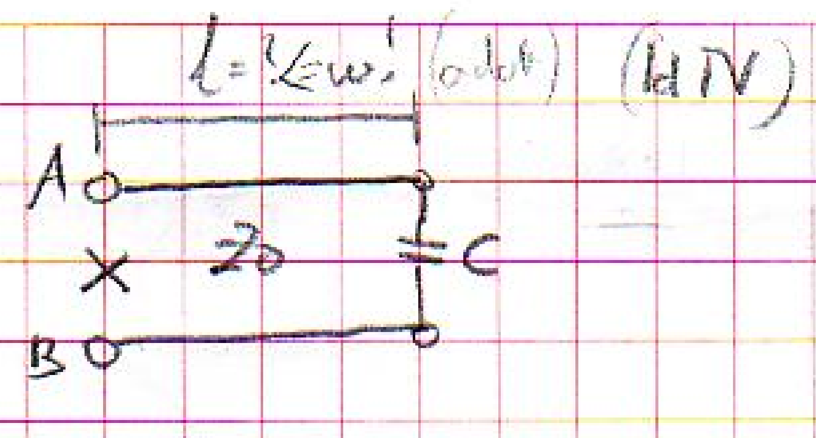
$$f = \ldots \quad (\ell \text{ adott})$$

c/2 párhuzamos rezonancia

$$Im(Y) = 0$$

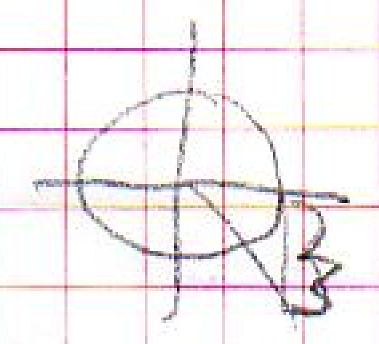
$$Im\left\{Y_1 + Y_{AB}^{be}\right\} = 0$$

példa (1)



$$\mathrm{Im}\quad Z_{er} = \infty = Z_1 (=\infty) + Z_{AB}^{be} = Z_0 \frac{Z_2 + jZ_0 \,tg\,\beta l}{Z_0 + jZ_2 \,tg\,\beta l} = Z_0 \frac{\frac{1}{j\omega C} + jZ_0 \,tg\,\beta l}{Z_0 + j\frac{1}{j\omega C}\,tg\,\beta l}$$

$\Rightarrow$ elég a nevező = 0 $\qquad Z_0 + \frac{1}{\omega C}\,tg\,\beta l = 0$

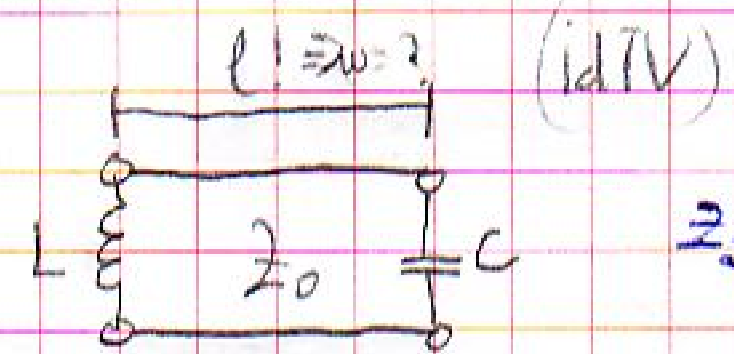


$$\Rightarrow tg\,\beta l = -Z_0\omega C$$

$$\beta l = arctg(-Z_0 \omega C)$$

$$\frac{2\pi}{\lambda_g} \cdot l = \ldots \Rightarrow l = \ldots$$

példa (2)

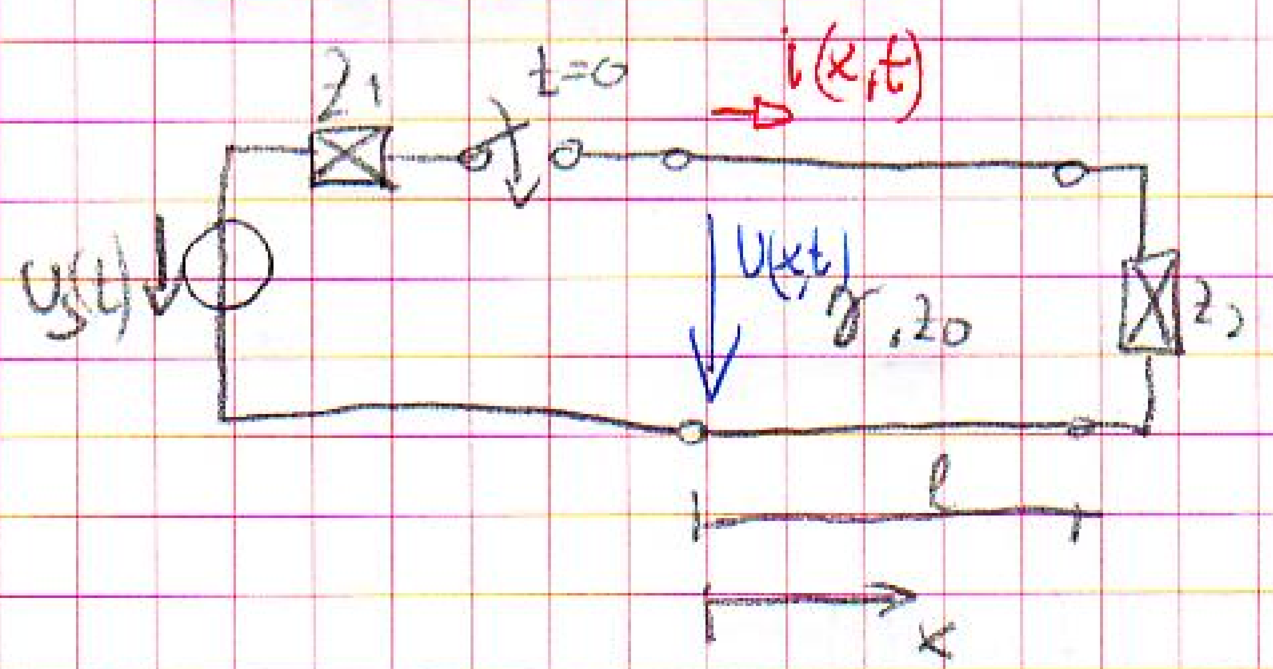


$$Z_{soros} = j\omega L + Z_0 \frac{\frac{1}{j\omega C} + jZ_0\,tg\,\beta l}{Z_0 + j\frac{1}{j\omega C}\,tg\,\beta l}$$

$$\mathrm{Im}(Z_{soros}) = 0 = \left(\omega L + \frac{\frac{Z_0}{\omega C} + Z_0^2\,tg\,\beta l}{Z_0 + \frac{1}{\omega C}\,tg\,\beta l}\right) \Rightarrow \beta l = ?$$

$\uparrow \frac{\omega}{v}$

Távvezetékek tranziensek:



haladó + visszavert

$$u(x,t) = f_1\left(t - \frac{x}{v}\right) + f_2\left(t + \frac{x}{v}\right)$$

$$i(x,t) = \frac{f_1\left(t - \frac{x}{v}\right)}{Z_0} - \frac{f_2\left(t + \frac{x}{v}\right)}{Z_0}$$

a) kezdeti érték feladat

$$t = 0 \quad \left.\begin{aligned} u(x,0) &= U(x) = f_1(x) + f_2(x) \\ i(x,0) &= I(x) = \frac{f_1(x)}{Z_0} - \frac{f_2(x)}{Z_0} \end{aligned}\right\} \quad \begin{aligned} f_1(x) &= \frac{1}{2}\left[U(x) + Z_0 I(x)\right] \\ f_2(x) &= \frac{1}{2}\left[U(x) - Z_0 I(x)\right] \end{aligned}$$

$$u(x,t)=\frac{1}{2}\Big[\underbrace{U\left(t-\frac{x}{v}\right)+z_0 I\left(t-\frac{x}{v}\right)}_{f_1}+\underbrace{U\left(t+\frac{x}{v}\right)-z_0 I\left(t+\frac{x}{v}\right)}_{f_2}\Big]$$

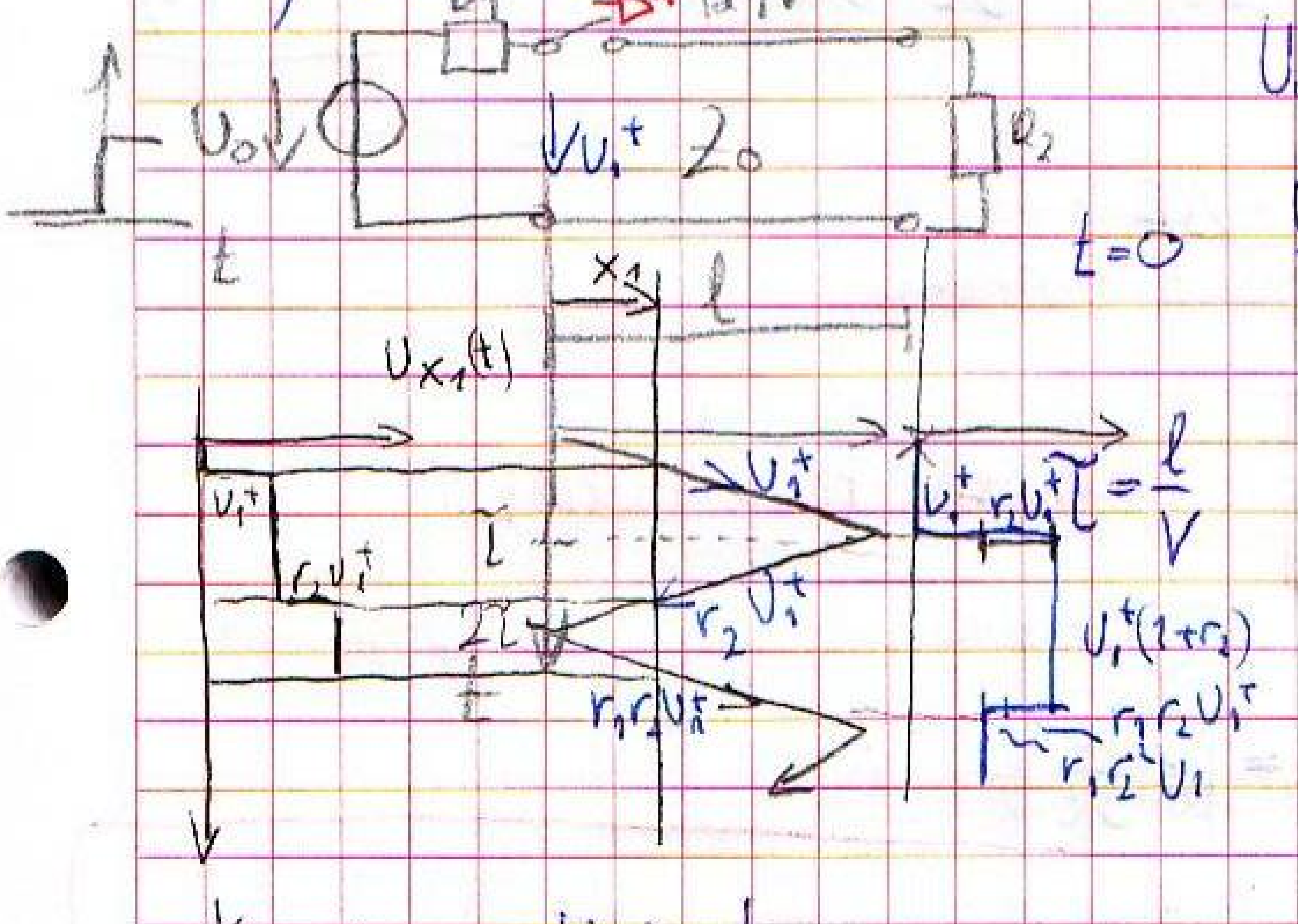
$$i(x,t)=\frac{1}{2z_0}\left[U\left(t-\frac{x}{v}\right)+z_0 I\left(t-\frac{x}{v}\right)-U\left(t+\frac{x}{v}\right)+z_0 I\left(t+\frac{x}{v}\right)\right]$$

b) peremérték feladat

$$x=0 \quad U_S \quad \left.\begin{aligned} u(0,t)&=U_0(t)=f_1(t)+f_2(t)\\ i(0,t)&=I_0(t)=\frac{f_1(t)}{z_0}-\frac{f_2(t)}{z_0}\end{aligned}\right\} \qquad \begin{aligned} f_1(t)&=\left[U_0(t)+z_0 I_0(t)\right]\cdot\frac{1}{2}\\ f_2(t)&=\frac{1}{2}\left[U_0(t)-z_0 I_0(t)\right]\end{aligned}$$

$$u(x,t)=\frac{1}{2}\left[U_0\left(t-\frac{x}{v}\right)\ z_0 I_0\left(t-\frac{x}{v}\right)+U_0\left(t+\frac{x}{v}\right)-z_0 I_0\left(t+\frac{x}{v}\right)\right]$$

c) Menetdiagram



$$U_0=R_1 I_1^+ + U_1^+ = U_1^+\left(\frac{R_1}{z_0}+1\right)$$

$$t=0 \quad \frac{U_1^+}{I_1^+}=z_0$$

$$U_1^+=U_0\frac{z_0}{R_1+z_0} \qquad r_2=\frac{R_2-z_0}{R_2+z_0}$$

$$r_1=\frac{R_1-z_0}{R_1+z_0}$$

$$U_2(\infty)=\frac{U_0 z_0}{z_0+R_1}\Big(1+r_2\underbrace{+r_1 r_2+r_1 r_2^2}_{r_1 r_2(1+r_2)}+r_1^2 r_2^2\ldots\Big)=U_0\left(\frac{z_0}{z_0+R_1}\right)(1+r_2)\left(1+r_1 r_2+(r_1 r_2)^2+\ldots\right)=$$

$$=U_0\frac{z_0}{z_0+R_1}\frac{(1+r_2)}{1-r_1 r_2}=\ldots=U_0\frac{R_2}{R_1+R_2}$$