2007.04.11. szerda XIV. Előadás (9. hét)

Elektromágneses hullám terjedés bemutatja :)

Síkhullámok (szabadon terjedő hullám)

(hullám forrástól messze)

$$\text{rot}\,\bar H = \sigma \bar E + \varepsilon \frac{\partial \bar E}{\partial t} \qquad \bar D = \varepsilon \bar E,\ \bar B = \mu \bar H$$

$$\text{rot}\,\bar E = -\mu \frac{\partial \bar H}{\partial t}$$

$$\text{div}\,\bar B = 0$$

$$\text{div}\,\bar D = 0,\ \rho = 0$$

Maxwell II.: $\underbrace{\text{rot}(\text{rot}\,\bar E)}_{\bar E\ (\text{vektor})} = -\mu\, \text{rot}\left(\frac{\partial \bar H}{\partial t}\right) = -\mu \frac{\partial}{\partial t} \underbrace{(\text{rot}\,\bar H)}_{\text{Maxwell I.}} = \underbrace{-\mu \frac{\partial}{\partial t}\left(\sigma \bar E + \varepsilon \frac{\partial \bar E}{\partial t}\right)}_{\bar E\ (\text{vektor})}$

Matematika: $\text{rot}\,\text{rot}\,\bar E = \text{grad}\, \underbrace{\text{div}\,\bar E}_{\text{IV.M.} \Rightarrow\, = 0} - \Delta \bar E$

$$\boxed{-\Delta \bar E + \mu \sigma \frac{\partial \bar E}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar E}{\partial t^2} = 0}$$

HOMOGÉN HULLÁMEGYENLET

(Hiperbolikus típusú parc. DE) ⇐ mat

vezető közeg: $|\bar J_v| \gg |\bar J_e| = \left|\varepsilon \frac{\partial \bar E}{\partial t}\right|$ (v: vezető, e: levegő)

$$-\Delta \bar E + \mu \sigma \frac{\partial \bar E}{\partial t} = 0 \Rightarrow \boxed{-\left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right) + \mu\sigma \frac{\partial E_x}{\partial t} = 0}$$

csak Descartes koord.!!! (x=x,y=y)

DIFFÚZIÓS EGYENLET (hővezetéshez)

(mat: parabolikus típusú parc. DE)

szigetelő $|\bar J_e| \gg |\bar J_v|$ $\sigma \sim 0$

$$-\Delta \bar E + \mu\varepsilon \frac{\partial^2 \bar E}{\partial t^2} = 0$$

$$\boxed{\Delta \bar E - \mu\varepsilon \frac{\partial^2 \bar E}{\partial t^2} = 0}$$

HULLÁMEGYENLET SZIGETELŐBEN

b) Maxwell I: $\text{rot rot } \bar{H} = \text{grad div } \bar{H} - \Delta \bar{H} = \sigma \text{ rot } \bar{E} + \varepsilon \frac{\partial}{\partial t} \text{rot } \bar{E} =$

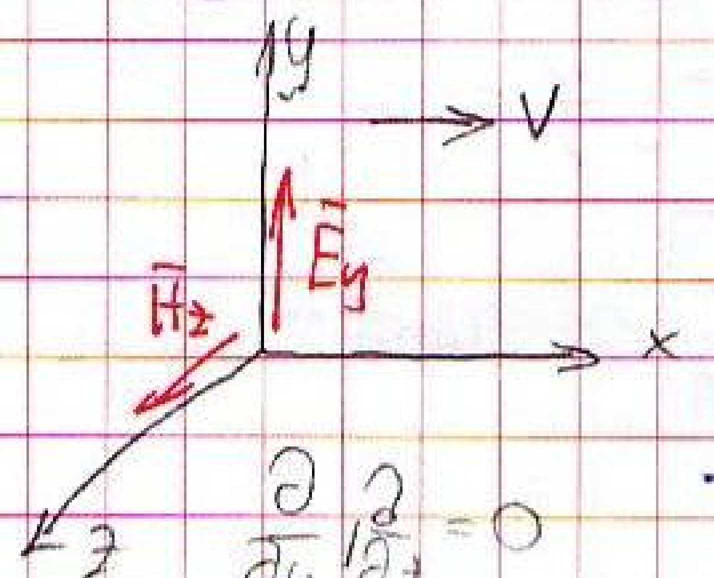
$$= -\mu\sigma \frac{\partial \bar{H}}{\partial t} - \mu\varepsilon \frac{\partial^2 \bar{H}}{\partial t^2}$$

$$\boxed{-\Delta \bar{H} + \mu\sigma \frac{\partial \bar{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{H}}{\partial t^2} = 0}$$

$|J_e| << |J_v| \quad -\Delta \bar{H} + \mu\sigma \frac{\partial \bar{H}}{\partial t} = 0$

$|J_e| >> |J_v| \quad \Delta \bar{H} - \mu\varepsilon \frac{\partial^2 \bar{H}}{\partial t^2} = 0$

szinuszos változás:



$$\bar{E}(x,t) = Re\{\bar{E}(x) e^{j\omega t}\}$$
$$\bar{H}(x,t) = Re\{\bar{H}(x) e^{j\omega t}\}$$

$\frac{\partial}{\partial y}, \frac{\partial}{\partial z} = 0$

$$-\Delta \bar{E}(x) + j\omega\mu\sigma \bar{E}(x) + j\omega\mu\, j\omega\varepsilon\, \bar{E}(x) = 0$$

/csak x irányba halad/

$$-\frac{\partial^2 \bar{E}(x)}{\partial x^2} + \gamma^2 E_x = 0 \quad \gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

$$\Rightarrow \gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \alpha + j\beta$$ TERJEDÉSI EGYÜTTHATÓ

Maxwell II $\Rightarrow \quad -j\omega\mu \bar{H}(x) = \text{rot } \bar{E}(x)$

$$\bar{H}(x) = -\frac{1}{j\omega\mu} \text{rot } \bar{E}(x) = -\frac{1}{j\omega\mu} \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ \frac{\partial}{\partial x} & 0 & 0 \\ E_x & E_y & E_z \end{vmatrix} =$$

$$= -\frac{1}{j\omega\mu} \frac{\partial}{\partial x} \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ 1 & 0 & 0 \\ E_x & E_y & E_z \end{vmatrix} = -\frac{1}{j\omega\mu} \frac{\partial}{\partial x} \left(\bar{e}_x \times \bar{E} \right)$$

(skalár...)

$\Rightarrow \bar{H}(x) \perp \bar{E}(x)$, $\bar{H}(x) \perp \bar{v}_x$, $\bar{E}(x) \cdot \bar{H}(x) = 0$

$\bar{E}(x) = E_y(x)\, \bar{e}_y$

$\bar{H}(x) = H_z(x)\, \bar{e}_z$

$$-\frac{\partial^2 E_y(x)}{\partial x^2} + \gamma^2 E_y(x) = 0 \Rightarrow$$

$$\Rightarrow E_y(x) = E^+ e^{-\gamma x} + E^- e^{\gamma x} \quad \gamma = \alpha + j\beta$$

$$E_y(x,t) = E^+ e^{-\alpha x} \underbrace{\cos\omega\left(t - \frac{x}{v}\right)}_{\cos(\omega t - \beta x)} + E^- e^{\alpha x} \cos\omega\left(t + \frac{x}{v}\right) = f_1\left(t - \frac{x}{v}\right) + f_2\left(t + \frac{x}{v}\right)$$

közeg hullámell. $\frac{j\omega\mu}{\gamma} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = Z_0$

$$H_z(x) = -\frac{1}{j\omega\mu}\frac{\partial}{\partial x}E_y(x) = \frac{\gamma}{j\omega\mu}\left[E^+ e^{-\gamma x} - E^- e^{+\gamma x}\right] = \frac{E^+}{Z_0}e^{-\gamma x} - \frac{E^-}{Z_0}e^{\gamma x}$$

új volt!

$$H_z(x,t) = \frac{E^+}{Z_0}e^{-\alpha x}\cos\omega\left(t-\frac{x}{v}\right) - \frac{E^-}{Z_0}e^{+\alpha x}\cos\omega\left(t+\frac{x}{v}\right) = \frac{f_1\left(t-\frac{x}{v}\right)}{Z_0} - \frac{f_2\left(t+\frac{x}{v}\right)}{Z_0}$$

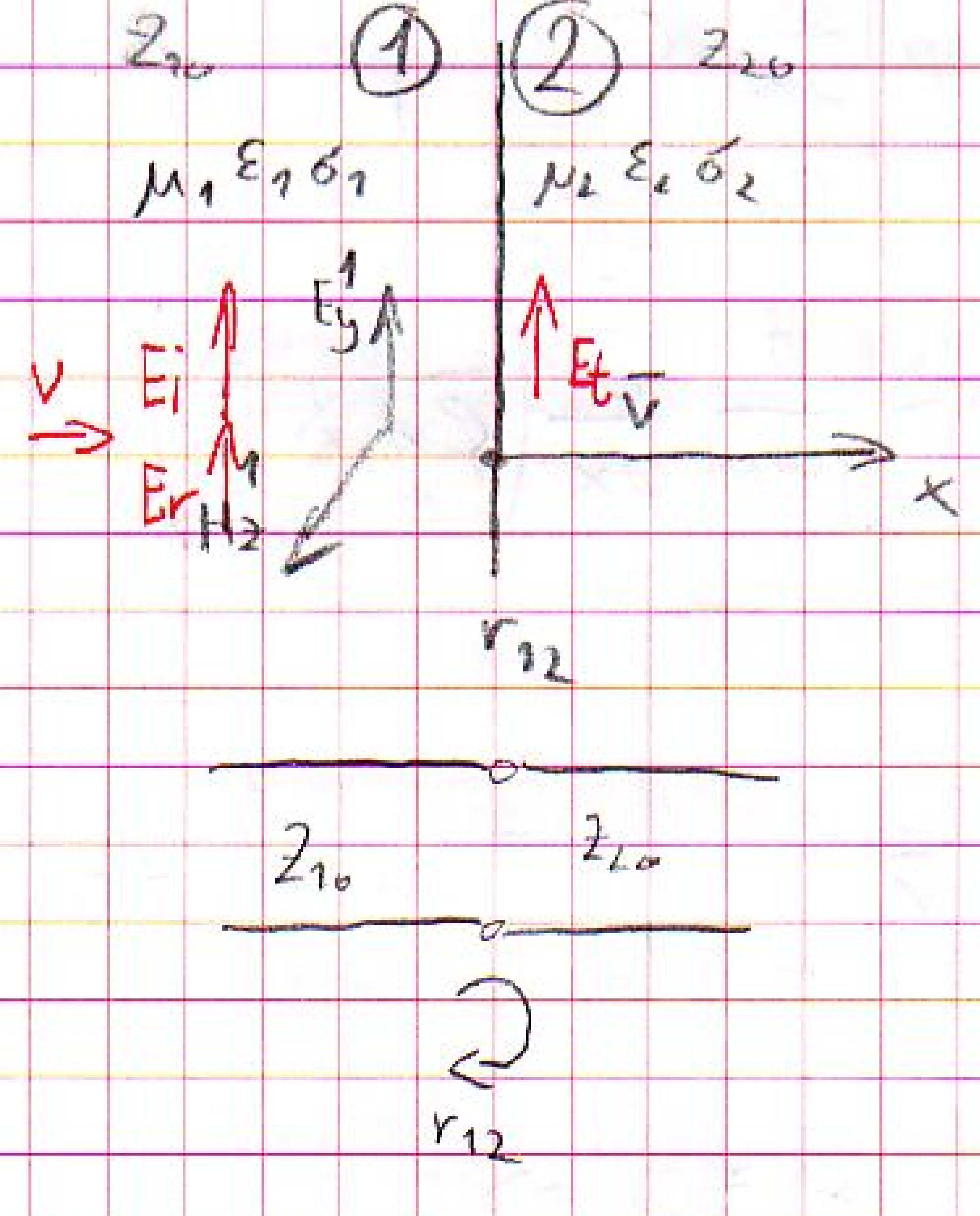
$$Z_0 \triangleq \frac{E_y^+(x)}{H_z^+(x)}$$ HULLÁMIMPEDANCIA DEFINÍCIÓJA

Szigetelő: $\sigma \cong 0$ $\quad \gamma = \sqrt{j\omega\mu\, j\omega\varepsilon} = j\omega\sqrt{\mu\varepsilon} = j\frac{\omega}{v} = j\beta$

$$Z_0 = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\cdot\sqrt{\frac{\mu_r}{\varepsilon_r}} = \sqrt{\frac{4\pi\cdot 10^{-7}}{\frac{10^{-9}}{4\pi 9}}}\sqrt{\frac{\mu_r}{\varepsilon_r}} \overset{\text{levegő}}{\sim} 120\pi \sim 377\,\Omega$$

Analógia: (távvez $\Leftrightarrow$ síkhullám)

TV	U	I	R	L	G	C	γ	Z_0
SH	E_y	H_z	–	μ	σ	ε	γ	Z_0



$E^1 = E^+ + E^- = E_i + E_r = E_i\cdot(1+r_{12})$

beesett, reflektált

$E_r = E_i r_{12}$

$$r_{12} = \frac{Z_{20}-Z_{10}}{Z_{20}+Z_{10}}$$

$E^2 = E_2^+ = E_t$ (transzmittív átvitt)

$x=0$: $E_{1\,tang} = E_{2\,tang}$

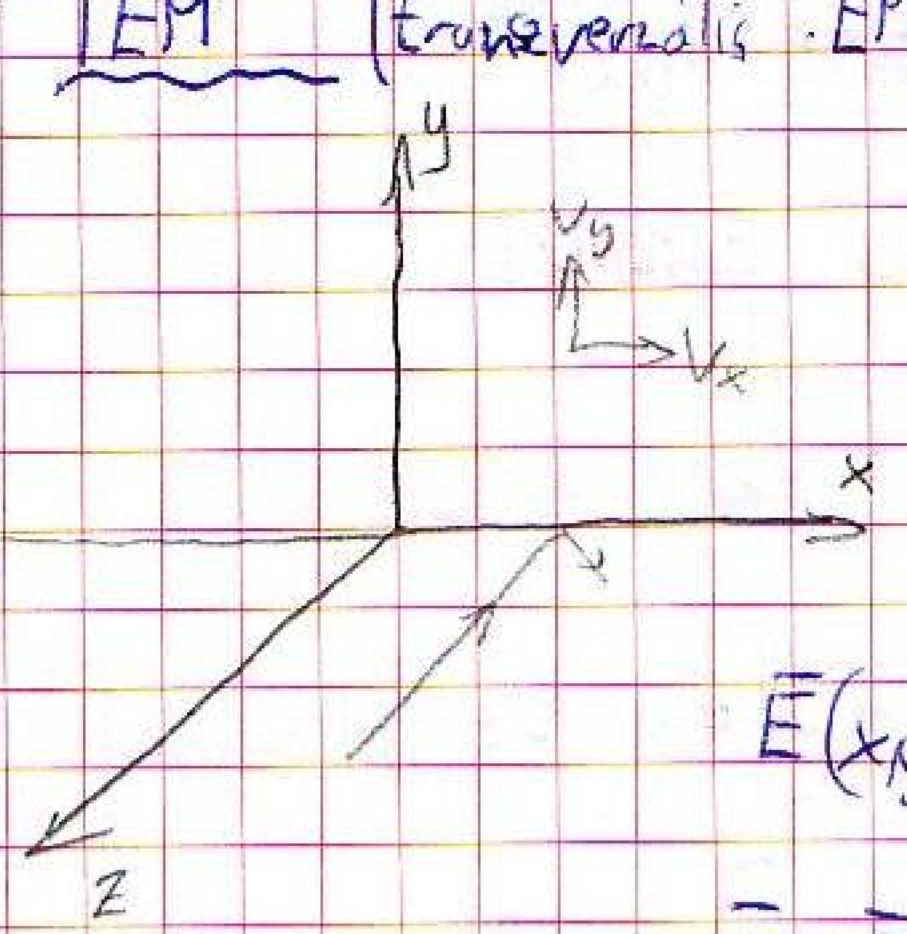
$E_i + E_r = E_t$

$t = 1 + r_{12}$

$x=0$: $H_{1\,tang} = H_{2\,tang}$ $\quad \frac{E_i}{Z_{10}} - \frac{E_r}{Z_{20}} = \frac{E_t}{Z_{20}}$

$E_i + E_r = E_t = E_i(1+r_{12}) = E_i\, t$ $\qquad t$: transzmissziós együttható

TEM (transzverzális EM) típusú hullámterjedés



v_x : $\beta = \frac{\omega}{v_x}$

v_y : $k = \frac{\omega}{v_y}$ (cirkulációs hullámszám)

$$\bar{E}(x,y,t) = E_0 e^{-j\beta x} e^{\mp jky} e^{j\omega t}$$

$\frac{\partial}{\partial z} = 0$

$\frac{\partial}{\partial x} = -j\beta$ $\quad \frac{\partial}{\partial y} = \mp jk$

$$\bar{E} = \bar{E}_x + \bar{E}_y + \bar{E}_z$$

$$\bar{H} = \bar{H}_x + \bar{H}_y + \bar{H}_z$$

!

$$\mathrm{rot}(\bar{E}) = -j\omega\mu\bar{H} = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ -j\beta & \mp jk & 0 \\ E_x & E_y & E_z \end{vmatrix} = \bar{e}_x(\mp jkE_z) - \bar{e}_y(-j\beta E_z) + \bar{e}_z(-j\beta E_y \pm jkE_x)$$

$\sigma = 0$

$$\mathrm{rot}(\bar{H}) = j\omega\varepsilon\bar{E} = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ -j\beta & \mp jk & 0 \\ H_x & H_y & H_z \end{vmatrix} = \bar{e}_x(\mp jkH_z) - \bar{e}_y(-j\beta H_z) + \bar{e}_z(-j\beta H_y \pm jkH_x)$$

$-j\mu\omega H_x = \mp jkE_z$ $\qquad j\omega\varepsilon E_x = \mp jkH_z$

$-j\mu\omega H_y = +j\beta E_z$ $\qquad j\omega\varepsilon E_y = +j\beta H_z$

$-j\omega\mu H_z = -j\beta E_y \pm jkE_x$ $\qquad j\omega\varepsilon E_z = -j\beta H_y \pm jkH_x$

$E_z \Rightarrow H_x, H_y \parallel \bar{v}$

$E_z \neq 0$ $\quad H_x = \pm \frac{k}{\mu\omega} E_z$ $\qquad H_z \neq 0$ $\quad E_x = \mp \frac{k}{\omega\varepsilon} H_z$

$E_x = 0$ $\quad H_y = \frac{-\beta}{\mu\omega} E_z$ $\qquad H_x = 0$ $\quad E_y = \frac{\beta}{\omega\varepsilon} H_z$

$E_y = 0$ $\qquad H_y = 0$ $\quad E_z \equiv 0$

$H_z \equiv 0$

transzverzális elektromos típusú terj $\qquad$ transzverzális mágneses típus terjedés

(TE típusú terj) $\qquad$ (TM típusú terj) $\Rightarrow$ $\boxed{3D}$ TEM = TE + TM

$\otimes$ TE: $j\omega\varepsilon E_z = -j\beta H_y \pm jkH_x = +j\frac{\beta^2}{\mu\omega}E_z + j\frac{k^2}{\mu\omega}E_z$

$$\boxed{\gamma^2 = \omega^2\mu\varepsilon = \beta^2 + k^2}$$ DISZPERZIÓS EGYENLET

Poynting vektor:

TE: $\bar{S} = \frac{1}{2}(\bar{E} \times \bar{H}^*) = \frac{1}{2}\begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ 0 & 0 & E_z \\ H_x^* & H_y^* & 0 \end{vmatrix} = \frac{1}{2}\left[\bar{e}_x\left(-E_z H_y^*\right) - \bar{e}_y\left(-E_z H_x^*\right)\right] = \bar{S}_x + \bar{S}_y$

$S_x = \frac{1}{2}(-E_z)\left(\frac{-\beta}{\omega\mu}E_z^*\right) = +\frac{1}{2}\frac{\beta}{\mu\omega}|E_z|^2 \Rightarrow P$ (hatásos)

$S_y = \frac{1}{2}E_z\left(\frac{\pm k^*}{\mu\omega}E_z^*\right) = \pm\frac{1}{2}\frac{k^*}{\mu\omega}|E_z|^2 \Rightarrow$
- k valós $\Rightarrow P$ (hatásos)
- k imagin. $(k = \pm jK)$ $\Rightarrow Q$ (meddő)

TM: $\bar{S} = \frac{1}{2}\bar{E}_x + \bar{H}^* = \frac{1}{2}\begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ E_x & E_y & 0 \\ 0 & 0 & H_z \end{vmatrix} = \frac{1}{2}\left[\bar{e}_x E_y H_z^* - \bar{e}_y E_x H_z^*\right] = \bar{S}_x + \bar{S}_y$

$S_x = \frac{1}{2}E_y H_z^* = \frac{1}{2}\frac{\beta}{\omega\varepsilon}|H_z|^2 \Rightarrow P$ (hatásos)

$S_y = -\frac{1}{2}E_x H_z^* = \pm\frac{1}{2}\frac{k}{\omega\varepsilon}|H_z|^2 \Rightarrow$
- k valós $\Rightarrow P$ (hatásos)
- $k = \pm jK$ $\Rightarrow Q$ (meddő)