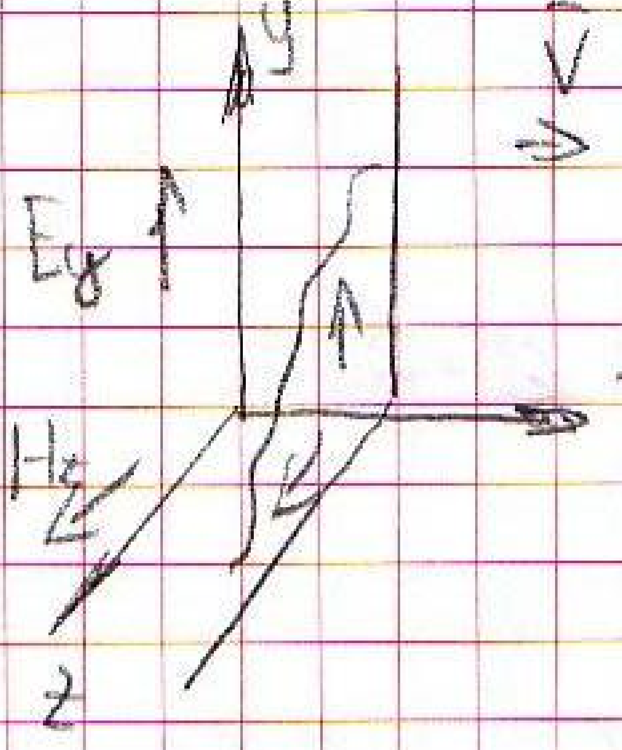
2007. 04. 17. kedd

XV. Előadás (10. hét)



TV	U	I	R	L	G	C	γ	Z_0
SH	E	H	-	μ	σ	ε	γ	Z_0

$$E_y(x) = E^+ e^{-\gamma x} + E^- e^{+\gamma x}$$
$$H_z(x) = \frac{E^+}{Z_0} e^{-\gamma x} + \frac{E^-}{Z_0} e^{+\gamma x}$$

$$E_y^+(x,t) = E^+ e^{-\alpha x} \underbrace{\cos(\omega t - \beta x)}_{\cos\omega\left(t - \frac{x}{v}\right)}$$

$\Rightarrow$ retordült, késleltetett

a) SH vizsgálata $\quad \sigma = 0 \quad -\Delta \bar{E} + j\omega\mu\, j\omega\varepsilon\, \bar{E} = 0$

$$\gamma = \sqrt{j\omega\mu \cdot j\omega\varepsilon} = j\omega\sqrt{\mu\varepsilon} = j\frac{\omega}{v} = j\frac{\omega}{c}\underbrace{\sqrt{\mu_r\varepsilon_r}}_{n} = j\beta, \ \alpha = 0, \ \beta = \frac{\omega}{v}$$

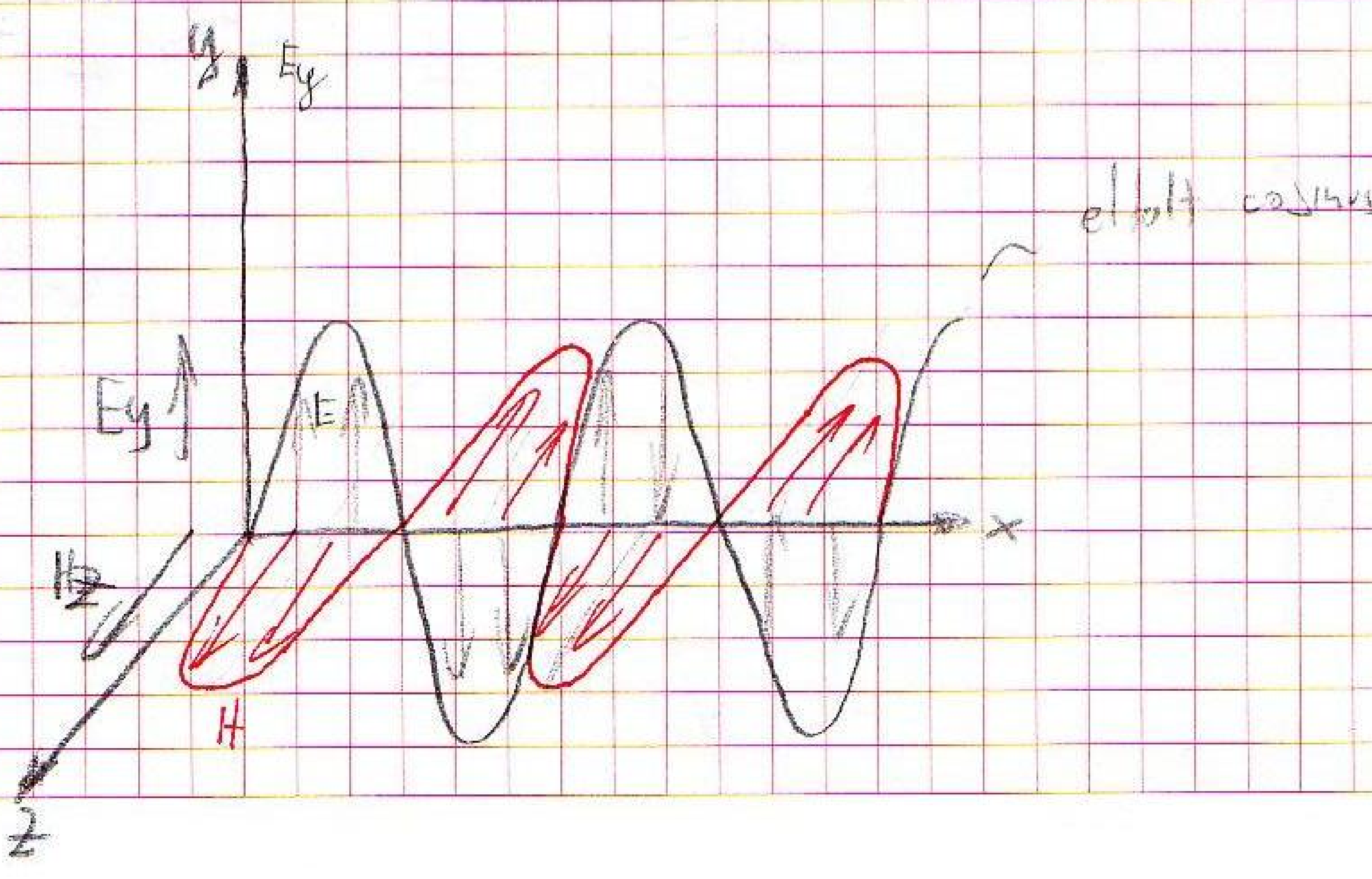
n: törésmutató

$$Z_0 = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = \sqrt{\frac{4\pi\cdot 10^{-7}}{\frac{10^{-9}}{4\pi 9}}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}}\ \Omega$$

ha $\mu_r = 1$, $\varepsilon_r = 1$ (levegő) $\Rightarrow Z_0 = 120\pi = 377\,\Omega$

$$\left.\begin{aligned} E_y^+(x,t) &= E^+ \cos\omega\left(t - \frac{x}{v}\right) \\ H_z^+(x,t) &= \frac{E^+}{Z_0}\cos\omega\left(t - \frac{x}{v}\right) \end{aligned}\right\}$$

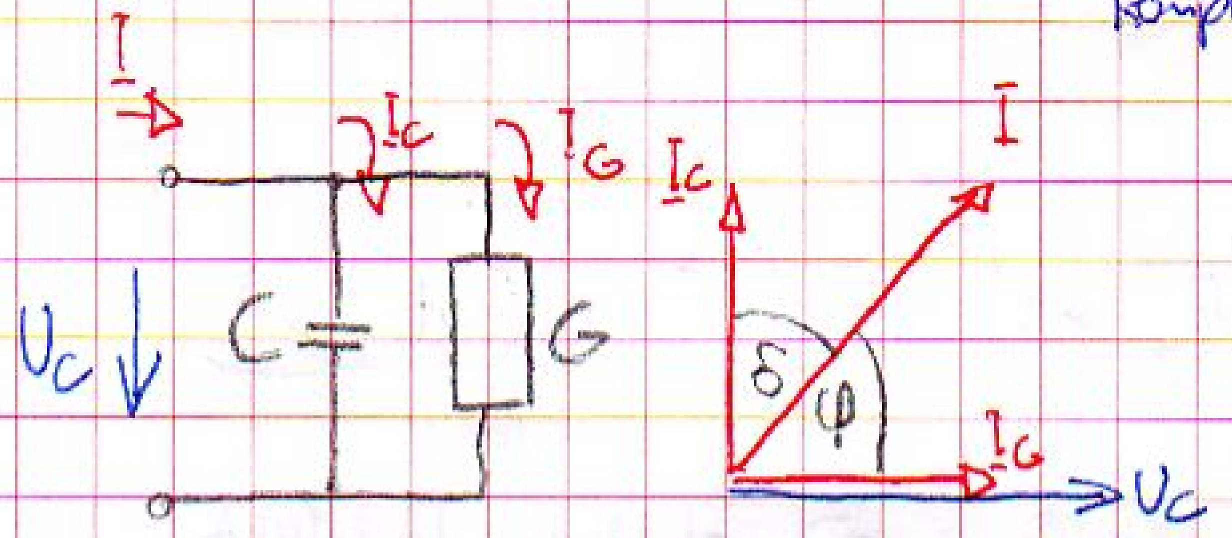
cirkulárisan polarizált hullámok



b) SH veszteséges szigetelő (korona kisülés + zizegő hang)

$$-\Delta \bar{E} + j\omega\mu(\sigma + j\omega\varepsilon)\bar{E} = 0$$

$$\sigma + j\omega\varepsilon = j\omega\varepsilon\left(1 + \frac{\sigma}{j\omega\varepsilon}\right) = j\omega\underbrace{\left[\varepsilon' - j\varepsilon''\right]}_{\text{komplex permittivitás}} \qquad \varepsilon' = \varepsilon \qquad \varepsilon'' = \frac{\sigma}{\omega}$$



veszteségi szög : δ

$$\underbrace{tg\,\delta}_{\text{eztudjuk meg!}} = \frac{I_G}{I_C} = \frac{U_C \cdot G}{U_C \cdot \omega C} \approx \frac{\sigma}{\omega\varepsilon} \ll 1$$

$$j\omega\varepsilon_k = j\omega\varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right) = j\omega\varepsilon\left(1 - j\,tg\,\delta\right)$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}} \cong j\omega\sqrt{\mu\varepsilon}\left(1 - j\frac{tg\,\delta}{2}\right) = \alpha + j\beta$$

Mat: $\sqrt{1-x} \sim 1 - \frac{x}{2} \pm \dots$ $\quad |x| \ll 1$

$$\beta = \frac{\omega}{v}$$

$$\alpha = \frac{\omega}{v}\frac{tg\,\delta}{2}$$

$$Z_0 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{\frac{1}{1 - j\frac{\sigma}{\omega\varepsilon}}} \approx \sqrt{\frac{\mu}{\varepsilon}}\left(1 + j\frac{tg\,\delta}{2}\right) = R_0 + jX_0$$

Mat: $\frac{1}{\sqrt{1-x}} \sim 1 + \frac{x}{2} + \dots$ $\quad |x| \ll 1$

$$R_0 = \sqrt{\frac{\mu}{\varepsilon}}$$

$$X_0 = \sqrt{\frac{\mu}{\varepsilon}}\frac{tg\,\delta}{2}$$

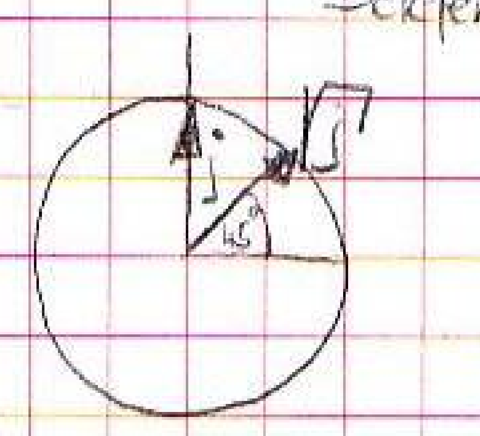
c) SH fémekben (jó vezetőkben)

$$|\bar{J}_v| >> |\bar{J}_e|$$

$$|\sigma \bar{E}| >> |\omega \varepsilon \bar{E}|$$

$|\sigma| >> |\omega \varepsilon|$ $\quad \omega << \frac{\sigma}{\varepsilon} \sim \frac{10^7}{10^{-9}} \sim 10^{16} \frac{rad}{sec}$ eddig igaz => erősítve [Schelkunoff =]

Tekintsük $\varepsilon = 0$ $\quad -\Delta \bar{E} + j\omega\sigma \bar{E} = 0$ $\quad \gamma = \sqrt{j\omega\mu\sigma} = e^{j\frac{\pi}{4}}\sqrt{\omega\mu\sigma} = \frac{1+j}{\sqrt{2}}\sqrt{\mu\omega\sigma} = \frac{1+j}{\delta}$



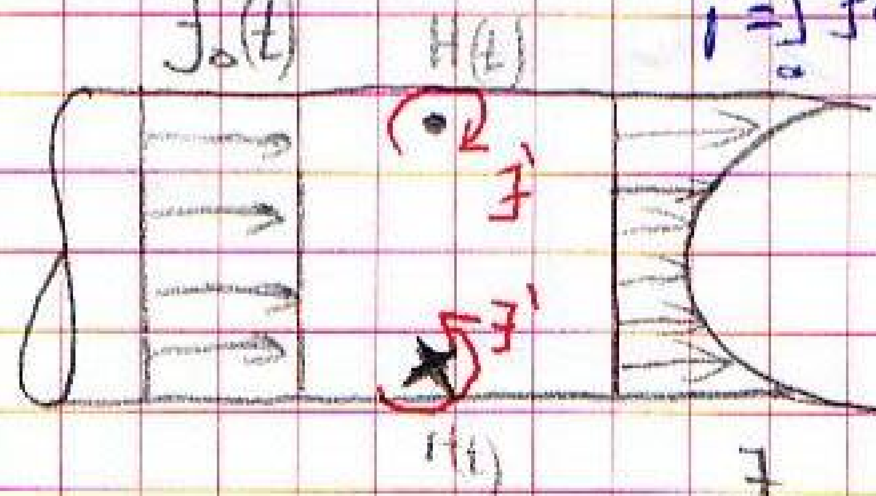
$$Z_0 = \sqrt{\frac{j\omega\mu}{\sigma}}\sqrt{\frac{\sigma}{\sigma}} = \frac{\gamma}{\sigma} = \frac{1+j}{\delta\sigma}$$

$$\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}} \; [m]$$ (BEHATOLÁSI) SKIN-MÉLYSÉG

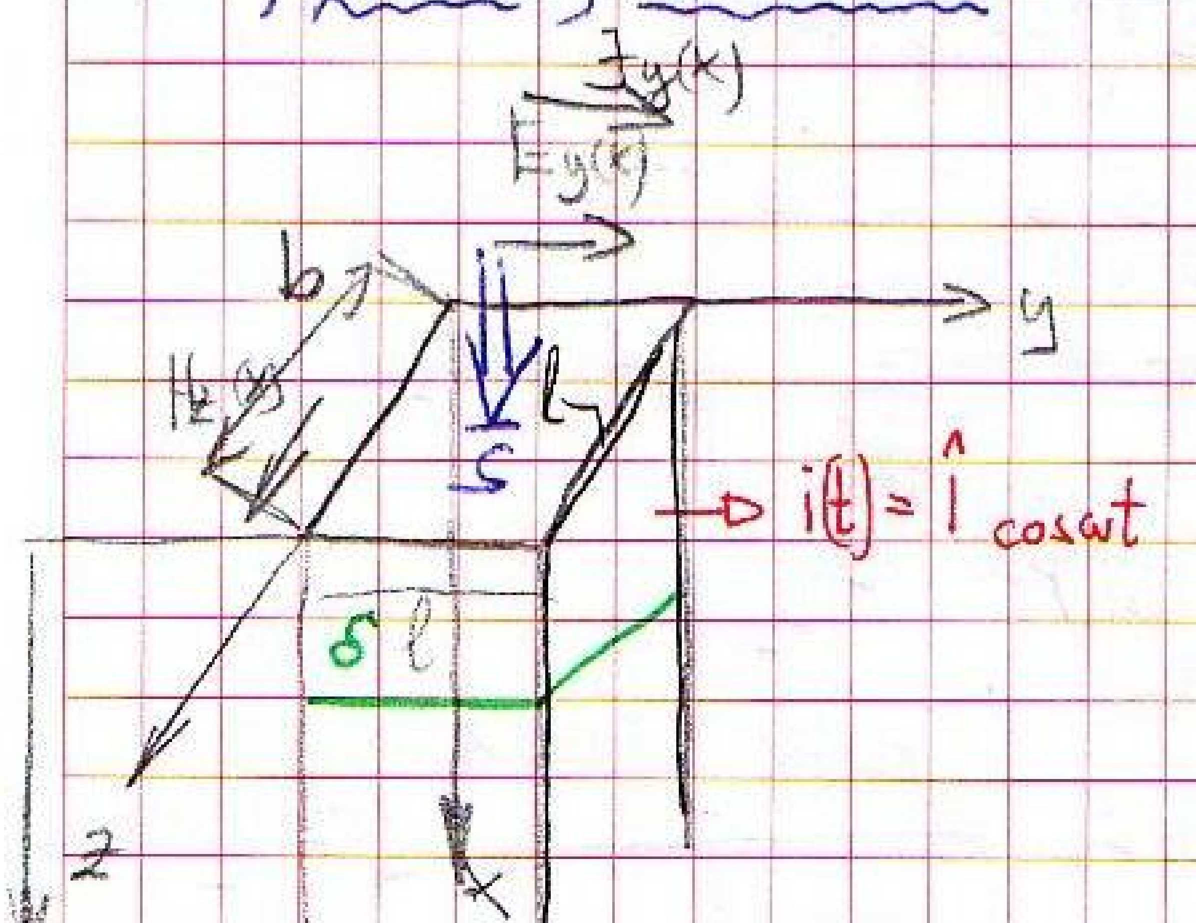
$\frac{r}{y}$ $\frac{V}{Am}$ $\frac{A}{Vm}$

$$\delta_{Cu} = \sqrt{\frac{2}{2\pi f_{Hz} \, 4\pi \cdot 10^{-7} \cdot 57 \cdot 10^{6}}} = \frac{66{,}6}{\sqrt{f_{Hz}}} \; [mm]$$

$$\hat{I} = \int \bar{J} d\bar{a}$$

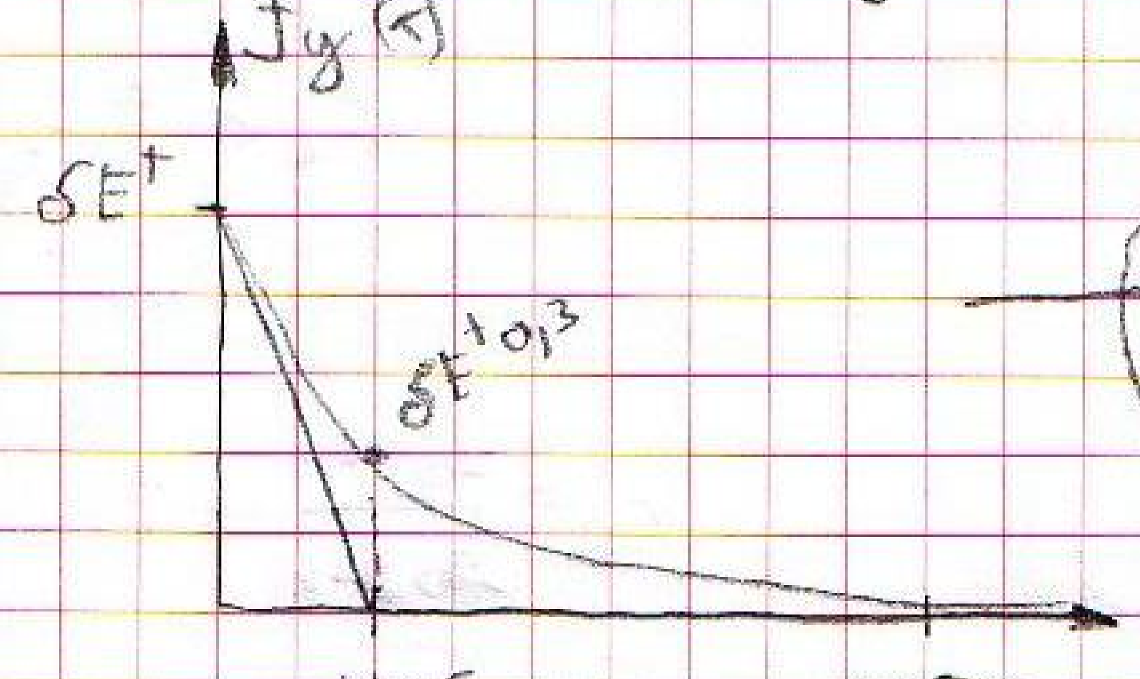


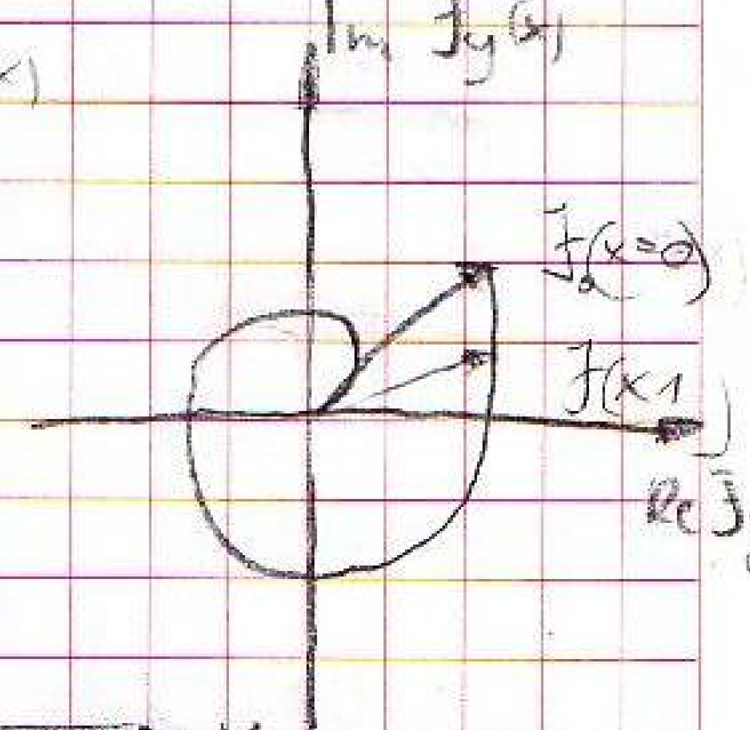
1) SH vastag hasábban



$x \Rightarrow \infty, \; E_y(x) = E^+ e^{-\gamma x} = E^+ e^{-\frac{1+j}{\delta}x} = E^+ e^{-(1+j)\frac{x}{\delta}}$

$$J_y(x) = \sigma E_y(x) = \underbrace{\sigma E^+ e^{-\frac{x}{\delta}}}_{J_y(x)} e^{-j\frac{x}{\delta}}$$

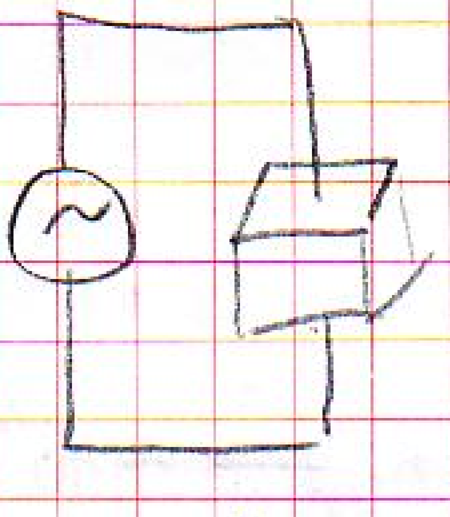




$E^+ = ?$ $\qquad \oint_l \bar{H}\, d\bar{l} = \hat{I} = H_y(x=0)\cdot b = b\frac{E^+}{Z_0} \Rightarrow E^+ = \frac{\hat{I} Z_0}{b}$

$E_y(x) = \hat{I}\frac{Z_0}{b} e^{-\gamma x}$

$H_z(x) = \frac{\hat{I}}{b} e^{-\gamma x}$



$Z_{belső} = ?$

$$P + jQ = \int_a \bar{S}\, d\bar{a} = Z_{belső}\frac{|\hat{I}|^2}{2} = \bar{S}(x=0)\, bl - \cancel{\bar{S}(d)}\, bl =$$

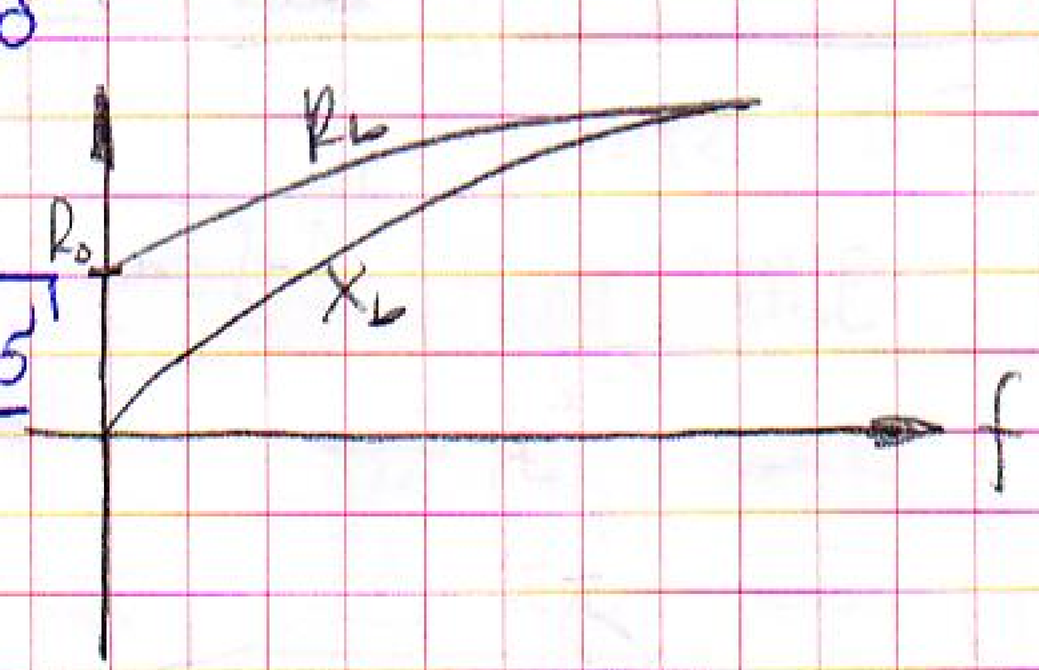
$$= \frac{1}{2} E_y(0) H_z^*(0)\, bl = \frac{1}{2}\frac{\hat{I} Z_0}{b}\cdot\frac{\hat{I}}{b}\cancel{b}l =$$

$$Z_{belső} = \frac{Z_0 l}{b} = \frac{l}{b\sigma}\cdot\frac{1+j}{\delta} = \frac{l}{\sigma b\delta}(1+j)$$

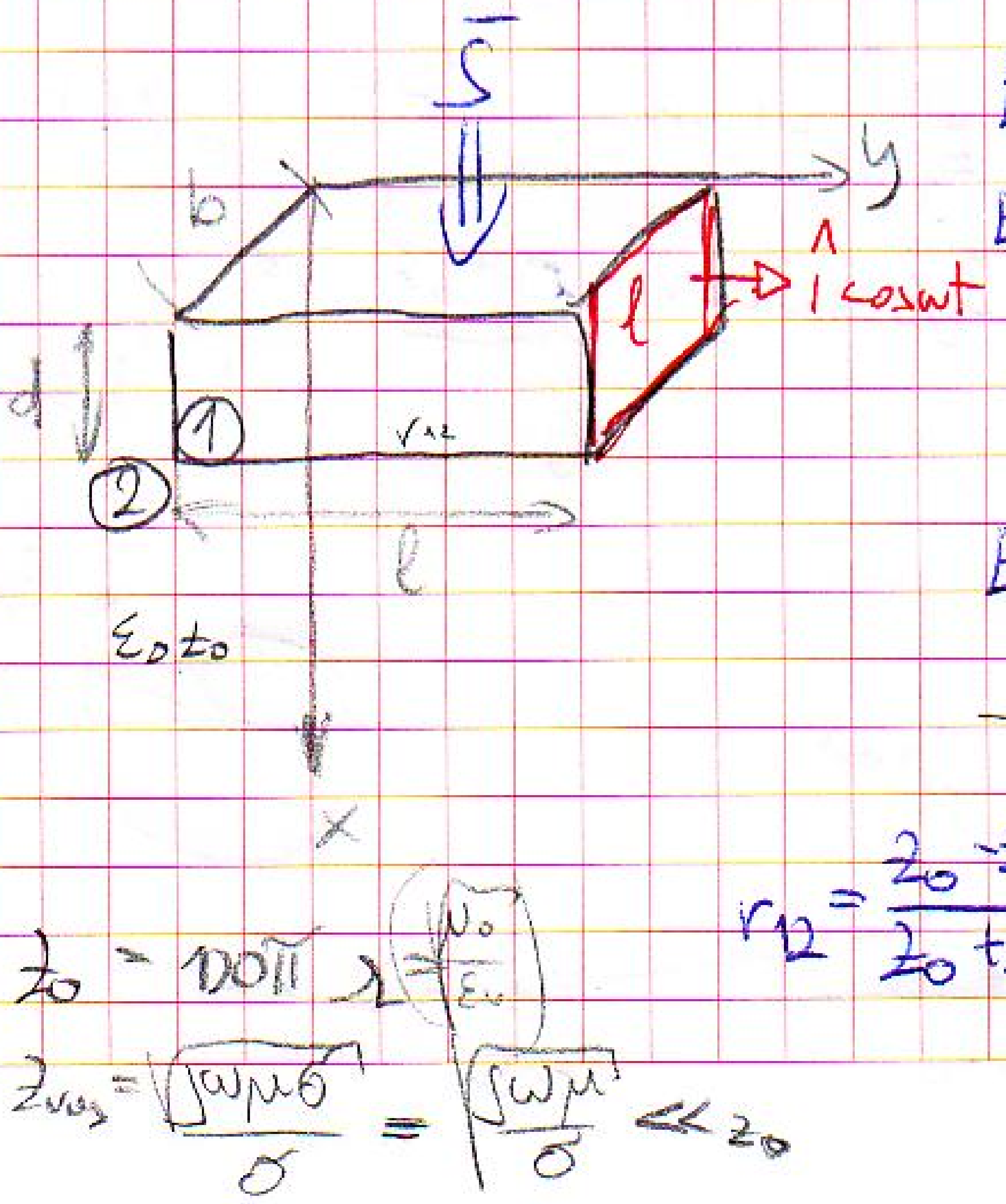
$$Z_0 = \sqrt{\frac{j\omega\mu}{\sigma}} = \frac{\gamma}{\sigma} = \frac{1}{\sigma}\frac{1+j}{\delta}$$

$Z_{belső} = R_b + jX_b$

$\omega L_b = X_b = R_b = \frac{l}{\sigma b\delta} = \frac{l}{\sigma b}\sqrt{\frac{\omega\mu\sigma}{2}}$



2) SH vékony hasábban: $\quad d < 5\delta$



$E_y(x) = E^+ e^{-\gamma x} + E^- e^{\gamma x}$

$E_y(x=d) = E^+ e^{-\gamma d} + E^- e^{+\gamma d}$

$r_{12} = \frac{E^- e^{\gamma d}}{E^+ e^{-\gamma d}} = 1 = \frac{E_2^-}{E_2^+} \Rightarrow E_2^- = E_2^+$

$E_y(x) = E_2^+ e^{+\gamma(d-x)} + E_2^- e^{-\gamma(d-x)} =$

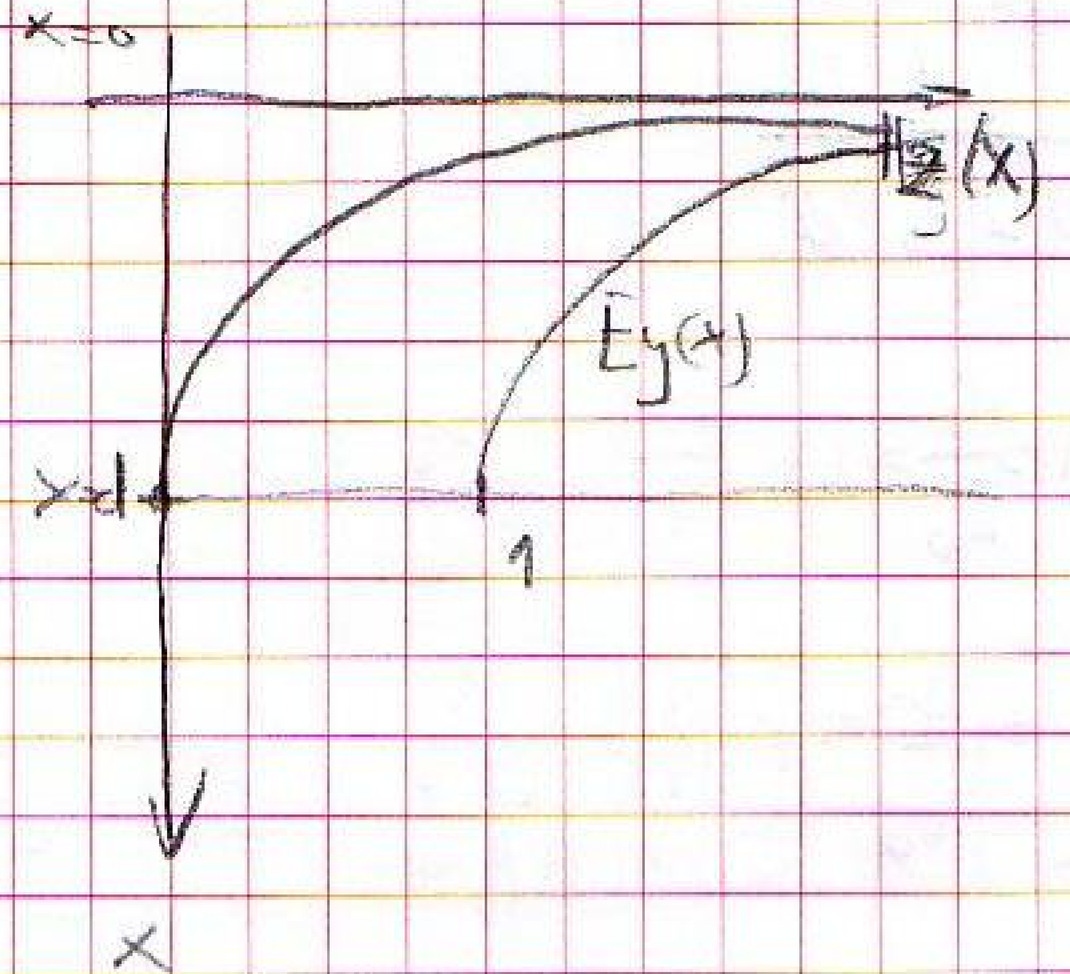
$= E_2^+\left(\frac{e^{\gamma(d-x)} + e^{-\gamma(d-x)}}{2}\right)2 = 2E_2^+ \operatorname{ch}\gamma(d-x)$

$r_{12} = \frac{Z_0 - Z_v}{Z_0 + Z_v} \sim 1$

$Z_0 = 120\pi\ \Omega = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

$Z_{vas} = \frac{\sqrt{j\omega\mu\sigma}}{\sigma} = \sqrt{\frac{j\omega\mu}{\sigma}} \ll Z_0$

$$H_z(x) = \frac{E^+ e^{-\gamma x}}{Z_0} - \frac{E^-}{Z_0} e^{\gamma x} = \frac{2E_z^+}{Z_0}\left(\frac{e^{\gamma(d-x)} - e^{-\gamma(d-x)}}{2}\right) = \frac{2E_z^+}{Z_0}\,\mathrm{ch}\,\gamma(d-x)$$



$\Rightarrow H = 0$ (nincs kilépő $\Rightarrow$ jól árnyékolt)

árnyékolás!

$$\underset{\text{árnyékolási faktor}}{K} = \left|\frac{E_y(x=0)}{E_y(x=d)}\right| = \left|\frac{\cancel{2E^+}\,\mathrm{ch}\,\gamma d}{\cancel{2E^+}}\right| = |\mathrm{ch}\,\gamma d|$$

$$\gamma d = \frac{1+j}{\delta} d = (1+j)\frac{d}{\delta} \qquad |\gamma d| << 1 \Rightarrow \mathrm{ch}\,\gamma d \sim 1 + \frac{\gamma d}{2}$$

$$K = \left|\mathrm{ch}\left(1 + \frac{\gamma d}{2}\right)\right| = \left|\left(1 + \frac{d}{\delta}\right) + j\frac{d}{\delta}\right| =$$

Impedancia

$$\oint_l H_z(x)\, d\bar{l} = \hat{I} = H_z(x=0)\, b - \cancel{H_z(x=d)}\, b$$

$$\hat{I} = b \cdot \frac{2E_z^+}{Z_0}\,\mathrm{sh}\,\gamma d$$

$$E_y(x) = \frac{\hat{I} Z_0}{b} \frac{\mathrm{ch}\,\gamma(d-x)}{\mathrm{sh}\,\gamma d}$$

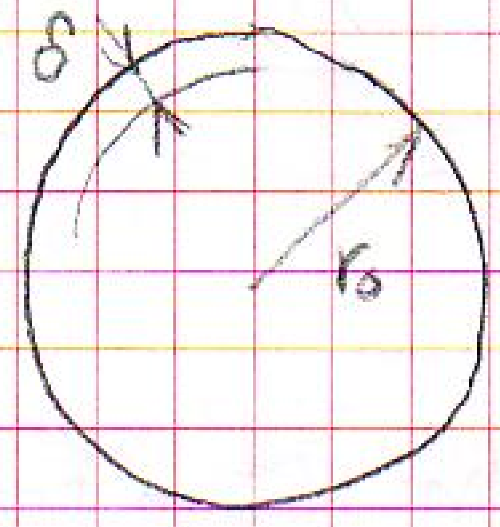
$$H_z(x) = \frac{\hat{I}}{b} \frac{\mathrm{ch}\,\gamma\, d-x}{\mathrm{sh}\,\gamma d}$$

$$P + jQ = Z_b \frac{|\hat{I}|^2}{2} = \oint_a \bar{S}\, d\bar{a} = S(x=0)\, b\, l - \cancel{S(x=d)\, b\, l} =$$

$$= \frac{1}{2} E_y(x=0) H_z^*(x=0)\, lb = \frac{\hat{I}}{b} Z_0 \frac{\mathrm{ch}\,\gamma d}{\mathrm{sh}\,\gamma d} \frac{\hat{I}^*}{\cancel{b}} \frac{l\cancel{b}}{2}$$

$$Z_b = \frac{Z_0 l}{b} \frac{\mathrm{ch}\,\gamma d}{\mathrm{sh}\,\gamma d}$$

3)



$5\delta < r_0 \Rightarrow$ vastag huzal

$$Z_b = R_b + jX_b = (1+j)\frac{l}{\sigma 2 r_0 \pi \delta}$$

$x = \frac{r_0}{2\delta}$, ha $x < 1 \Rightarrow \frac{R_b}{R_0} = 1 + \frac{x^4}{4}$ $\quad \frac{X_b}{R_0} = x^2 \rightarrow L_b = \frac{\mu l}{8\pi}$

segédvált

ha $x > 1 \Rightarrow \frac{R_b}{R_0} = x + \frac{1}{4} + \frac{3}{64x}$, $\frac{X_b}{R_0} = x - \frac{3}{64x}$