2007.04.24. kedd   XVI. Előadás (11. hét)

Gerjesztett hullámok (inhomogén)

$$\text{rot}\,\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \qquad \text{div}\,\bar{B} = 0 \qquad \bar{D} = \varepsilon \bar{E} \qquad \bar{J} = \sigma \bar{E}$$

$$\text{rot}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad \text{div}\,\bar{D} = \varrho \qquad \bar{B} = \mu \bar{H} \qquad \bar{E}_b = 0$$

$\bar{J}(\bar{r},t)$, $\varrho(\bar{r},t)$ adott/ismert

mágn. vektor pot

III $\text{div}\,\bar{B} = 0 \Rightarrow \bar{B} = \text{rot}\,\bar{A} \qquad \bar{A}(r,t)$

II $\text{rot}\,\bar{E} = -\frac{\partial}{\partial t}\text{rot}\,\bar{A} = -\text{rot}\,\frac{\partial \bar{A}}{\partial t} \Rightarrow \text{rot}\left(\underbrace{\bar{E} + \frac{\partial \bar{A}}{\partial t}}_{-\text{grad}\,\phi}\right) = 0 \qquad \bar{E} + \frac{\partial \bar{A}}{\partial t} = -\text{grad}\,\phi$ elektr. skalár pot.

$$\boxed{\bar{E} = -\frac{\partial \bar{A}}{\partial t} - \text{grad}\,\phi}$$

$$U_{12} = \int_{r_1}^{r_2} \bar{E}\,d\bar{l} = \underbrace{\int_{r_1}^{r_2} -\text{grad}\,\phi\,d\bar{l}} - \int_{r_1}^{r_2} \frac{\partial \bar{A}}{\partial t}\,d\bar{l}$$

$$U_{12} = \phi(r_1) - \phi(r_2) - \int_{r_1}^{r_2} \frac{\partial \bar{A}}{\partial t}\,dl \Rightarrow \text{a tér NEM potenciális}$$

I) $$\text{rot}\left(\frac{\text{rot}\,\bar{A}}{\mu}\right) = \underbrace{\text{grad}\left(\frac{1}{\mu}\,\text{div}\,\bar{A}\right) - \frac{1}{\mu}\,\Delta\bar{A}} = \bar{J} + \varepsilon\frac{\partial}{\partial t}\left(-\frac{\partial \bar{A}}{\partial t} - \text{grad}\,\phi\right) =$$

$$= \bar{J} - \varepsilon \frac{\partial^2 \bar{A}}{\partial t^2} - \varepsilon \,\mathrm{grad}\, \frac{\partial \phi}{\partial t}$$

$$\mathrm{IV)}\quad \mathrm{div}\, \varepsilon \left(-\frac{\partial \bar{A}}{\partial t} - \mathrm{grad}\, \varepsilon \right) = \varrho \quad \Rightarrow \quad -\Delta \phi - \frac{\partial}{\partial t} \left(\mathrm{div}\, \bar{A} \right) = \frac{\varrho}{\varepsilon}$$

$\mu, \varepsilon =$ áll

a) $\mathrm{div}\, \bar{A} = ?$

Coulomb-mérték $\quad \mathrm{div}\, \bar{A} = 0$

$$\mathrm{I)}\quad -\Delta A + \mu\varepsilon \frac{\partial^2 \bar{A}}{\partial t^2} = \mu \bar{J} - \mu \varepsilon \,\mathrm{grad}\, \frac{\partial \phi}{\partial t}$$

$$\mathrm{IV)}\quad -\Delta \phi = \frac{\varrho}{\varepsilon}$$

b) Lorentz-feltétel

$$\frac{\mathrm{div}\, \bar{A}}{\mu} = -\varepsilon \frac{\partial \phi}{\partial t} \Rightarrow \boxed{\mathrm{div}\, \bar{A} = -\mu \varepsilon \frac{\partial \phi}{\partial t}}$$

$$\boxed{\begin{aligned} -\Delta \bar{A} + \mu\varepsilon \frac{\partial^2 \bar{A}}{\partial t^2} &= \mu \bar{J} \\ -\Delta \phi + \mu\varepsilon \frac{\partial^2 \phi}{\partial t^2} &= \frac{\varrho}{\varepsilon} \end{aligned}}$$

Inhomogén hullámegyenletek
(Hiperbolikus típusú PDE-k)

$$\left. \begin{aligned} \bar{A}^* &= \bar{A} + \mathrm{grad}\, \Psi \\ \phi^* &= \phi - \frac{\partial \Psi}{\partial t} \end{aligned} \right\} \text{Lorentz-feltétel}$$

$$\mathrm{div}\, A^* = \mathrm{div}\, A^* + \mathrm{div}\, \mathrm{grad}\, \Psi =$$
$$= \mu\varepsilon \left(\frac{\partial}{\partial t} - \frac{\partial^2 \Psi}{\partial t^2} \right)$$

$$\Delta \Psi - \mu\varepsilon \frac{\partial^2 \Psi}{\partial t^2} = 0$$

Szinuszos gerjesztés (frekvencia tartományban):

$$\bar{H}(\bar{r},t) = Re\{\bar{H}(\bar{r})e^{j\omega t}\}$$
$$\bar{A}(\bar{r},t) = Re\{\bar{A}(\bar{r})\cdot e^{j\omega t}\}$$
$$\phi(\bar{r},t) = Re\{\phi(\bar{r})\, e^{j\omega t}\}$$

$\boxed{e^{j\omega t}:}$

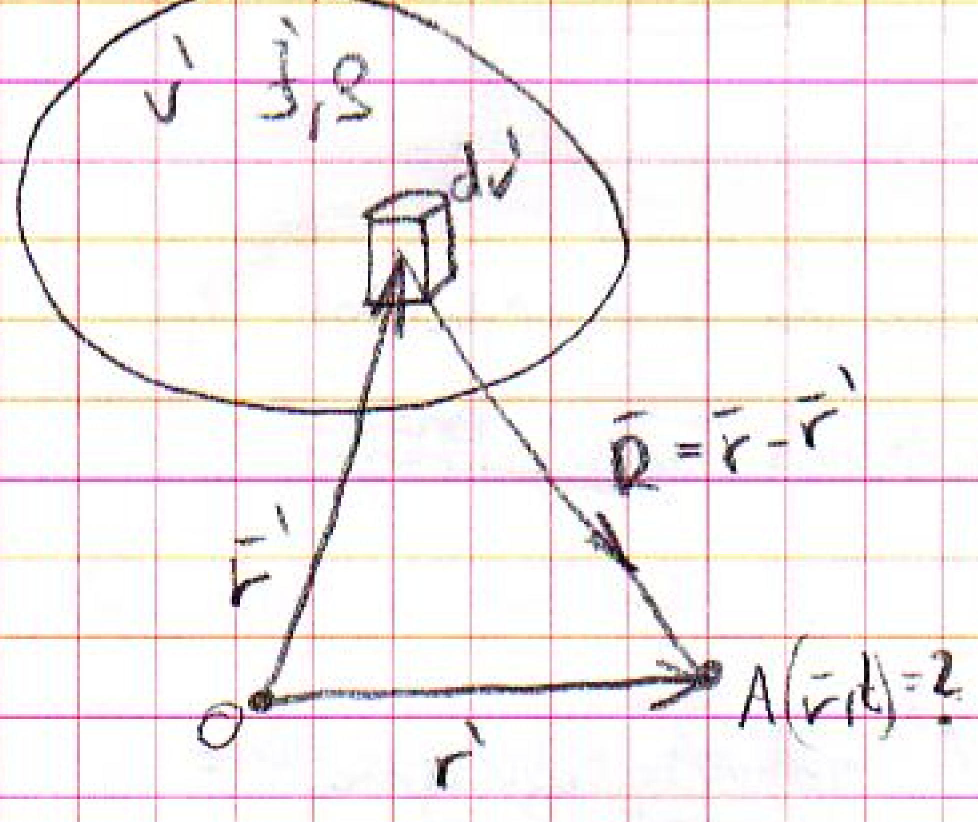
$$\bar{B}(\bar{r}) = rot\,\bar{A}(\bar{r})$$
$$\bar{E}(\bar{r}) = -j\omega\bar{A}(\bar{r}) - grad\,\phi(\bar{r}) = -j\omega\bar{A}(\bar{r}) + \frac{1}{j\omega\mu\varepsilon} grad(div\,\bar{A}(\bar{r}))$$
$$div\,\bar{A}(\bar{r}) = -j\omega\mu\varepsilon\,\phi(\bar{r})$$

$$\left.\begin{aligned} -\Delta\bar{A}(\bar{r}) + j\omega\mu j\omega\varepsilon\,\bar{A}(\bar{r}) &= \mu\bar{J}(\bar{r}) \\ -\Delta\phi(\bar{r}) + j\omega\mu j\omega\varepsilon\,\phi(\bar{r}) &= \frac{\rho(\bar{r})}{\varepsilon} \end{aligned}\right\}$$

$A(\,,t) = A\left(t - \frac{r}{v}\right)$ retardált/késleltetett potenciál

HELMHOLTZ EGYENLET

(elliptikus típusú PDE)



$$\bar{A}(\bar{r},t) = \frac{\mu}{4\pi}\int_{V'} \frac{\bar{J}\left(\bar{r}', t - \frac{R}{v}\right)}{R}\, dV' = \frac{\mu}{4\pi}\int_{V'} \frac{\bar{J}\left(\bar{r}', t - \frac{|\bar{r}-\bar{r}'|}{v}\right)}{|\bar{r}-\bar{r}'|}\, dV$$

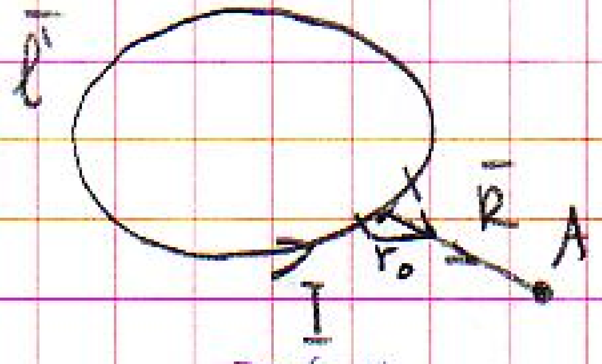
$$\phi(\bar{r},t) = \frac{1}{4\pi\varepsilon}\int_{V'} \frac{\rho\left(\bar{r}', t - \frac{R}{v}\right)}{R}\, dV' = \ldots\ldots$$

$R = |\bar{r} - \bar{r}'| << \lambda$ majdnem stacionárius a tér

(kvázistacionárius), örvényáram terek

$$Re\left\{\bar{A}(\bar{r},t) = \frac{\mu}{4\pi}\int_{V'} \frac{\bar{J}(\bar{r}')}{|\bar{r}-\bar{r}'|}\, dV'\, e^{j\omega t}\right\} = A(\bar{r},t) \text{ valós időfv.}$$

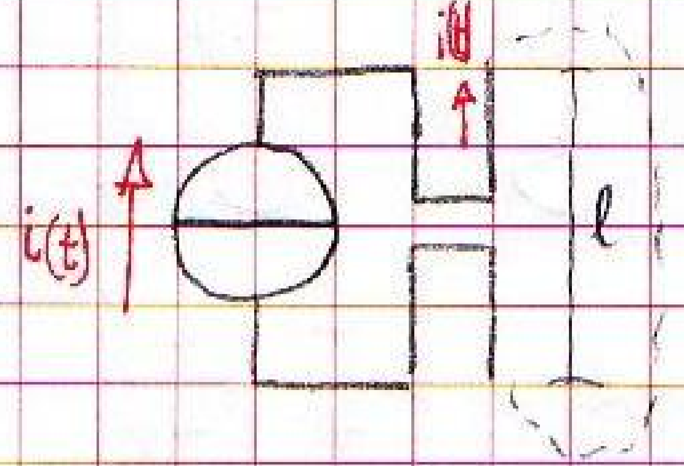
Vonalas vezető:

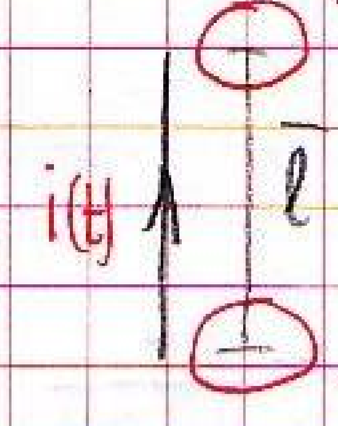


$$\bar{A}(\bar{r}) = \frac{\mu I}{4\pi}\oint_{l'} \frac{dl'}{|\bar{r}-\bar{r}'|} \Rightarrow H(\bar{r}) = \frac{I}{4\pi}\oint_{l'} \frac{dl' \times \bar{r}_0}{R^2}$$

$$\bar{J}(\bar{r}')dV' = I\,dl'$$

Hertz dipólus (antenna)





$+Q(t) = Q_0 e^{j\omega t}$

$-Q(t)$

$\bar{p} = q\bar{l} = \bar{l}\,\mathrm{Re}\{Q_0 e^{j\omega t}\}$

$\frac{d\bar{p}}{dt} = \bar{l}\frac{dq}{dt} = \bar{l}\, i(t) = \mathrm{Re}\{\underbrace{j\omega \bar{l} Q_0 e^{j\omega t}}_{j\omega \bar{p}_0}\} =$

$i(t) = I_0 \cos\omega t$

$= \mathrm{Re}\{I_0 \bar{l} e^{j\omega t}\}$

$\bar{p}_0 = \frac{I_0}{j\omega}\bar{l}$

valódi antenna $d\bar{A}$-ból

$|\bar{r}| \gg |\bar{l}|$ $\quad \bar{A} = \frac{\mu}{4\pi}\frac{i\left(t-\frac{r}{v}\right)}{r}\bar{l} = \frac{\mu}{4\pi}\frac{I_0\bar{l}}{r}e^{j\omega\left(t-\frac{r}{v}\right)}$

gömbi koord. rendszer

$$\bar{A}(\bar{r},t) = \underbrace{\frac{\mu I_0 \bar{l}}{4\pi r} e^{-j\frac{\omega}{v}r}}_{\bar{A}(\bar{r})}\, e^{j\omega t}$$

$\frac{\omega}{v} = \beta$, $\quad v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$

$\boxed{e^{j\omega t}}$ $\quad \bar{H}(\bar{r}) = \frac{1}{\mu}\mathrm{rot}\,\bar{A}(\bar{r}) = \frac{I_0}{4\pi}\mathrm{grad}\left(\frac{e^{-j\beta r}}{r}\right)\times\bar{l} = \frac{I_0}{4\pi}\underbrace{\left[-\frac{1}{r^2} - j\frac{\beta}{r}\right]}_{\text{rögzített } \bar{r}} e^{-j\beta r}\underbrace{\bar{e}_r\times\bar{l}}_{1\cdot l\sin\vartheta\,\bar{e}_\varphi}$

$\bar{A}(\bar{r}) = f(\bar{r})\bar{l}$, $\quad \mathrm{rot}\,\bar{A}(\bar{r}) = f(\bar{r})\cancel{\mathrm{rot}\,\bar{l}} + \mathrm{grad}\,f(\bar{r})\times\bar{l}$

$\bar{H} = \bar{H}_r + \bar{H}_\vartheta + \bar{H}_\varphi$

$H_r = 0$
$H_\vartheta = 0$ $\quad \bar{e}_r \times \bar{l} = -\bar{l}\times\bar{e}_r$

$$H_\varphi = \frac{I_0 l}{4\pi}\left[\frac{1}{r^2} + j\frac{\beta}{r}\right]\sin\vartheta\, e^{-j\beta r}$$

⇒ akkor max, ha $\sin\vartheta = 1$ ⇒ ha antennára merőleges

közeltér $\quad$ távoli tér

⇒ villámcsapás ⇒ szegép ferkenelet

$$\mathrm{I)}\quad \mathrm{rot}\,\bar{H} = j\omega\varepsilon\bar{E} \Rightarrow \bar{E}(\bar{r}) = \frac{1}{j\omega\varepsilon}\,\mathrm{rot}\,\bar{H} = \frac{1}{j\omega\varepsilon}\begin{vmatrix} \frac{\bar{e}_r}{r^2\sin\vartheta} & \frac{\bar{e}_\vartheta}{r\sin\vartheta} & \frac{\bar{e}_\varphi}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\vartheta} & 0 \\ 0 & 0 & r\sin\vartheta H_\varphi \end{vmatrix} =$$

$$= \frac{1}{j\omega\varepsilon}\left[\frac{\bar{e}_r}{r^2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(r\sin\vartheta H_\varphi\right) - \frac{\bar{e}_\vartheta}{r\cancel{\sin\vartheta}}\frac{\partial}{\partial r}\left(r\cancel{\sin\vartheta}H_\varphi\right)\right] = \bar{E}_r + \bar{E}_\vartheta$$

$$E_r = \frac{1}{j\omega\varepsilon r\cancel{\sin\vartheta}}\frac{\partial}{\partial\vartheta}\left(\frac{I_0 l}{4\pi}\left(\frac{1}{r^2} + j\frac{\beta}{r}\right)\underbrace{\sin^2\vartheta}_{2\cancel{\sin\vartheta}\cos\vartheta}\right)e^{-j\beta r} = \frac{I_0 l}{4\pi}\left(\frac{1}{j\omega\varepsilon r^3} + \frac{j\beta}{j\omega\varepsilon r^2}\right)2\cos\vartheta\, e^{-j\beta r}$$

még közelebbi tér — közeli tér

$$E_\vartheta = \frac{-1}{j\omega\varepsilon}\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{I_0 l}{4\pi}\left(\frac{1}{r} + j\beta\right)e^{-j\beta r}\sin\vartheta\right) = \frac{1}{j\omega\varepsilon r}\left[-\frac{1}{r^2} + \left(\frac{1}{r} + j\beta\right)(-j\beta)\right]\frac{I_0 l}{4\pi}\sin\vartheta\, e^{-j\beta r} =$$

$$= \frac{I_0 l}{4\pi}\left(-\frac{1}{j\omega\varepsilon r^3} + \frac{j\beta}{j\omega\varepsilon r^2} + \frac{(j\beta)^2}{j\omega\varepsilon r}\right)\sin\vartheta\, e^{-j\beta r}$$

nagyon közeli — közeli — távoli

Poynting-vektor

$$\bar{S} = \frac{1}{2}\bar{E}\times\bar{H}^* = \frac{1}{2}\begin{vmatrix} \bar{e}_r & \bar{e}_\vartheta & \bar{e}_\varphi \\ E_r & E_\vartheta & 0 \\ 0 & 0 & H_\varphi^* \end{vmatrix} = \frac{1}{2}\bar{e}_r E_\vartheta H_\varphi^* - \bar{e}_\vartheta\frac{1}{2}E_r H_\varphi^*$$

Távoli tér:

$$\left.\begin{aligned} H_\varphi &= \frac{I_0 l}{4\pi} j\frac{\beta}{r}\sin\vartheta\, e^{-j\beta r} \\ E_\vartheta &= \frac{I_0 l}{4\pi}\frac{(j\beta)^2}{j\omega\varepsilon r}\sin\vartheta\, e^{-j\beta r} \end{aligned}\right\}\ \bar{S}_r \text{ valós } \begin{pmatrix}\text{távoliterben} \\ \text{hatásos telj}\end{pmatrix}$$

csak sugárirányú energia áramlás