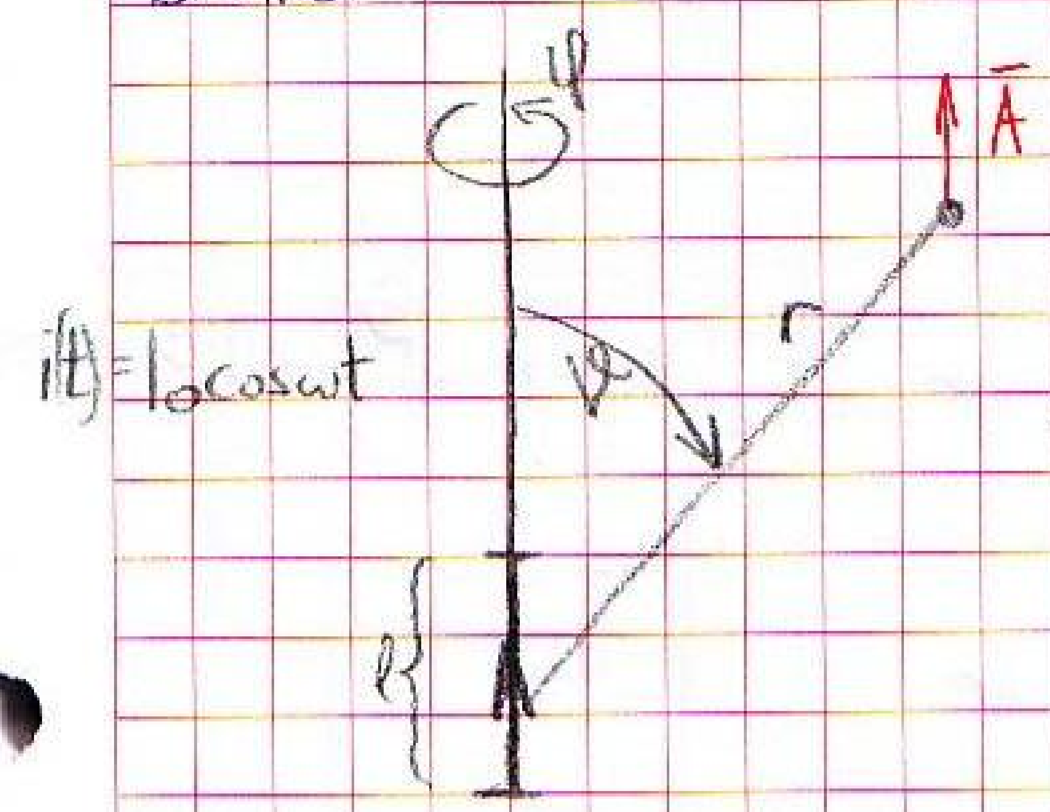
2007.04.25.-szerda XVII. Előadás (11. hét)

Hertz-dipolus antenna (folyt)

~ismétlés~



$$\bar{A}(\bar{r},t)=\frac{\mu}{4\pi}\frac{i\left(t-\frac{|\bar{r}|}{v}\right)}{r}\bar{\ell}=\mathrm{Re}\left\{\frac{I_0\mu}{4\pi}\frac{e^{j\omega\left(t-\frac{r}{v}\right)}}{r}\bar{\ell}\right\}=$$

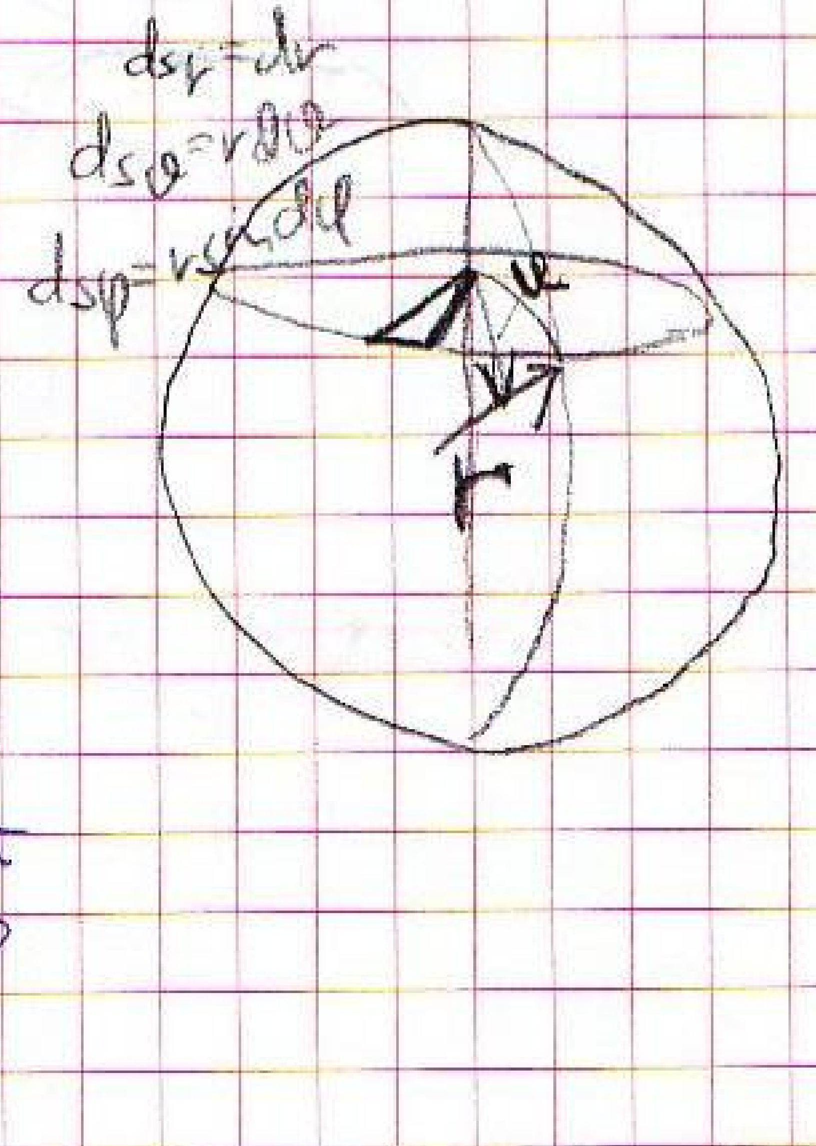
$$\overset{\frac{\omega}{v}r=\beta r}{=}\mathrm{Re}\left\{\underbrace{\frac{\mu I_0}{4\pi}\frac{e^{-j\beta r}}{r}\bar{\ell}}_{\bar{A}(\bar{r})}e^{j\omega t}\right\}=\bar{A}(r,\vartheta,\varphi,t)$$

$$\bar{H}(\bar{r})=\frac{\mathrm{rot}\,\bar{A}(\bar{r})}{\mu}=\frac{1}{\mu}\frac{\mu I_0}{4\pi}\underbrace{\mathrm{rot}\left[\left(\frac{e^{-j\beta r}}{r}\right)\bar{\ell}\right]}$$

$$\mathrm{rot}\,f(r)\bar{\ell}=f(r)\underbrace{\mathrm{rot}\,\bar{\ell}}_{\substack{\text{hosszvektor}\\ \text{változik}}}+\mathrm{grad}\,f(r)\times\bar{\ell}$$

$$\bar{H}(\bar{r})=\frac{I_0}{4\pi}\left[\frac{\partial}{\partial r}\left(\frac{e^{-j\beta r}}{r}\right)\times\bar{\ell}\right]=\frac{I_0}{4\pi}\left(-\frac{1}{r^2}-j\frac{\beta}{r}\right)e^{-j\beta r}\underbrace{\bar{e}_r\times\bar{\ell}}_{-\ell\times\bar{e}_r=-\bar{e}_\varphi\ell\sin\vartheta}=$$

$$=\frac{I_0}{4\pi}\left(\frac{1}{r^2}+j\frac{\beta}{r}\right)e^{-j\beta r}\ell\sin\vartheta\,\bar{e}_\varphi$$



$\Rightarrow$ $H_r=0$, $H_\vartheta=0$ $\quad H_\varphi=\frac{I_0\ell}{4\pi}\left[\frac{1}{r^2}+\frac{j\beta}{r}\right]\sin\vartheta\, e^{-j\beta r}$

I) $\mathrm{rot}\,\bar{H}=j\omega\varepsilon\bar{E}\Rightarrow\bar{E}=\frac{1}{j\omega\varepsilon}\mathrm{rot}\,\bar{H}=$

$$=\frac{1}{j\omega\varepsilon}\begin{vmatrix}\frac{\bar{e}_r}{r^2\sin\vartheta} & \frac{\bar{e}_\vartheta}{r\sin\vartheta} & \frac{\bar{e}_\varphi}{r}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\vartheta} & 0\\ 0 & 0 & r\sin\vartheta H_\varphi\end{vmatrix}=$$

folyt $\Rightarrow$

$$= \frac{1}{j\omega\varepsilon}\left[\frac{\bar{e}_r}{r^2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(r\sin\vartheta\, H_\varphi\right) - \frac{\bar{e}_\vartheta}{r\cancel{\sin\vartheta}}\frac{\partial}{\partial r}\left(r\cancel{\sin\vartheta}\, H_\varphi\right)\right] = E_r + E_\vartheta$$

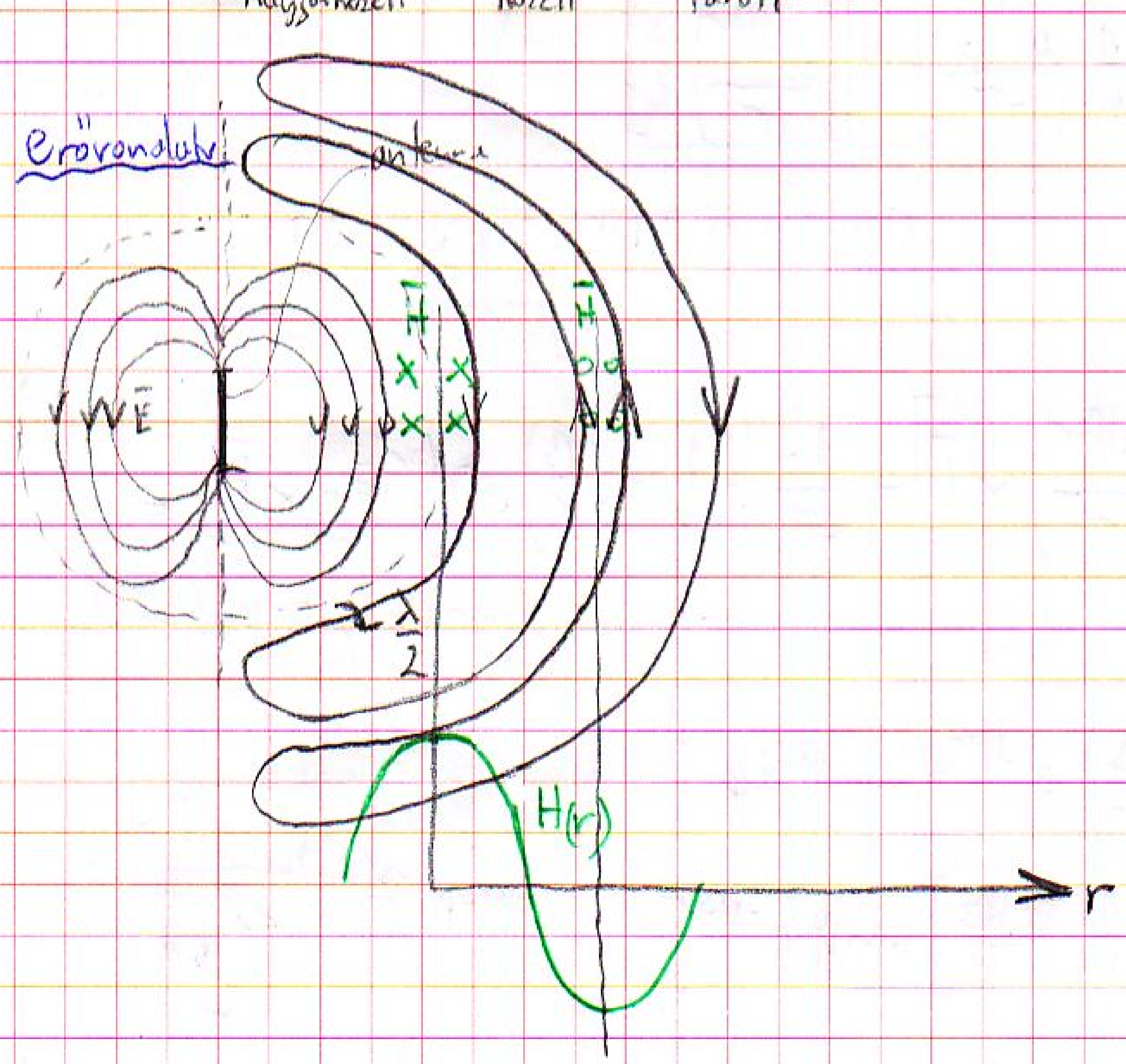
$$E_r = \frac{1}{j\omega\varepsilon\, r\cancel{\sin\vartheta}}\frac{\partial}{\partial\vartheta}\left(\frac{I_0 l}{4\pi}\left[\frac{1}{r^2}+\frac{j\beta}{r}\right]e^{-j\beta r}\underset{\Downarrow}{\sin^2\vartheta}\right) = \frac{I_0 l}{4\pi}\left(\frac{1}{j\omega\varepsilon r^3}+\frac{j\beta}{j\omega\varepsilon r^2}\right)2\cos\vartheta\, e^{-j\beta r}$$

$$2\cancel{\sin\vartheta}\cos\vartheta$$

$$E_\vartheta = -\frac{1}{j\omega\varepsilon\cdot r}\frac{\partial}{\partial r}\left(\frac{I_0 l}{4\pi}\left(\frac{1}{r}+j\beta\right)e^{-j\beta r}\sin\vartheta\right) = \frac{-1}{j\omega\varepsilon r}\frac{I_0 l}{4\pi}\left[-\frac{1}{r^2}+\left(\frac{1}{r}+j\beta\right)(-j\beta)\right]e^{-j\beta r}\sin\vartheta =$$

$$= \frac{I_0 l}{4\pi}\left[\underbrace{\frac{1}{j\omega\varepsilon r^3}}_{\text{nagyon közeli}}+\underbrace{\frac{j\beta}{j\omega\varepsilon r^2}}_{\text{közeli}}+\underbrace{\frac{(j\beta)^2}{j\omega\varepsilon r}}_{\text{távoli}}\right]\sin\vartheta\, e^{-j\beta r}$$

~ ismétlés vége ~



Távoli tér $|l| \ll r$

$H_r = H_\vartheta = 0$

$H_\varphi = \frac{I_0 l}{4\pi} \frac{j\beta}{r} \sin\vartheta \, e^{-j\beta r}$

$E_r = 0, \ E_\varphi = 0$

$E_\vartheta = \frac{I_0 l}{4\pi} \frac{(j\beta)^2}{j\omega\varepsilon} \cdot \frac{1}{r} \sin\vartheta \, e^{-j\beta r}$

tulajdonsága: síkhullámok!

1) $\bar{H} \parallel$ (φ irányú) $(\bar{e}_r \times \bar{l})$, $\bar{E} \perp \bar{H}$,

2) $\frac{E_\vartheta}{H_\varphi} = \frac{|\bar{E}|}{|\bar{H}|} = Z_0 = \frac{j\beta}{j\omega\varepsilon} = \frac{\sqrt{j\omega\mu j\omega\varepsilon}}{j\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \sqrt{\frac{\mu_r}{\varepsilon_r}} \overset{\text{levegő}}{=} \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi = 377\,\Omega$

$\gamma = j\beta = \sqrt{j\omega\mu j\omega\varepsilon}$

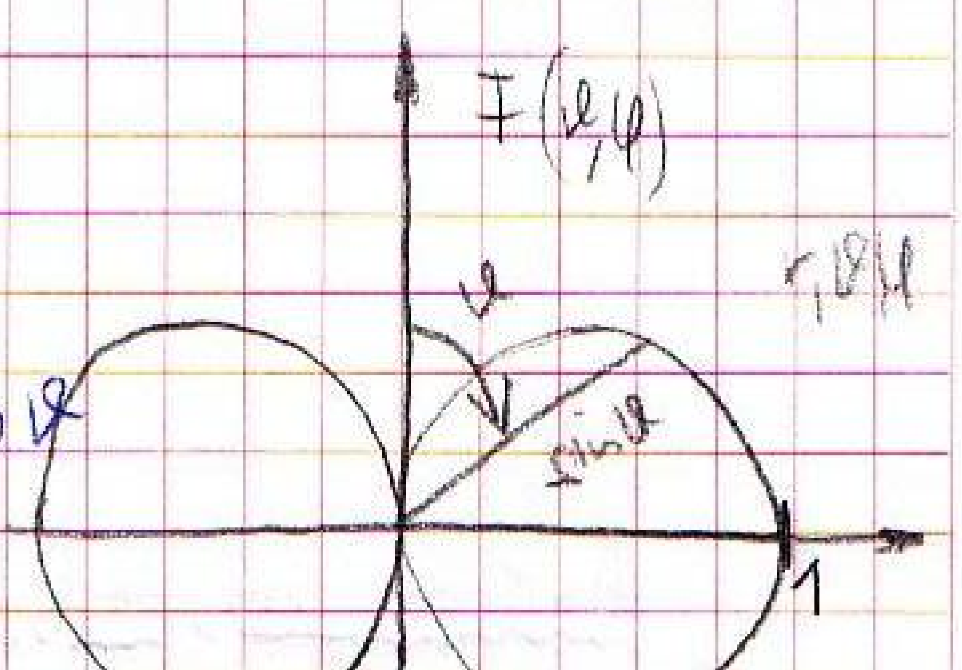
3) $H_\varphi = j \frac{I_0 l}{4\pi} \cdot \frac{2\pi}{\lambda} \cdot \frac{\sin\vartheta}{r} e^{-j\beta r} = j \frac{I_0}{2} \left(\frac{l}{\lambda}\right) \frac{\sin\vartheta}{r} e^{-j\beta r} = H_\varphi(r, \vartheta, \varphi)$

$\beta = \frac{2\pi}{\lambda}$

$E_\vartheta = Z_0 H_\varphi = j \frac{I_0}{2} \left(\frac{l}{\lambda}\right) \sqrt{\frac{\mu}{\varepsilon}} \frac{\sin\vartheta}{r} e^{-j\beta r} = j \frac{I_0}{2} \left(\frac{l}{\lambda}\right) 120\pi \cdot \frac{\sin\vartheta}{r} e^{-j\beta r} = E_\vartheta(r, \vartheta, \varphi)$

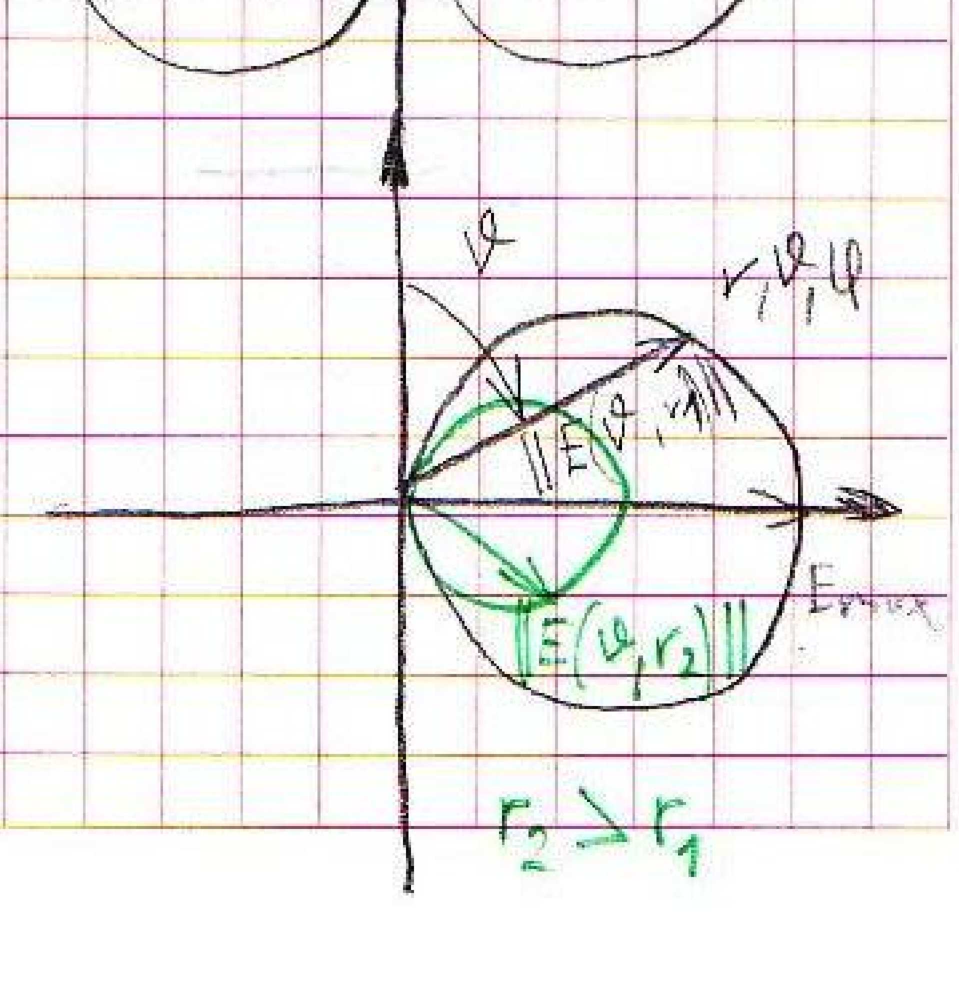
Antenna sugárzási irány karakterisztikája:

$\| E_\vartheta(r, \vartheta, \varphi) \|_{max} \Big|_{\vartheta = 90°} = \frac{I_0}{2} \left(\frac{l}{\lambda}\right) \frac{120\pi}{r}$



iránykarakterisztika $\Rightarrow F(\vartheta, \varphi) = \frac{\| E_\vartheta(r, \vartheta, \varphi) \|}{\| E_\vartheta(r, \vartheta, \varphi) \|_{max}} = \sin\vartheta$

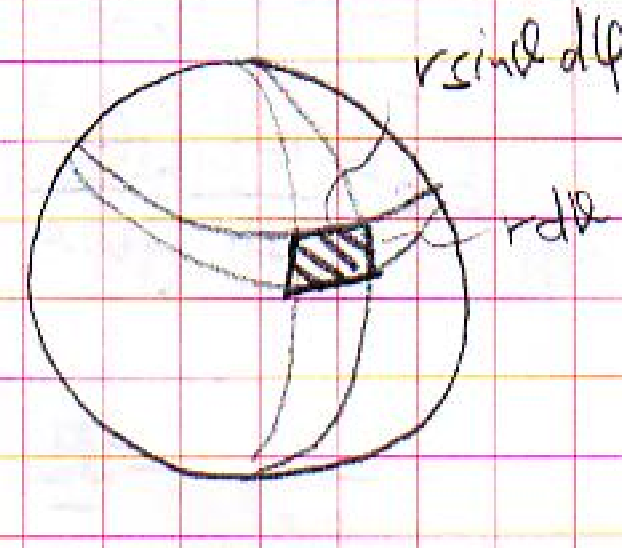
$\boxed{E_\vartheta(r, \vartheta) = E_{max} \sin\vartheta}$



Kisugárzott teljesítmény:

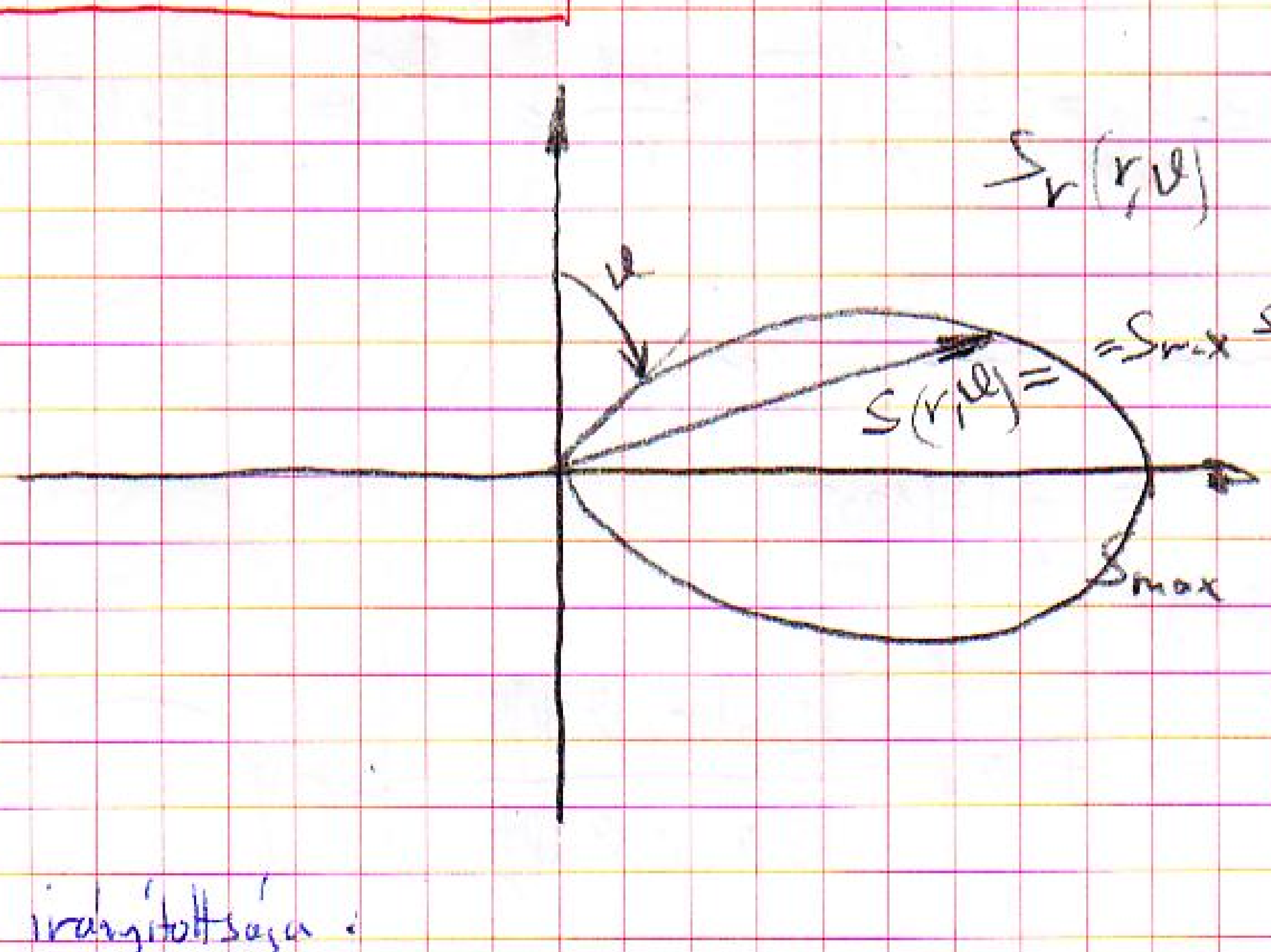
$$\bar S = \frac{1}{2}\bar E \times \bar H = \frac{1}{2}\begin{vmatrix} \bar e_r & \bar e_\vartheta & \bar e_\varphi \\ 0 & E_\vartheta & 0 \\ 0 & 0 & H_\varphi^* \end{vmatrix} = \bar e_r \frac{1}{2} E_\vartheta H_\varphi^* = \frac{1}{2}\left(\frac{I_0}{2}\right)^2\left(\frac{l}{\lambda}\right)^2 120\pi \frac{\sin^2\vartheta}{r^2}\bar e_r \text{ (valós)}$$

$$P = \oint_a \bar S \, d\bar a = \frac{1}{2}\underbrace{\frac{I_0^2}{2}}_{I_{eff}^2}\left(\frac{l}{\lambda}\right)^2 60\pi \int_{\varphi=0}^{2\pi}\int_{\vartheta=0}^{\pi}\frac{\sin^2\vartheta}{r^2}\, r\,d\vartheta\, r\sin\vartheta\, d\varphi =$$



$$= \frac{\cancel{1}}{\cancel{2}}\frac{I_0^2}{2}\left(\frac{l}{\lambda}\right)^2 60\pi\cdot\cancel{2}\pi\underbrace{\int_{\vartheta=0}^{\pi}\sin^3\vartheta\, d\vartheta}_{4/3} = \frac{I_0^2}{2}\left(\frac{l}{\lambda}\right)^2 80\pi^2 = \underbrace{\frac{I_0^2}{2}}_{I_{eff}^2} R_{sugárzási}$$

$$\boxed{R_{sug} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 \quad [\Omega]}$$ SUGÁRZÁSI ELLENÁLLÁS

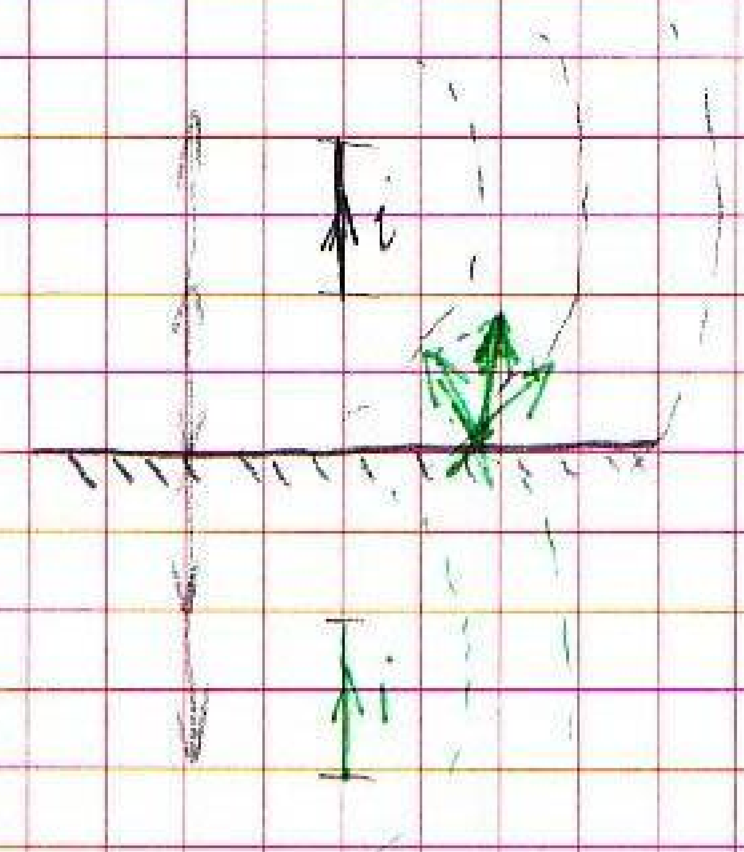


Antenna irányítottsága:

$$D = \frac{S_{max}}{S_{átl}} \qquad S_{max}\Big|_{\vartheta=\frac{\pi}{2}} = \frac{I_0^2}{2}\left(\frac{l}{\lambda}\right)^2\frac{30\pi}{r^2}$$

$$S_{átl} = \frac{P}{A} = \frac{P}{4r^2\pi} = \frac{\frac{I_0^2}{2}\left(\frac{l}{\lambda}\right)^2 80\pi^2}{4r^2\pi}$$

$$\underline{\underline{D}} = \frac{30\pi}{20\pi} = \underline{\underline{1{,}5}}$$



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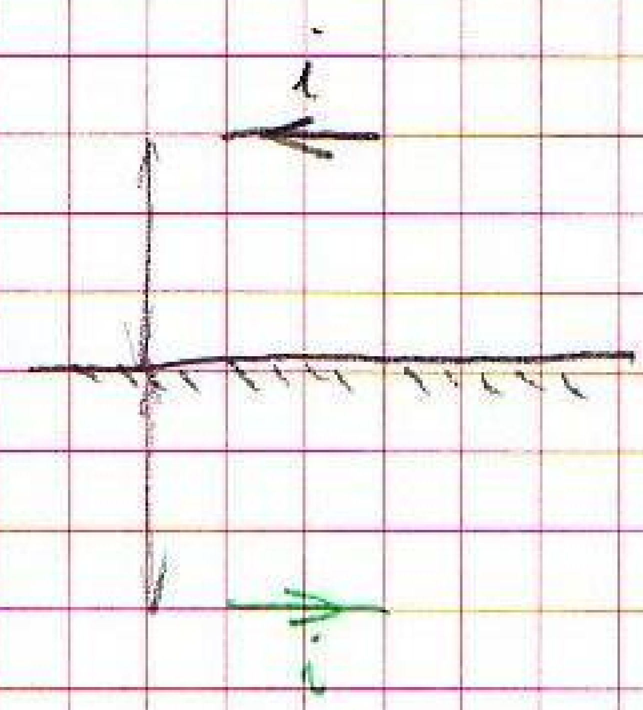

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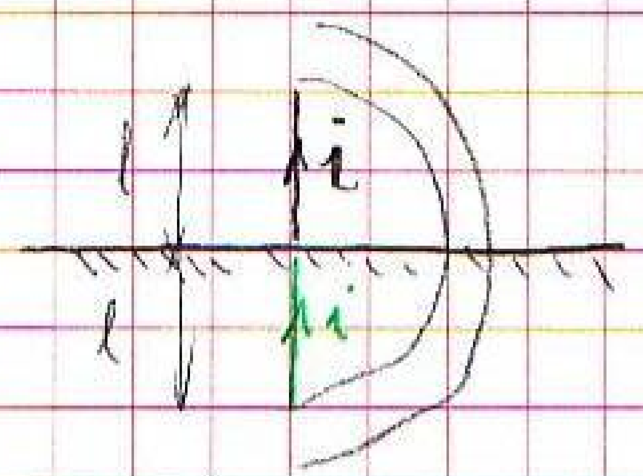

$$P = \frac{1}{2}\,\frac{I_0^2}{2}\left(\frac{2l}{\lambda}\right)^2 80\pi^2 = \frac{I_0^2}{2} R_{sug}\,,\quad R_{sug} = 160\pi^2\left(\frac{l}{\lambda}\right)^2$$

feltér