2007. 05. 15. kedd   XX. Előadás (14. hét)

Csillapítás mentes átvitel $\alpha = 0$

Diszperziós egyenlet

$$\gamma^2 + \omega^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma^2 = (j\beta)^2 = \left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \varepsilon\right) < 0$$

$$\omega^2 \mu \varepsilon \geq \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$(2f\pi)^2 \underbrace{\mu_0 \varepsilon_0}_{\frac{1}{c^2}} \mu_r \varepsilon_r = \left(2\pi \frac{f}{c}\right)^2 N^2 \geq \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$N^2 = \varepsilon_r \mu_r$ (törésmutató)

$$(j\beta)^2 = j^2 \left(\omega^2 \mu \varepsilon - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]\right)$$

$$\beta^2 = \left(\frac{2\pi}{\lambda}\right)^2 N^2 - \left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right) \qquad \left(\frac{2\pi}{\lambda}\right)^2 \geq \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

Határhullámhossz (λ_h)

$$\left(\frac{2\cancel{\pi}}{\lambda^h}\right)^2 N^2 = \left(\frac{m\cancel{\pi}}{a}\right)^2 + \left(\frac{n\cancel{\pi}}{b}\right)^2 \Rightarrow \lambda^h_{m,n} = \frac{2N}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$$\boxed{\lambda_{m,n} < \lambda^h_{m,n}} \qquad \frac{c}{\lambda^h_{m,n}} = f^h_{m,n} < f_{m,n}$$

Fázistényező

$$\beta = \sqrt{\left(\frac{2\pi}{\lambda}\right)^2 N^2 \underbrace{- \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}_{-\left(\frac{2\pi}{\lambda^h}\right)^2 N^2}} = \frac{2\pi N}{\lambda}\sqrt{1-\left(\frac{\lambda}{\lambda^h}\right)^2}$$

$$N = \sqrt{\varepsilon_r \mu_r}$$

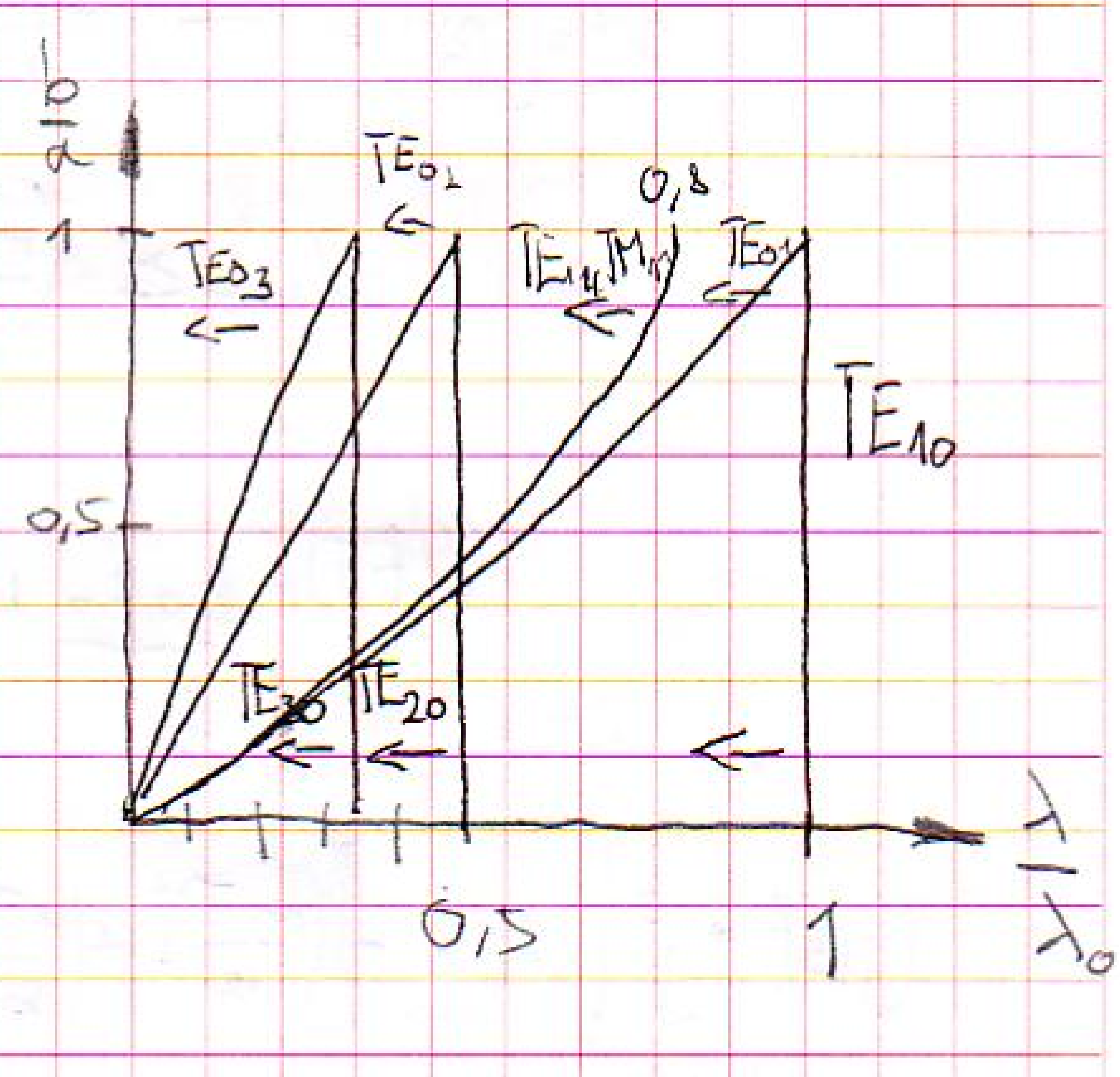
$$\beta_{m,n} = \frac{2\pi N}{\lambda_{m,n}}\sqrt{1-\left(\frac{\lambda}{\lambda^h_{m,n}}\right)^2} = \frac{2\pi}{\Lambda_{m,n}}$$

$$\boxed{\Lambda_{m,n} = \frac{\lambda_{m,n}}{N\sqrt{1-\left(\frac{\lambda_{m,n}}{\lambda^h_{m,n}}\right)^2}}}$$ CSŐBEN TERJEDŐ HULLÁMHOSSZ

TE_{10} $m=1$, $n=0$ $\quad \lambda^h_{1,0} = 2Na = \lambda_0$

TE_{20} $m=2$, $n=0$ $\quad \lambda^h_{2,0} = 2N\frac{a}{2} = \frac{\lambda_0}{2}$

TE_{30} $m=3$, $n=0$ $\quad \lambda^h_{3,0} = 2N\frac{a}{3} = \frac{\lambda_0}{3}$



TE_{11}, TM_{11}

$$\lambda^h_{11} = \frac{2N}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}} = \frac{2Na}{\sqrt{1+\left(\frac{a}{b}\right)^2}} = \frac{\lambda_0}{\sqrt{1+\left(\frac{a}{b}\right)^2}}$$

$TE_{0,1}$

$$\lambda^h_{0,1} = \frac{2N b}{1} = \frac{2Na}{a} b = \lambda_0 \frac{b}{a} \quad \text{(egyenes)}$$

példa: csőtápvonal levegővel kitöltve

$f = 3\,GHz$ $\quad a = ?$ $\quad$ $\boxed{\text{csak}}$ TE_{10} terjedjen! $\left[b = \frac{a}{2} \right]$ megkövetelésének, nem kell majd hozzá

$$\gamma^2 + \omega^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma^2 = (j\beta_{10})^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon \overset{\text{éppen terjed}}{=} 0$$

$$\left(\frac{1\pi}{a}\right)^2 + \cancel{\left(\frac{0\pi}{b}\right)^2} = \left(\frac{2\pi f}{c}\right)^2 \Rightarrow a = \frac{c}{2f} = \frac{3\cdot 10^8}{2\cdot 3\cdot 10^9} = 0{,}05\,m = 5\,cm$$

TE_{20}

$$\left(\frac{2\pi}{a}\right)^2 + \cancel{\left(\frac{0\pi}{b}\right)^2} \leq \left(\frac{2\pi f}{c}\right)^2 \Rightarrow a \leq \frac{c}{f} = 0{,}1\,m = 10\,cm$$

$$\boxed{5\,cm \leq a < 10\,cm}$$

$$TE_{10} < a < TE_{20}$$

indul $\qquad$ már nem lehet

Hullámimpedancia :

$$Z_0 = \frac{E_x}{H_y} = -\frac{E_y}{H_x} \quad (TE, TM)$$

$\boxed{TM}$

$$E_x = j\,\frac{k_x \overset{j\beta}{\gamma}}{\omega\varepsilon}\, M \cos k_x x \sin k_y y \; e^{-\gamma z}$$

$$H_y = -k_x M \cos k_x x \sin k_y y \; e^{-\gamma z}$$

$$Z_0^{TM} = \frac{\beta_{m,n}}{\omega\varepsilon} = \frac{2\pi}{\lambda_{m,n}} \cdot \frac{1}{\omega\varepsilon} = \frac{\frac{2\pi}{\omega\varepsilon} N \sqrt{1-\left(\frac{\lambda_{m,n}}{\lambda^h_{m,n}}\right)^2}}{\lambda_{m,n}} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-\left(\frac{\lambda_{m,n}}{\lambda^h_{m,n}}\right)^2}$$

$$\frac{2\pi N}{\omega\varepsilon} = \frac{\cancel{2\pi}\sqrt{\varepsilon_r\mu_r}}{\cancel{2\pi} \underset{c/\lambda}{\cancel{f}}\, \varepsilon_0\varepsilon_r} = \frac{\lambda_{m,n}\sqrt{\varepsilon_r\mu_r}}{\frac{1}{\sqrt{\varepsilon_0\mu_0}}\,\varepsilon_0\varepsilon_r} = \lambda_{m,n}\sqrt{\frac{\mu}{\varepsilon}}$$

$\boxed{TE}$

$$Z_0^{TE} = \frac{\mu\omega}{\beta_{m,n}} = \sqrt{\frac{\mu}{\varepsilon}}\,\frac{1}{\sqrt{1-\left(\frac{\lambda_{m,n}}{\lambda^h_{m,n}}\right)^2}}$$