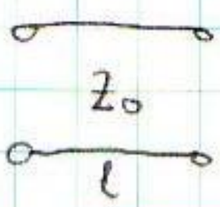
2007. 04. 04 Szerda — IV Gyakorlat (8. hét)

Távvezeték paraméterek

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta$$ terjedési ellenállás

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$ hullám impedancia

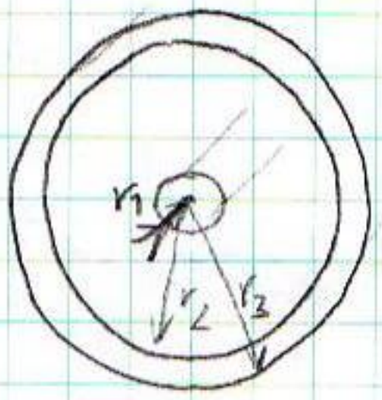


- **Kettős vezeték**

id IV: $C' = 0, R = 0$

$$L' = \frac{\mu}{2\pi} \ln \frac{d}{r_0} \qquad C' = \frac{\pi \cdot \varepsilon}{\ln \frac{d}{r_0}} \qquad \gamma = j\beta$$ (csak fázis fordul ampl. nem csökken)

- **Koaxiális kábel** (ált id IV)

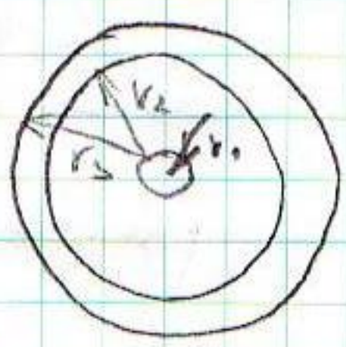


$$L' = \frac{\mu}{2\pi} \ln \frac{r_2}{r_1} \; ; \quad C' = \frac{2\pi\varepsilon}{\ln \frac{r_2}{r_1}}$$

példa (1) vörösréz koax

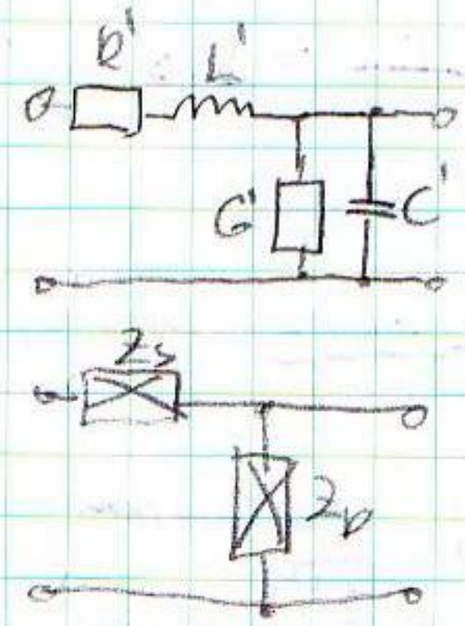
$f = 50\,Hz$

$r_1 = 7{,}2\,mm$
$r_2 = 41{,}6\,mm$
$r_3 = 41{,}8\,mm$



$\varepsilon_r = 3{,}5 ; 10^4 \cdot \operatorname{tg}\delta = 80$

$\varrho = \frac{1}{57} \cdot \Omega \frac{mm^2}{m}$



- **R' kiszámítása**

$A_1 = r_1^2 \pi = 162{,}86\,mm^2$
(belső vezető keresztmetszete)

$A_2 = (r_3^2 - r_2^2)\pi = 162{,}86\,mm^2$
(külső vezető keresztmetszete)

km-kénti ellenállás

$$R' = \frac{\varrho l}{A_1} + \frac{\varrho l}{A} + \ldots = 0{,}2155 \frac{\Omega}{km}$$

$$L' = \frac{\mu_0}{2\pi}\left[\underbrace{\ln\frac{r_2}{r_1}}_{\text{külsőirányú}} + \underbrace{\frac{1}{4}}_{\text{belsőirányú}}\right] = \ldots = 0{,}398\ \frac{H}{km}$$

$$\omega L' = 314 \cdot 0{,}398 \cdot 10^{-3}\ \frac{\Omega}{km} = 0{,}1247\ \frac{\Omega}{km}$$

soros impedancia: $Z_s = R + j\omega L = \ldots = 0{,}2485\, e^{j30{,}48^\circ}\ \frac{\Omega}{km}$

kapacitás

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{2\pi \cdot 10^{-9} \cdot 3{,}5}{4\pi \cdot 9 \ \ln\left(\frac{41}{7{,}2}\right)} = 0{,}1125 \cdot 10^{-9}\ \frac{F}{m}$$

$$\omega C = 314 \cdot 0{,}1115 \cdot 10^{-9} = 3{,}5 \cdot 10^{-6}\ \frac{S}{km}$$

$$G' = \omega C \cdot tg\,\delta = 0{,}28 \cdot 10^{-6}\ \frac{S}{km}$$

$$Y_p = G' + j\omega C' \cong j35 \cdot 10^{-6}\ \frac{S}{km}$$

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \ldots = (1{,}47 + j2{,}56) \cdot 10^{-3}\ \frac{1}{km} \Rightarrow$$

$$\Rightarrow \alpha = 1{,}47 \cdot 10^{-3}\ \frac{1}{km}$$

$$\beta = 2{,}56 \cdot 10^{-3}\ \frac{1}{km}$$

nem id. ⇒ túlnagy fázis forgatás (100 - 1000 Ω)

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = (818 - j145)\,\Omega = 833 \cdot e^{-j10{,}9^\circ}\ \Omega$$

(komplex!!!)

példa(2) Szabadvez. (kettős vez.)

$d = 30\,mm \quad r_0 = 2{,}5\,mm \quad f = 10\,MHz$

$$Z_0 = \sqrt{\frac{j\omega L'}{j\omega C'}} = \sqrt{\frac{L'}{C'}} = \frac{1}{\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \ln\left(\frac{d}{r_0}\right) = 120 \ln\frac{30}{2{,}5} = 300\,\Omega$$

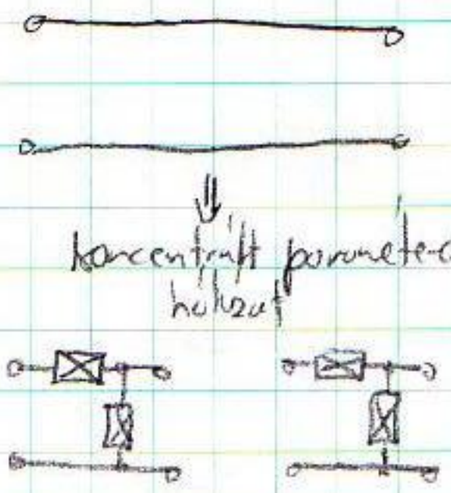
$$\gamma = j\beta = j0{,}209\ \frac{1}{m}$$

$$j\omega\sqrt{L'C'} =$$

$$\beta = 0{,}209\ \frac{1}{m} = 0{,}209\ \frac{1}{10^{-3}km} = 0{,}209 \cdot 10^{3}\ \frac{1}{km}$$ (jóval nagyobb mint az előzőnél)

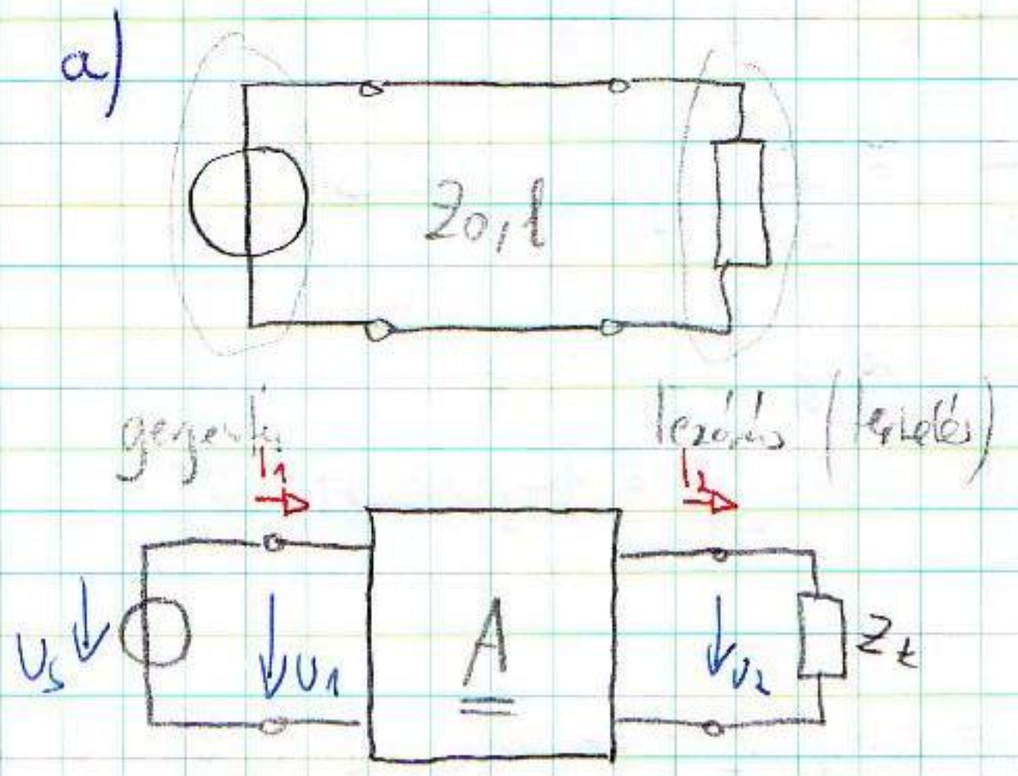
Kettős vezeték vagy koax kábel

+ kis β	− nagy β
+ kis Z_0	− nagy Z_0
− EMT nagy (H)	− nagy csillapítás



kb. 5-10 elemre bontani

felbontás elég jó közelítéssel



$$U_1 = ch\,\gamma l\; U_2 + Z_0\, sh\,\gamma l \cdot I_2$$
$$I_1 = \frac{sh\,\gamma l}{Z_0} \cdot U_2 + ch\,\gamma l \cdot I_2$$

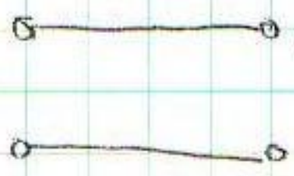
példa(3)

Távvezeték, mint Létkapu

TV par. $\begin{cases} r = 1{,}1 \cdot 10^{-3} e^{j\,79^\circ} \frac{1}{km} = (9{,}193 + j\,1{,}025) \cdot 10^{-3} \frac{1}{km} \\ Z_0 = (818 - j\,145{,}7)\,\Omega = 833\, e^{-j\,10{,}1^\circ}\,\Omega \end{cases}$

fogyasztót $U_2 = 90\,kV$ $I_2 = 400 \cdot e^{-j\,10^\circ}\,A$

$l = 20\,km$ $U_1 = ?$ $I_1 = ?$



$$U(z) = U_1^+ e^{-\gamma z} + U_1^- e^{+\gamma z}$$
$$I(z) = \frac{U_1^+}{Z_0} e^{-\gamma z} - \frac{U_1^-}{Z_0} e^{\gamma z}$$

$U_2 = U(l)$, $I_2 = I(l)$

$U_1 = ch\,g\; U_2 + Z_0\, sh\,g\; I_2$

$I_1 = \frac{sh\,g}{Z_0} U_2 + ch\,g\; I_2$

$g = \gamma l$

hálgi kicsi

$sh\,g \approx g$

$ch\,g \approx 1 + \frac{1}{2} g^2$

$$Z_0 \cdot sh\,g \approx Z_0 \cdot g = 18{,}32\, e^{j69{,}8^\circ}\ \Omega$$

$$\frac{sh\,g}{Z_0} \approx \frac{g}{Z_0} = j\,26{,}4\,10^{-4}\ S$$

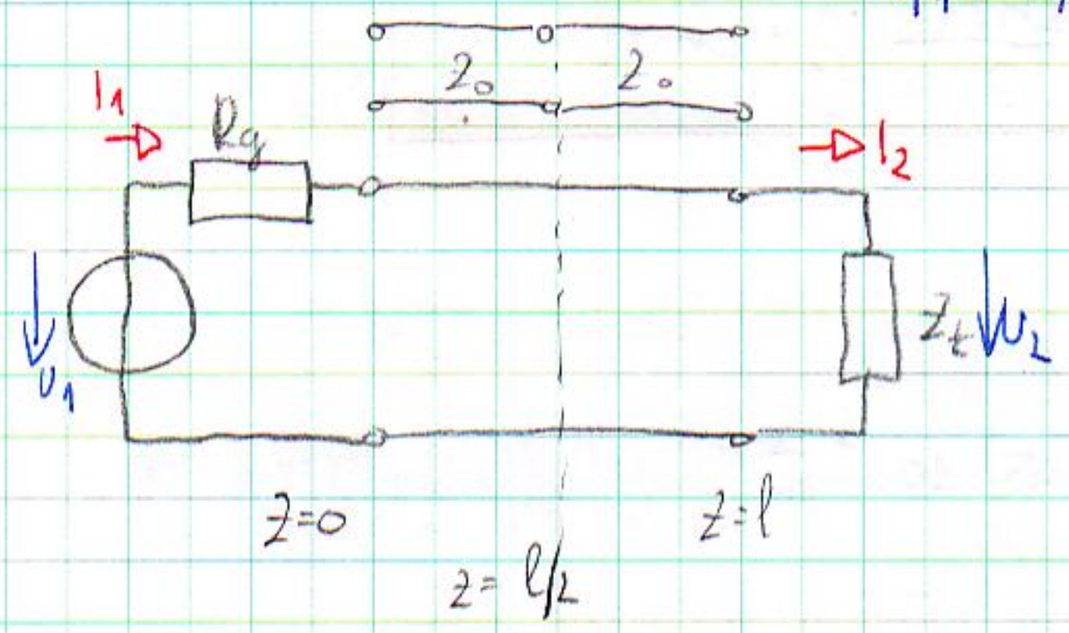
$$sh\,g \approx g = 2{,}2 \cdot 10^{-2}\, e^{j\,79{,}9^\circ}$$

$$ch\,g \approx 1 + \frac{1}{2}g^2 = 1 + j\,0{,}837 \cdot 10^{-4}$$

$$U_1 = ch\,g\ U_2 + Z_0\, sh\,g\ I_2 = 94 \cdot 10^3\, e^{j\,3{,}85^\circ}\ V$$

$$I_1 = \frac{sh\,g}{g}\ U_2 + ch\,g\ I_2 = 399{,}9\, e^{-j\,9{,}6^\circ}\ A$$

$$\varphi_1 = 3{,}85^\circ - (-9{,}6^\circ) \approx 13{,}45^\circ \quad (> \varphi_2)$$



$$U_{\frac{l}{2}} = ch\,\frac{g}{2}\ U_2 + Z_0\, sh\,\frac{g}{2}\ I_2$$

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példa(4)

$Z_0 = 160\,\Omega \quad l_1 = 500\,m$

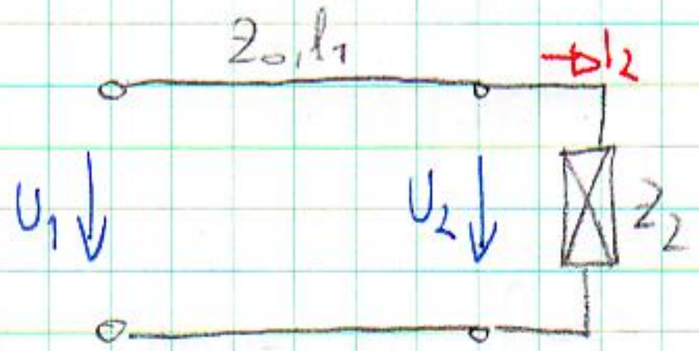
id TV

$Z_2 = (100 + j\,10)\,\Omega$

$U_1 = 100\,V$

$U_2 = ? \quad I_2 = ?$

$f = 1\,MHz$



$$U_2 = Z_2 \cdot I_2$$

$$I_2 = \frac{U_2}{Z_2}$$

láncparaméteres alapján: $U_1 = \overbrace{ch\,\gamma l_1}^{\cos\beta l}\ U_2 + \overbrace{j\,sh\,\gamma l_1}^{j\sin\beta l} \cdot Z_0 \cdot \frac{U_2}{Z_2}$

$$\beta = \frac{\omega}{c} = \frac{2\pi \cdot 10^6}{3 \cdot 10^8} = \frac{2}{3}\pi \cdot 10^{-2}\,\frac{1}{m} \qquad \beta l = \frac{2\pi}{3} \cdot 10^{-2} \cdot 500 = 3\pi + \frac{\pi}{3}$$

id. TV. $\gamma = j\beta$ $g = j\beta l$

$\operatorname{ch} j\beta l = \cos \beta l$

$\operatorname{sh} j\beta l = j \sin \beta l$

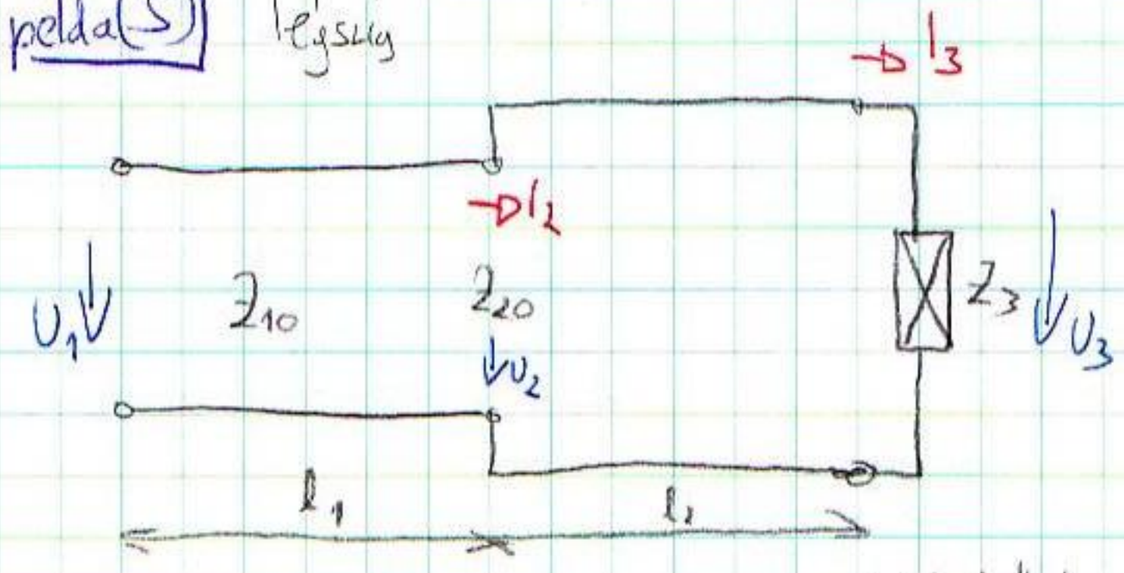
$\cos \beta l = \ldots = -\frac{1}{2} = 0{,}5$

$\sin \beta l = \ldots = -\frac{\sqrt{3}}{2} \approx -0{,}866$

$$U_2 = \frac{U_1}{\cos \beta l + j \frac{z_0}{z_2} \sin \beta l} = 66\, e^{j\, 245{,}05^\circ}\ V$$

$$I_2 = \frac{U_2}{Z_2} = 0{,}27 e^{j\, 239{,}25^\circ}\ A$$

példa(S) legyen



$z_{10} = 100\,\Omega$, $l_1 = 10\,m$

$z_{20} = 160\,\Omega$, $l_2 = 0{,}5\,m$

$Z_3 = (120 + j40)\,\Omega$

$U_3 = ?$ $I_3 = ?$

$U_1 = 60\,V$

$f = 50\,MHz$

Szabadtéri kábelben

$$\lambda = \frac{c}{f} = 6\,m$$

$$\beta = \frac{2\pi}{\lambda} = \frac{\pi}{3}\ \frac{1}{m}$$

$$g_1 = \beta l_1 = \frac{10}{3}\pi = 3\pi + \frac{1}{3}\pi$$

$$g_2 = \beta l_2 = \frac{\pi}{6}$$

folyt

$\Rightarrow$

$$U_1 = ch\gamma l_1 \cdot U_2 + z_{10} \cdot sh\gamma l_1 \cdot I_2 = \underbrace{ch\gamma l_1}_{\cos\beta l_1}\left(\underbrace{ch\gamma l_2}_{\cos\beta l_2} U_3 + z_{20}\underbrace{sh\gamma l_2}_{j\sin\beta l_2}\cdot I_3\right) + \underbrace{z_0 sh\gamma l_1}_{j\sin\beta l_1}\left(\frac{U_3}{z_{20}}\underbrace{sh\gamma l_2}_{j\sin\beta l_2} + \underbrace{ch\gamma l_2}_{\cos\beta l_2} I_3\right) =$$

U_3/z_3

$$=$$

id TV $\Rightarrow$ $ch j\beta l_1 = \cos\beta l_1$

$sh j\beta l_1 = j\sin\beta l_1$

$$= \left(U_3 \cos\beta l_1 \cdot \cos\beta l_2 - \frac{z_{10}}{z_{20}} \sin\beta l_1 \cdot \sin\beta l_2\right) + j U_3 \left(\frac{\cos\beta l_1}{z_3} \cdot z_{20} \sin\beta l_2 + \frac{z_{10}}{z_3} \cdot \sin\beta l_1 \cdot \cos\beta l_2\right)$$

$$\underline{U_3} = \frac{U_1}{\cos\beta l \cdot \cos\beta l_2 - \frac{z_{10}}{z_{20}} \sin\beta l_1 \sin\beta l_2 + j\left(\frac{z_{20}}{z_3} \cos\beta l_1 \cdot \sin\beta l_2 + \frac{z_{10}}{z_3} \sin\beta l_1 \cos\beta l_2\right)} = 693\, e^{j 115{,}85^\circ}\ V$$

$$\underline{I_3} = \frac{\underline{U_3}}{z_3} = 0{,}493\, e^{j 97{,}24^\circ}\ A$$