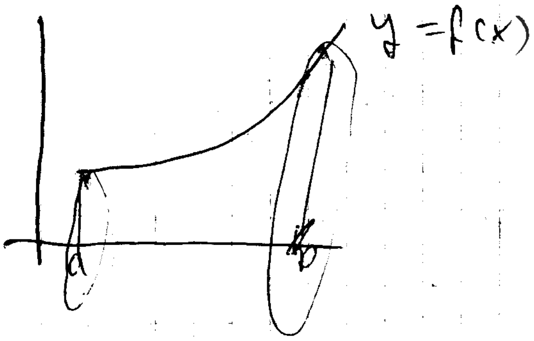
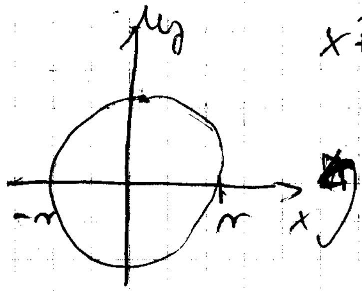
2007. 03. 02.



$$\sum_{i=1}^{n} \Delta x \cdot f^2(x) \cdot \pi \to \int_a^b \pi \, y^2(x)\, dx$$



$$x^2 + y^2 = r^2$$

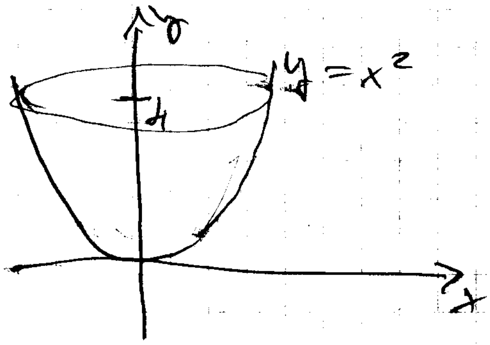
$$y = \sqrt{r^2 - x^2}$$

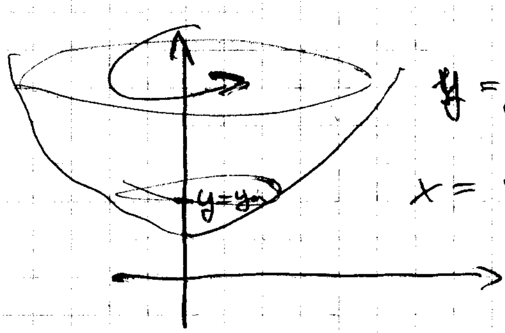
$$\int_{-r}^{r} (r^2 - x^2) \cdot \pi \, dx =$$

$$= \pi \left[r^2 x - \frac{x^3}{3} \right]_{x=-r}^{r} =$$

$$= \pi \left[r^2 \cdot r - \frac{r^3}{3} + r^3 - \frac{r^3}{3} \right] =$$

$$= \frac{4 r^3 \pi}{3}$$





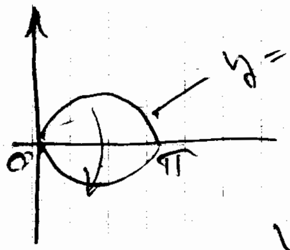
$y = f(x)$

$x = \sqrt{y}$

$f^{-1}(x)$

$\cancel{\int \Delta y (\sqrt{y})^2 \cdot \pi}$

$$\sum_{i=1}^{n} \Delta y \cdot (\sqrt{y})^2 \cdot \pi \xrightarrow[\Delta y \to 0]{} \int_{y=0}^{4} y \pi \, dy = \pi \left. \frac{y^2}{2} \right|_{y=0}^{4} = 8\pi$$



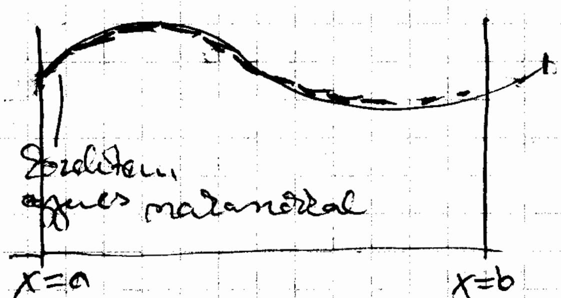
$$V_x = \pi \cdot \int_0^{\pi} \sin^2 x \, dx = \pi \int_0^{\pi} \frac{1-\cos 2x}{2} \, dx =$$

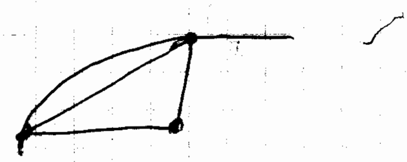
$\sin^2 x + \cos^2 x = 1$

$\cos^2 x - \sin^2 x = \cos 2x$

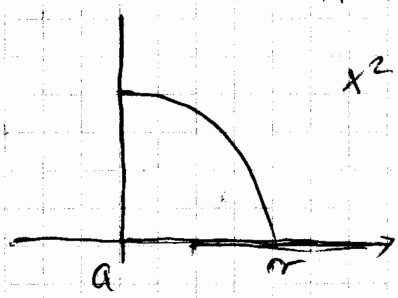
$$= \frac{\pi}{2}\left[x - \frac{\sin 2x}{4}\right]_{x=0}^{\pi} = \frac{\pi^2}{2}$$

Ívhossz





$$s = \int_{x=a}^{b} \sqrt{1+y'^2} \, dx$$



$x^2 + y^2 = r^2$

$y = \sqrt{r^2 - x^2}$

x	0	r
t	0	$\pi/2$

$$= \int_{t=0}^{\pi/2} \frac{r}{\sqrt{r^2 - r^2\sin^2 t}} \cdot r\cos t \, dt$$

$$y' = \frac{1}{2\sqrt{r^2 - x^2}} \cdot (-2x)$$

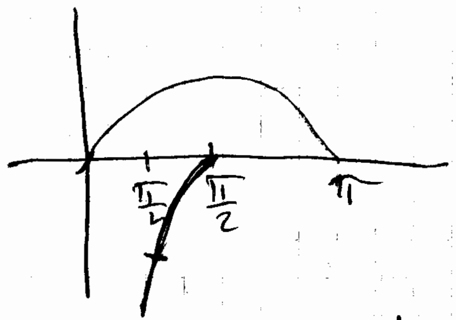
$$\int_{t=0}^{\pi/2} r\,dt = \boxed{r \cdot \frac{\pi}{2}}$$

$$y'^2 = \frac{x^2}{r^2 - x^2}$$

$$s = \int_{x=0}^{r} \sqrt{1 + \frac{x^2}{r^2 - x^2}} \, dx = \int_{x=0}^{r} \sqrt{\frac{r^2 - x^2 + x^2}{r^2 - x^2}} \, dx =$$

$$= \int_{x=0}^{r} \frac{r}{\sqrt{r^2 - x^2}} \, dx = \left[\begin{array}{l} x = r \cdot \sin t \\ dx = r \cdot \cos t \, dt \end{array}\right.$$

$$y = \ln \sin x \qquad \frac{\pi}{4} \leq x \leq \frac{\pi}{2}$$



$$y' = \frac{1}{\sin x} \cdot \cos x$$

$$s = \int_{x=\frac{\pi}{4}}^{\pi/2} \sqrt{1 + \frac{\cos^2 x}{\sin^2 x}}\, dx = \int_{x=\frac{\pi}{4}}^{\pi/2} \frac{1}{\sin x}\, dx$$

$$\left[\begin{array}{l} \operatorname{tg} \frac{x}{2} = t \\ \frac{x}{2} = \operatorname{arctg} t \\ \frac{dx}{2} = \frac{1}{1+t^2}\, dt \end{array}\right.$$

$$\frac{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}} = \frac{\operatorname{tg}^2 \frac{x}{2} + 1}{2 \operatorname{tg} \frac{x}{2}}$$

x	$\pi/4$	$\pi/2$
t	$\operatorname{tg} \frac{\pi}{8}$	$\operatorname{tg} \frac{\pi}{4}$

$$= \int_{t=\operatorname{tg}\frac{\pi}{8}}^{1} \frac{t^2+1}{2t} \cdot \frac{2\,dt}{1+t^2} = \ln t \Big]_{t=\operatorname{tg}\frac{\pi}{8}}^{1} =$$

$$= \ln 1 - \ln \operatorname{tg} \frac{\pi}{8}$$

$$\left.\begin{array}{l} x = x(t) \\ y = y(t) \end{array}\right\}$$

$$\int_a^b \sqrt{1+y'^2(x)}\, dx \quad \Bigg| \quad y' = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\dot{y}}{\dot{x}}$$

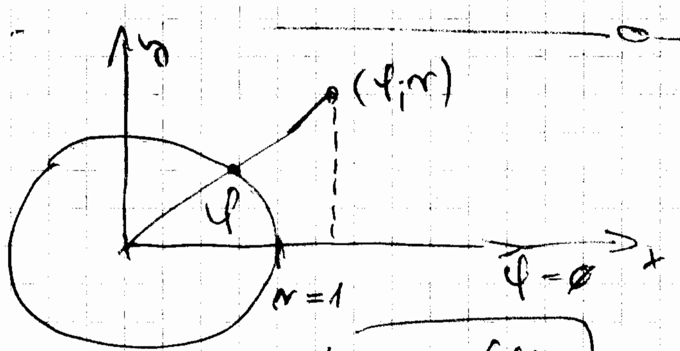
$$dx = \dot{x}(t)\, dt$$

$$\int_{t=t_1}^{t_2} \sqrt{1+\left(\frac{\dot{y}}{\dot{x}}\right)^2} \cdot \dot{x}(t)\, dt =$$

$$= \int_{t_1}^{t_2} \sqrt{\dot{x}^2(t) + \dot{y}(t)}\, dt =$$

$$\left.\begin{array}{l} x = \cos t \\ y = \sin t \end{array}\right\} \qquad \begin{array}{l} \dot{x} = -\sin t \\ \dot{y} = \cos t \end{array} \qquad \dot{x}^2 + \dot{y}^2 = 1$$

$$\int_{t=0}^{2\pi} \sqrt{1}\, dt = \underline{2\pi}$$



$$\boxed{r = r(\varphi)}$$

$$\left.\begin{array}{l} x = r \cdot \cos\varphi \\ y = r \cdot \sin\varphi \end{array}\right\}$$

$$x^2 + y^2 = r^2$$

$$\frac{y}{x} = \operatorname{tg}\varphi \qquad \varphi = \operatorname{arctg}\frac{y}{x}$$

$$s = \int_{t_1}^{t_2} \sqrt{\dot{x}^2 + \dot{y}^2}\, dt$$

$$\left.\begin{array}{l} x = r(\varphi)\cos\varphi \\ y = r(\varphi)\sin\varphi \end{array}\right\}$$

$$\frac{dx}{d\varphi} = r'(\varphi)\cdot\cos\varphi - r(\varphi)\sin\varphi$$

$$\frac{dy}{d\varphi} = r'\varphi \sin\varphi + r(\varphi)\cdot\cos\varphi$$

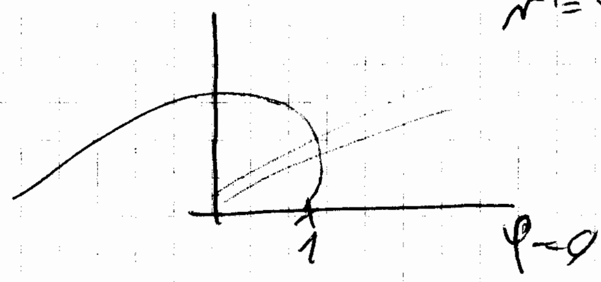
$$\dot{x}^2 = r'^2\cos^2\varphi - 2rr'\cos\varphi\sin\varphi + r^2\cdot\sin^2\varphi$$

$$\dot{y}^2 = r'^2\cdot\sin^2\varphi + 2rr^2\cdot\sin\varphi\cos\varphi + r^2\cos^2\varphi$$

$$\dot{x}^2 + \dot{y}^2 = r'^2 + r^2$$

$r' = e^\varphi$

$$s = \int_{\varphi_A}^{\varphi_B} \sqrt{r^2 + r'^2}\, d\varphi$$

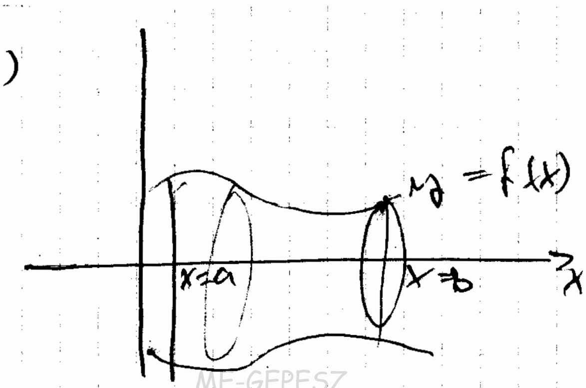


$r = e^\varphi \qquad \emptyset \le \varphi \le \pi \qquad s = ?$ $\qquad \sqrt{r^2 + r'^2} = \sqrt{e^{2\varphi} + e^{2\varphi}}$

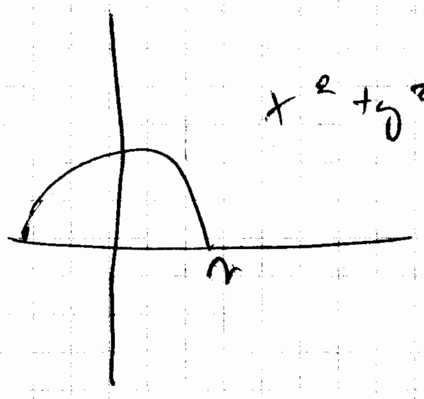
$$s = \int_{\varphi=\emptyset}^{\pi} e^\varphi \cdot \sqrt{2}\, d\varphi = \sqrt{2}\, e^\varphi \Big]_{\varphi=\emptyset}^{\pi} = \sqrt{2}\,(e^\pi - 1)$$

Palástfelszín

$y = f(x)$



$$F_x \approx \sum \pi 2 f(x_i)\,\Delta s_i \to \int_{x=a}^{b} 2\,y(x)\cdot\sqrt{1+y'^2(x)}\,dx$$



$$x^2+y^2=r^2$$

$$y=\sqrt{r^2-x^2}$$

$$y'=\frac{-2x}{2\sqrt{r^2-x^2}}$$

$$F_x=2\pi\int_{x=0}^{r}\sqrt{r^2-x^2}\cdot\sqrt{1+\frac{x^2}{r^2-x^2}}\,dx=$$

$$=2\pi\int_{x=0}^{r} r\,dx=2\pi\Big[r x\Big]_{x=0}^{r}=\underline{\underline{2\pi r^2}}$$

Többváltozós függvény

$$z=f(x,y)$$

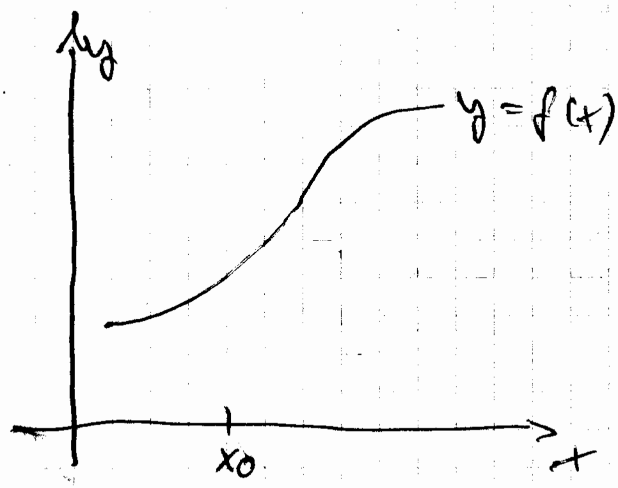
Határérték

$z = f(x,y)$ korlátos, ha van olyan K, hogy
$$|f(x,y)| \leq K$$

$y = f(x)$, x_0 $\lim_{x \to x_0} f(x) = A$, ha tetszőleges $\varepsilon > 0$,

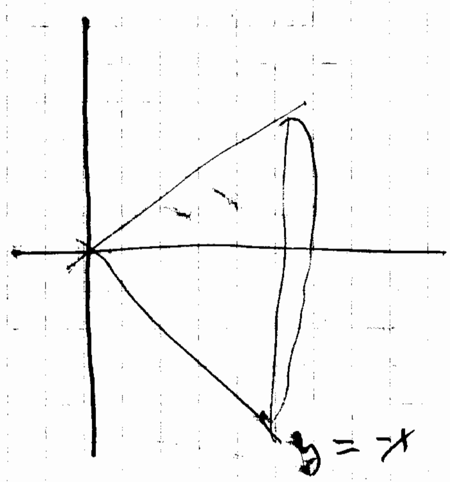
$\exists$ $\delta > 0$, ha $|x - x_0| < \delta \Rightarrow |f(x) - A| < \varepsilon$

van olyan



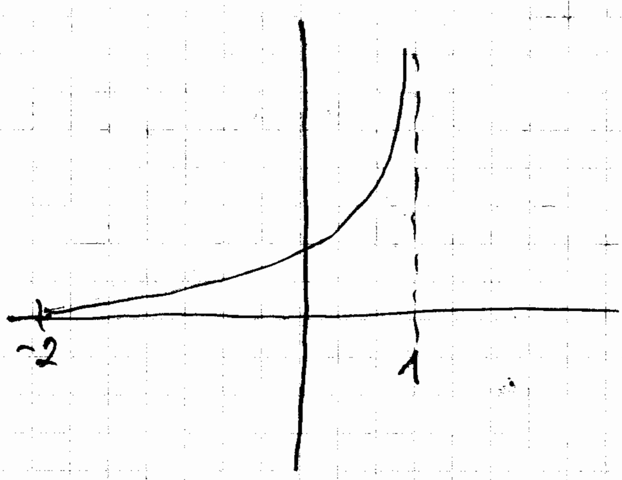
$$F_x = \int_{x=a}^{b} 2\pi \, |y(x)| \sqrt{1 + y'^2(x)} \, dx$$

$$F_y = \int_{y=c}^{d} 2\pi \, |x(y)| \sqrt{1 + x'^2(y)} \, dy$$



$$\int_{-2}^{1} \frac{dx}{\sqrt{1-x}}$$

$$\lim \int_{-2}^{1-\varepsilon} \frac{dx}{\sqrt{1-x}} = \int_{-2}^{1} \frac{dx}{\sqrt{1-x}}$$



$$\int_{-\infty}^{\infty} f(x)\,dx \quad \Big| \quad \int_{-\infty}^{c} f(x)\,dx = \lim_{R\to\infty} \int_{-R}^{c} f(x)\,dx$$

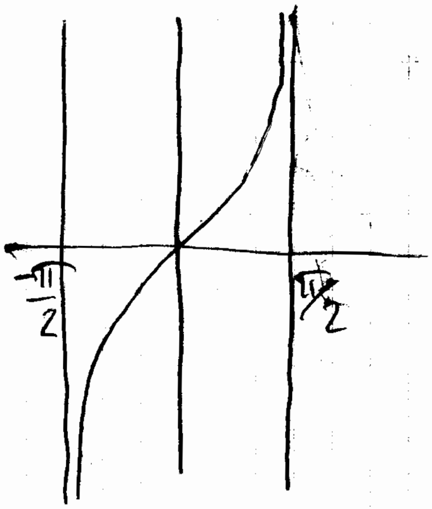
Improprius

$$\lim_{\varepsilon\to+0} \int_{-2}^{1-\varepsilon} \frac{dx}{\sqrt{1-x}} = \lim_{\varepsilon\to+0} \left[-2\sqrt{1-x}\right]_{x=-2}^{1-\varepsilon} =$$

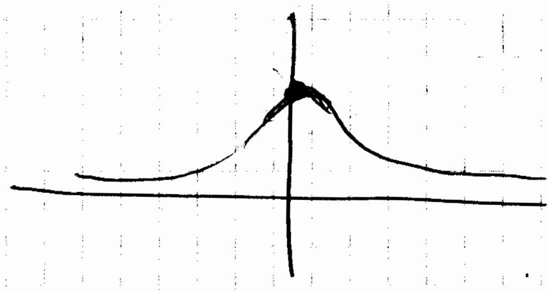
$$\lim_{\varepsilon\to+0} \left[-2\sqrt{1-(1-\varepsilon)} + \underbrace{2\sqrt{1-(-2)}}_{2\sqrt{3}}\right] = 2\sqrt{3}$$

ME-GEPESZ
NME

$$\int_{-\infty}^{+\infty} \frac{1}{1+x^2}\,dx = \lim_{\substack{a \to -\infty \\ b \to +\infty}} \int_a^b \frac{dx}{1+x^2} = \lim_{\substack{a \to -\infty \\ b \to +\infty}} \Big[\operatorname{arctg} x\Big]_{x=a}^{b} =$$



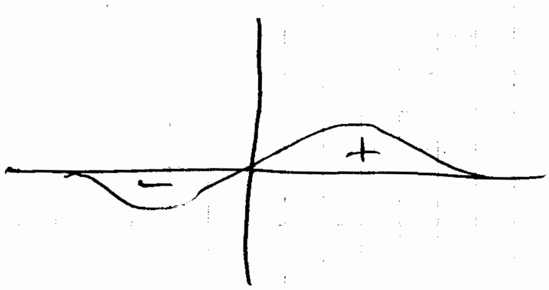
$$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \underline{\underline{\pi}}$$



$$\int_{-\infty}^{\infty} \frac{x}{1+x^2}\,dx = \lim_{\substack{a \to -\infty \\ b \to +\infty}} \int_a^b \frac{x\,dx}{1+x^2} = \boxed{\frac{1}{2}\ln(1+x^2)}$$

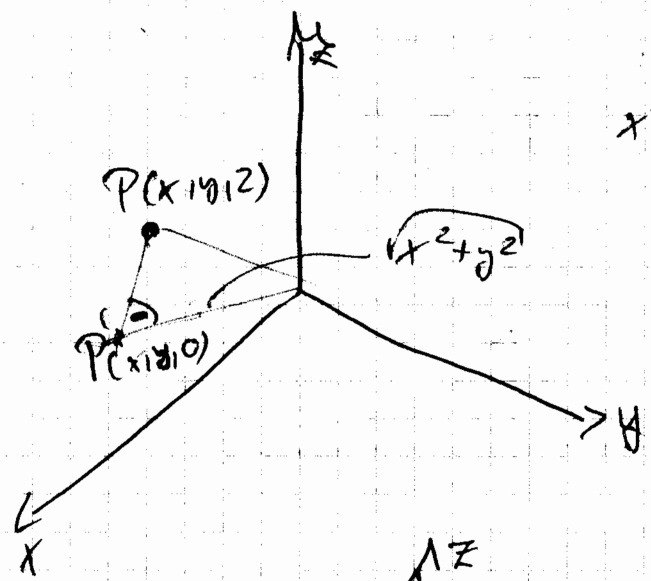
$$= \lim_{\substack{a \to -\infty \\ b \to +\infty}} \left[\frac{1}{2}\ln(1+x^2)\right]_{x=a}^{b} = \cancel{\lim_{b \to +\infty} \frac{1}{2}\ln(1+b^2)}$$

$$= \underbrace{\lim_{b \to +\infty} \frac{1}{2}\ln(1+b^2)}_{+\infty} - \underbrace{\lim_{a \to -\infty} \frac{1}{2}\ln(1+a^2)}_{+\infty}$$



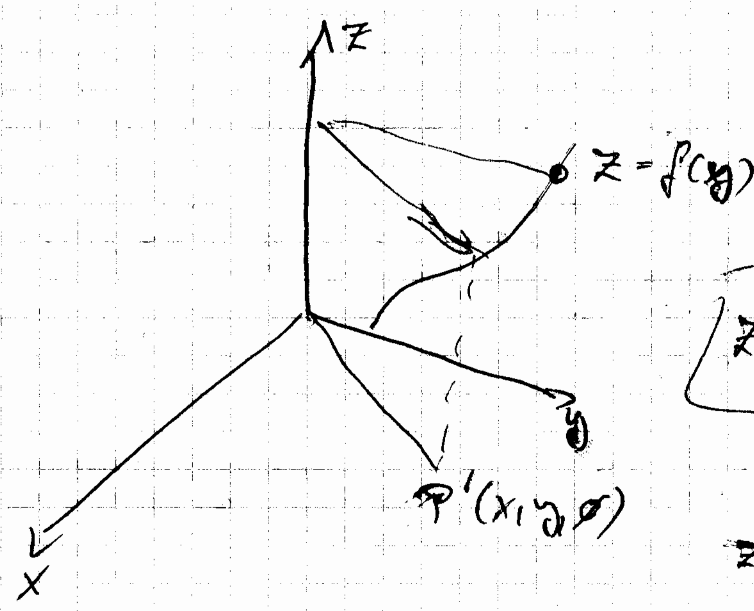
Improprius integrál Cauchy-féle főértéke

$$\int_{-\infty}^{\infty} f(x)\,dx = \lim_{R\to\infty} \int_{-R}^{R} f(x)\,dx$$



$x^2+y^2+z^2=R^2$

$\overline{OR} = \sqrt{z^2+x^2+y^2} = R$



$$\boxed{z = f\left(\sqrt{x^2+y^2}\right)}$$

$z=y$ $\quad z=\sqrt{x^2+y^2}$

$z^2=x^2+y^2$

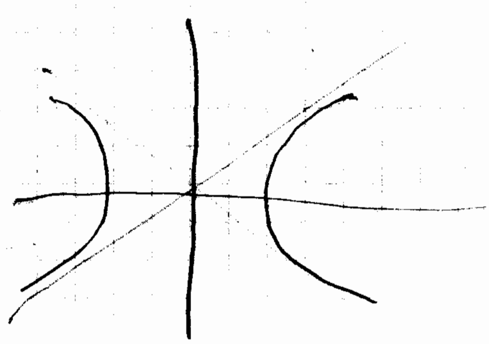
$$\frac{z^2}{c^2}+\frac{y^2}{b^2}=1$$

$z=y^2$

<u>forgási paraboloid</u> $z=x^2+y^2$

$$\frac{z^2}{c^2}+\frac{x^2+y^2}{b^2}=1$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \Big| : \frac{x^2}{a^2}$$

$$1 - \frac{\left(\frac{y^2}{b^2}\right)}{\left(\frac{x^2}{a^2}\right)} = \frac{1}{\frac{x^2}{a^2}}$$

$$\frac{\left(\frac{y^2}{x^2}\right)}{\left(\frac{a^2}{b^2}\right)} \Rightarrow 1$$

$$\frac{x^2}{a^2} - \frac{y^2+z^2}{b^2} = 1 \Rightarrow$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$z = xy$

$y = a$

$z = a \cdot x$ $\quad x = b$

$y = x$

$z = x^2$

$z = c \quad x \cdot y = c$

$\overline{P_0P} < \delta$

(x_0, y_0) $\quad \sqrt{(x-x_0)^2 + (y-y_0)^2} \leqq \delta^2$

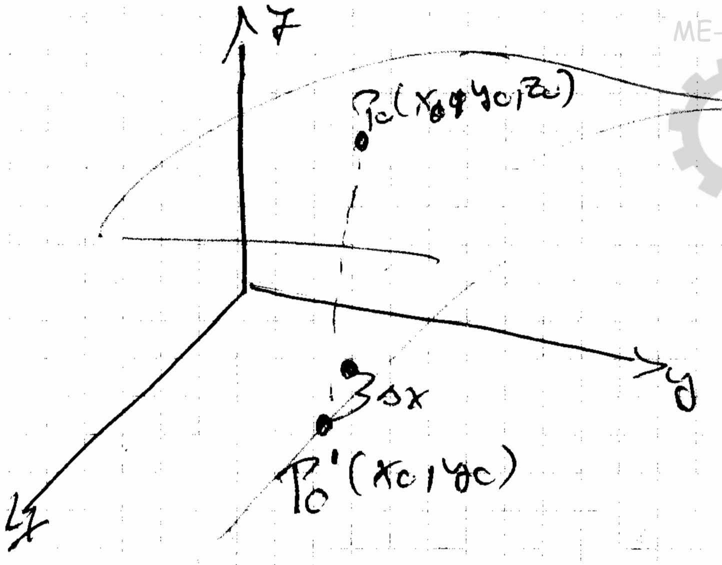
(x, y)

$z = f(x, y) \quad P_0'(x_0, y_0)$

$\cancel{P_0(x_0, y_0, f(x_0, y_0))}$ $\quad P_0(x_0, y_0, z_0)$

$\lim\limits_{P \to P_0} f(x, y) = A$ ha tetszőleges pozitív ε-hoz van olyan pozitív δ, ha

$\overline{PP_0} < \delta \Rightarrow |f(x,y) - A| < \varepsilon$



$y=y_0$ síkban

parciális derivált

$$\left.\frac{\partial f(x,y)}{\partial x}\right|_{P_0'}$$

$$\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x_0,y_0)-f(x_0,y_0)}{\Delta x}$$

$$\frac{\partial f(x,y)}{\partial y}=\lim_{\Delta y\to 0}\frac{f(x_0,y_0+\Delta y)-f(x_0,y_0)}{\Delta y}$$

$$\vec{r}=\{x_0,y_0,f(x_0,y_0)\}+\{1;y_0;z'_x(x_0,y_0)\}\cdot t+$$

$$+u\{x_0;1;z'_y(x_0,y_0)\}$$

$$\left.\frac{\partial z}{\partial x}\right|_{P_0(x_0,y_0)}=\left.z'_x\right|_{P_0}$$

$$z=x^2+3yx+4y^2$$

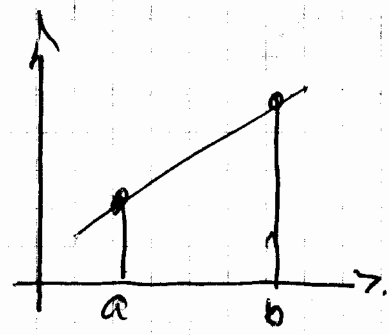
$$z'_x=2x+3y$$

$$z'_y=3x+8y$$

Szélsőérték

Stacionárius pontok

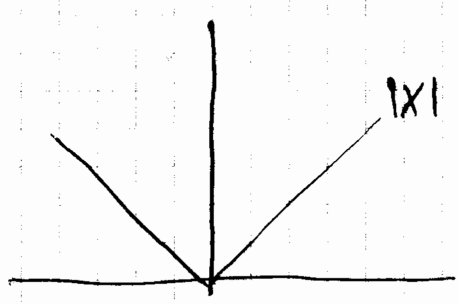
$$\left.\begin{array}{l} z'_x = 0 \\ z'_y = 0 \end{array}\right\}$$



$$z = x^2 + y^2$$

$$\left.\begin{array}{l} z'_x = 2x = 0 \\ z'_y = 2y = 0 \end{array}\right\} \quad \begin{array}{l} x = 0 \\ y = 0 \end{array}$$

$$P_0(0;0)$$



$$\frac{\partial^2 f(x,y)}{\partial x^2} = f''_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f(x,y)}{\partial x}\right)$$

$$\left.\frac{\partial^2 f(x,y)}{\partial x\,\partial y} = f''_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f(x,y)}{\partial x}\right)\right\}$$

Schwarz-tétel

$$f''_{xy} = f''_{yx}$$

$$f''_{yx}$$

$$D = \begin{vmatrix} f''_{xx} & f''_{xy} \\ f''_{yx} & f''_{yy} \end{vmatrix}$$

| Stacionárius pontban

$D(P_0) < 0$

$D(P_0) = 0$ -> további vizsg. szükséges

$f''_{xx}(P_0) > 0$ minimum. $D(P_0) > 0$ van sz. é.

$f''_{xx}(P_0) < 0$ maximum

$z''_{xx} = 2$ $\quad z''_{xy} = z''_{yx} = 0$

$z''_{yy} = 2$

$$D = \begin{vmatrix} 2 & \omega \\ 0 & 2 \end{vmatrix}_{|B} = 4 > 0$$

> 0