④ $1+\sqrt{2x}+\left(\sqrt{2x}\right)^2+\left(\sqrt{2x}\right)^3+\ldots+\left(\sqrt{2x}\right)^n+\ldots = \sum_{n=0}^{\infty}\left(\sqrt{2x}\right)^n$

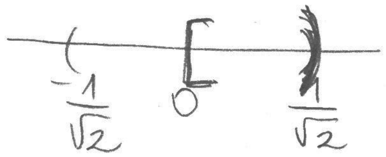
$\left(\sqrt{2x}\right)^n$ — a_n

$\left(\sqrt{2}\right)^n \cdot \left(\sqrt{x}\right)^n$ — a_n

konvergenciatartomány ?

$$r=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\left|\frac{\left(\sqrt{2}\right)^n}{\left(\sqrt{2}\right)^{\cancel{n}+1}}\right| = \underline{\underline{\frac{1}{\sqrt{2}}}}$$

$x \geq 0$



$$x=\frac{1}{\sqrt{2}} \quad \sum_{n=0}^{\infty}\left(\sqrt{2\cdot\frac{1}{\sqrt{2}}}\right)^n$$

$$\lim_{n\to\infty}\frac{\left(\sqrt{2\cdot\frac{1}{\sqrt{2}}}\right)^{\cancel{n+1}}}{\cancel{\left(\sqrt{2\cdot\frac{1}{\sqrt{2}}}\right)^n}} = 1{,}18 > 1 \quad \text{divergens}$$

$$I=\left[0;\frac{1}{\sqrt{2}}\right)$$

⑤ $f(x,y)=e^{y-x}+y\ln x^2$ $\quad \underline{a}(1;2)$

$$f'_x=e^{y-x}\cdot -1+y\cdot\frac{1}{x^{\cancel{2}}}\cdot\cancel{2x} = \underline{-e^{y-x}+\frac{2y}{x}}$$

$$f'_y=e^{y-x}\cdot 1+1\cdot\ln x^2$$

$$\text{grad } f=\left(-e^{y-x}+\frac{2y}{x}\,;\ e^{y-x}+\ln x^2\right)$$

$$\text{grad } f(\underline{a})=\left(-e+4\,;\ e\right)$$

$$f''_{xx}=-e^{y-x}\cdot -1+2y\cdot -1\cdot x^{-2}=e^{y-x}-\frac{2y}{x^2}$$

$$f''_{xy}=-e^{y-x}\cdot 1+\frac{2}{x}\cdot 1 = -e^{y-x}+\frac{2}{x}$$

$$f''_{yy}=e^{y-x}\cdot 1 \qquad H=\begin{bmatrix} e^{y-x}-\frac{2y}{x^2} & -e^{y-x}+\frac{2}{x} \\ -e^{y-x}+\frac{2}{x} & e^{y-x}\end{bmatrix}=\begin{bmatrix} e-4 & -e+2 \\ -e+2 & e\end{bmatrix}$$

2004. june. 24

1)

$$y'' + 2y' - 8y = e^x \qquad y(0)=0 \quad y'(0)=1$$

$$r^2 + 2r - 8 = 0$$

$$r_{1,2} = \frac{-2 \pm \sqrt{4+32}}{2} = \begin{cases} 2 \\ -4 \end{cases}$$

$$y_{h,á} = c_1 e^{2x} + c_2 e^{-4x}$$

$$\begin{cases} y_{i,p} = A \cdot e^x \\ y' = A \cdot e^x \\ y'' = A \cdot e^x \end{cases}$$

$$A \cdot e^x + 2 \cdot A \cdot e^x - 8 \cdot A \cdot e^x = e^x$$

$$-5A\, e^x = e^x$$

$$-5A = 1 \rightarrow A = \frac{-1}{5}$$

$$y_{i,p} = -\frac{1}{5} e^x$$

$$y_{i,á} = c_1 e^{2x} + c_2 e^{-4x} - \frac{1}{5} e^x \qquad y(0)=0$$

$$0 = c_1 e^0 + c_2 e^0 - \frac{1}{5} e^0$$

$$0 = c_1 + c_2 - \frac{1}{5}$$

$$y' = c_1 \cdot e^{2x} \cdot 2 + c_2 e^{-4x} \cdot -4 - \frac{1}{5} e^x \qquad (y'(0)=1)$$

$$1 = c_1 e^0 \cdot 2 + c_2 e^0 \cdot -4 - \frac{1}{5} e^0$$

$$1 = 2c_1 - 4c_2 - \frac{1}{5}$$

$$\boxed{y_{i,á} = \frac{1}{3} e^{2x} - \frac{2}{15} e^{-4x} - \frac{1}{5} e^x}$$

$$\left.\begin{array}{l} 0 = c_1 + c_2 - \frac{1}{5} \rightarrow c_1 = \frac{1}{5} - c_2 \\ 1 = 2c_1 - 4c_2 - \frac{1}{5} \end{array}\right\} \quad 1 = \frac{2}{5} - 2c_2 - 4c_2 - \frac{1}{5}$$

$$c_1 = \frac{1}{5} - \frac{-2}{15} = \frac{3+2}{15} = \frac{5}{15} = \frac{1}{3}$$

$$1 = \frac{1}{5} - 6c_2 \quad \Big/ -\frac{1}{5}$$

$$\frac{4}{5} = -6c_2 \quad \Big/ : -6 \rightarrow c_2 = \frac{4}{5 \cdot -6} = \frac{4}{-30} = \frac{-2}{15}$$

2004. máj. 20

5) $\overline{\text{grad}}\, f = ?$ $\qquad f(x,y) = e^{x-y} + x \cdot \ln y^2$

$\overline{\text{grad}}\, f(\bar{a}) = ?$ $\qquad \bar{a} = (2,1)$

$\underline{\underline{H}} = ?$

$\underline{\underline{H}}(\bar{a}) = ?$

$$\overline{\text{grad}}\, f = (f'_x \;;\; f'_y)$$

$$\underline{\underline{H}} = \begin{bmatrix} f''_{xx} & f''_{xy} \\ f''_{xy} & f''_{yy} \end{bmatrix}$$

$f''_{xy} = f''_{yx}$

$$\left.\begin{aligned} f'_x &= e^{x-y} \cdot 1 + 1 \cdot \ln y^2 \\ f'_y &= e^{x-y} \cdot -1 + x \cdot \frac{1}{\cancel{y^2}_{\,y^{-1}}} \cdot 2\cancel{y} \end{aligned}\right\} \quad \begin{aligned} \overline{\text{grad}}\, f &= \left(e^{x-y} + \ln y^2 \;;\; e^{x-y} \cdot -1 + \frac{2x}{y}\right) \\ \overline{\text{grad}}\, f(\bar{a}) &= (e^1 + \underbrace{\ln 1}_{0} \;;\; e^1 \cdot -1 + 4) \\ &= (e \;;\; -e + 4) \end{aligned}$$

$$f''_{xx} = e^{x-y}$$

$$f''_{xy} = e^{x-y} \cdot -1 + \frac{1}{y^{\cancel{2}}} \cdot 2\cancel{y} = -e^{x-y} + \frac{2}{y}$$

$$f''_{yy} = -1 \cdot e^{x-y} \cdot -1 + 2x \cdot -1 \cdot y^{-2} = e^{x-y} - \frac{2x}{y^2}$$

$$\underline{\underline{H}} = \begin{bmatrix} e^{x-y} & -e^{x-y} + \frac{2}{y} \\ -e^{x-y} + \frac{2}{y} & e^{x-y} - \frac{2x}{y^2} \end{bmatrix}$$

$$\underline{\underline{H}}(\bar{a}) = \begin{bmatrix} e^1 & -e^1 + 2 \\ -e^1 + 2 & e^1 - 4 \end{bmatrix} = \begin{bmatrix} e & -e + 2 \\ -e + 2 & e - 4 \end{bmatrix}$$

2004. máj. 27.

3) $y' = \frac{-3xy^2}{1+x^2}$ $\quad y(0)=1$

$$\frac{dy}{dx} = \frac{-3x \cdot y^2}{1+x^2}$$

$\frac{f'}{f} \to \ln|f|$

$$\int \underbrace{\frac{1}{y^2}}_{y^{-2}} dy = \int \frac{-3x}{1+x^2} dx = -3\int \frac{1}{2} \frac{2x}{1+x^2} dx$$

$$\frac{y^{-1}}{-1} = \frac{-3}{2} \cdot \ln|1+x^2| + C$$

$$\frac{1}{-y} = -\frac{3}{2} \ln|1+x^2| + c \quad /\cdot -1$$

$$\frac{1}{y} = \frac{3}{2} \ln|1+x^2| - c \quad /\text{reciprok}$$

$$y = \frac{1}{\frac{3}{2} \ln|1+x^2| - c}$$

$y(\overset{x}{0}) = \overset{y}{1}$

$$1 = \frac{1}{\underbrace{\frac{3}{2}\ln|1|}_{0} - C}$$

$$1 = \frac{1}{-C} \quad /\cdot -1$$

$$-1 = \frac{1}{C}$$

$$-1 = C$$

$$y = \frac{1}{\frac{3}{2}\ln|1+x^2| + 1}$$

2004. 06. 17.

3)

$$y'' - 4y = x$$

$$r^2 - 4 = 0$$

$$r^2 = 4$$

$$r = \pm 2$$

$$y_{h,á} = C_1 e^{2x} + C_2 e^{-2x}$$

$$y_{i,p} = Ax + B$$

$$y' = A$$

$$y'' = 0$$

$$0 - 4 \cdot (Ax + B) = x$$

$$-4Ax - 4B = x$$

$$-4A = 1 \rightarrow A = -\frac{1}{4}$$

$$-4B = 0 \rightarrow B = 0$$

$$y_{i,p} = -\frac{1}{4}x$$

$$y_{i,á} = y_{h,á} + y_{i,p} = C_1 e^{2x} + C_2 e^{-2x} - \frac{1}{4}x$$

4 Taylor sor

$$f(x) = e^x \qquad x_0 = 0$$

$$f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

$$f(0) = e^0 = 1$$

$$f'(x) = e^x = f'' = f''' = \dots$$

$$f'(0) = 1 = f'' = f''' = \dots$$

$$= 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots + \frac{1}{n!}x^n + \dots$$

4 pufr.

$$= \int_{x=0}^{2} \left[4\cdot(2-x) - 2x\cdot(2-x) - (2-x)^2 \right] dx = \left[\frac{x^3}{3} - 4\frac{x^2}{2} + 4x \right]_{x=0}^{2} =$$

$$8 - 4x - \cancel{4x} + 2x^2 - 4 + \cancel{4x} - x^2$$

$$x^2 - 4x + 4$$

$$= \left[\frac{8}{3} - 8 + 8 \right] = \underline{\underline{\frac{8}{3}}}$$

$$V = \frac{T_a \cdot m}{3} = \frac{\frac{2\cdot 2}{\cancel{2}} \cdot 4}{3} = \underline{\underline{\frac{8}{3}}}$$

5) $f(x,y,z) = \left(2xy^3;\ 3x^2y^2;\ \frac{1}{z}\right)$

$x \quad y \quad z$

$\text{div}\ \underline{v} = 2y^3 + 3x^2\cdot 2y + -1\cdot z^{-2} \neq 0$ forrásos

divergencia $\text{div}\ \underline{v} = 0$ forrásmentes

rot $\underline{v}$ = rotáció

$$\text{rot}\ \underline{v} = \begin{vmatrix} \underset{+}{\underline{i}} & \underset{-}{\underline{j}} & \underset{+}{\underline{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy^3 & 3x^2y^2 & \frac{1}{z} \end{vmatrix} = \underline{i}(0-0) - \underline{j}(0-0) + \underline{k}\left(3\cdot 2x\cdot y^2 - 2x\cdot 3y^2\right)$$

$$= (0;0;0)$$

rot $\underline{v} = 0$ örvénymentes

2004. 05. 27

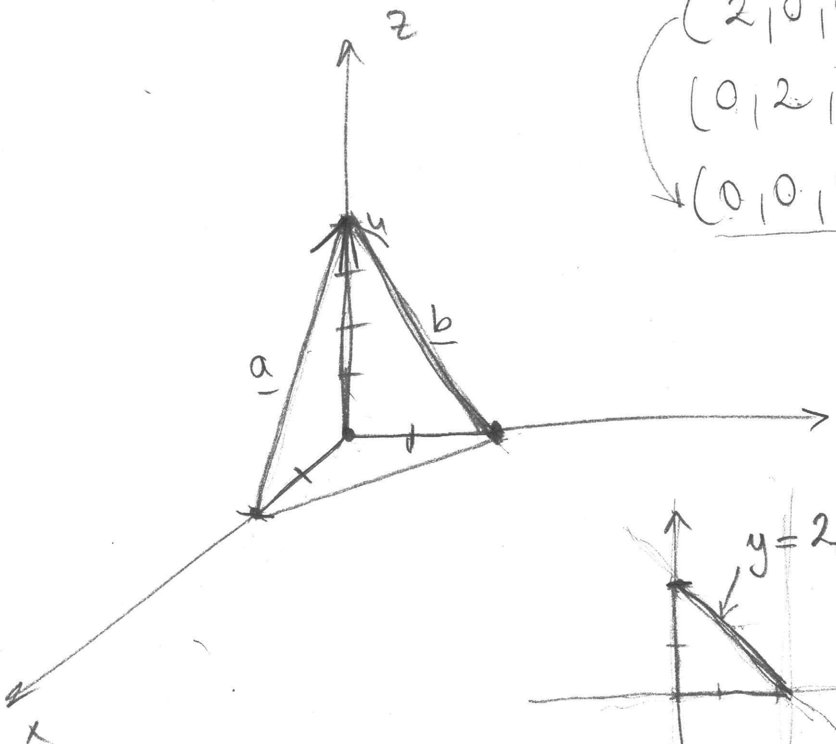
4) $\iiint_V 1 \, dx\, dy\, dz$

$(0,0,0)$

$(2,0,0)$

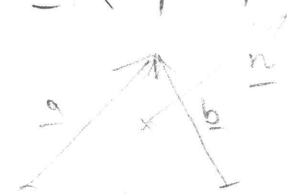
$(0,2,0)$

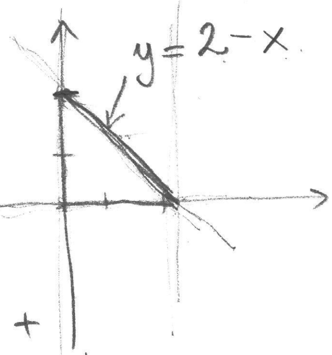
$(0,0,4)$



$\underline{a}\,(-2;0;4)$

$\underline{b}\,(0;-2;4)$





$$\underline{a}\times\underline{b} = \begin{array}{ccc} + & - & + \\ \end{array}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -2 & 0 & 4 \\ 0 & -2 & 4 \end{vmatrix} = \underline{i}\,(0+8) - \underline{j}\,(-8-0) + \underline{k}\,(4-0) =$$

$$= (\underset{A}{8}\,;\underset{B}{8}\,;\underset{C}{4}) = \underline{n}$$

$Ax + By + Cz = Ax_0 + By_0 + Cz_0$

$8x + 8y + 4z = 8\cdot 0 + 8\cdot 0 + 4\cdot 4$

$8x + 8y + 4z = 16 \quad /:4$

$\underline{2x + 2y + z = 4}$ sík egyenlete $\to z = 4-2x-2y$

$$\int_{x=0}^{2}\int_{y=0}^{2-x}\left(\int_{z=0}^{4-2x-2y} 1 \, dz\right)dy\,dx = \int_{x=0}^{2}\int_{y=0}^{2-x}\left[z\right]_{z=0}^{4-2x-2y} dy\,dx =$$

$$= \int_{x=0}^{2}\left(\int_{y=0}^{2-x} 4-2x-2y \; dy\right)dx = \int_{x=0}^{2}\left[4y - 2x\cdot y - \cancel{2}\frac{y^2}{\cancel{2}}\right]_{y=0}^{2-x} dx =$$

2004. jun. 17

(5) $f(x,y,z)=x^2+y^2+z^2-yz+2x-6z$

$f'_x=2x+2=0 \to 2x=-2$, $\underline{x=-1}$

$f'_y=2y-z=0 \to \underline{2y=z}$

$f'_z=2z-y-6=0$

$4y-y-6=0$

$3y=6$

$\underline{y=2}$ $\quad \underline{z=4}$

$P(-1;\ 2;\ 4)$

$f''_{xx}=2$

$f''_{xy}=0$

$f''_{xz}=0$

$f''_{yx}=f''_{xy}=0$

$f''_{yy}=2$

$f''_{yz}=-1$

$f''_{zz}=2$

$$\underline{\underline{H}}=\begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \end{array}$$

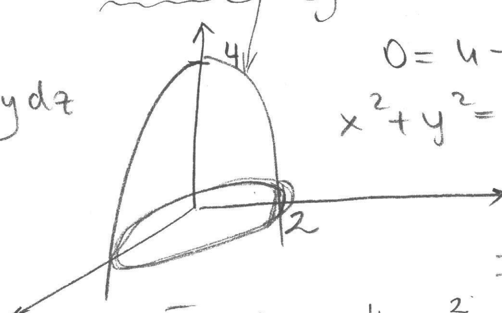
$|\underline{\underline{H}}|=2\cdot(4-1)=6>0$ ne!

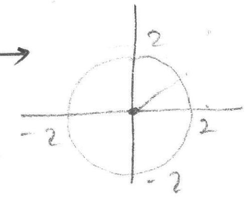
(6) $\underline{w}=(y,3x,7z)$ $\quad V:$ $z=4-x^2-y^2 = 4-(x^2+y^2)$ $\quad z=0$

$0=4-x^2-y^2$

$x^2+y^2=4$

$$\oiint_F w\, dF = \iiint_V \operatorname{div}\vec{w}\, dx\,dy\,dz$$





$\operatorname{div}\underline{w}=0+0+7=\underline{\underline{7}}$

$$\iiint_{x=\ y=\ z=} 7\, dx\,dy\,dz \overset{\text{henger}}{=} \int_{\varphi=0}^{2\pi}\int_{r=0}^{2}\int_{z=0}^{4-r^2} 7\cdot r\, dz\,dr\,d\varphi$$

$$=\int_{\varphi=0}^{2\pi}\int_{r=0}^{4} \underbrace{7r\,[z]_{z=0}^{4-r^2}}_{7r(4-r^2)=28r-7r^3} dr\,d\varphi = \int_{\varphi=0}^{2\pi}\left[28\frac{r^2}{2}-7\frac{r^4}{4}\right]_{r=0}^{4} d\varphi = -224\cdot 2\pi$$

$14\cdot 16$, $28\cdot 8$, $224-448$, $7\cdot 64$, -224

$=\underline{\underline{-448\pi}}$