2. házi feladatot kell leadni -> !!!!!!!!!

1 héttel hamarabb a 3. alkalom előtt.

május 4. !!!!! (Péntek) 12-ig Tanszék

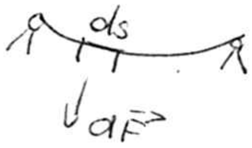
(M/(I) megjelent NEPTUN)

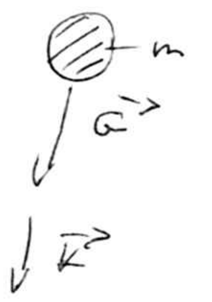
— ○ —

Megoszló erőrendszerek

Def. A támadáspontok folytonos ponthalmazt alkotnak

- térfogaton megoszló $d\vec{F} = \vec{q}\, dV$ pl: súly $dG = dm\vec{g} = \underbrace{\rho g}_{\gamma} \vec{k}\, dV$
- felületen –"– : $d\vec{F} = \vec{p}\, dA$
- vonalon –"– : $d\vec{F} = \vec{f}\, ds$

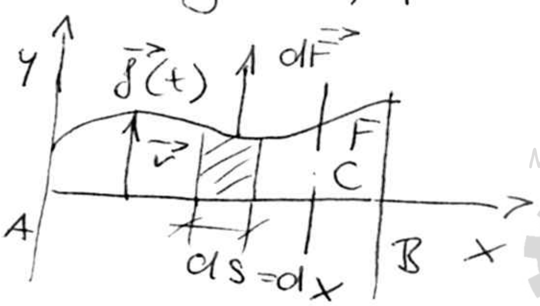




Eredő: $\vec{F} = \int d\vec{F}$

- Vonalon megoszló, párhuzamos ER

Pl:



$\vec{f}(t) = f(s)\vec{j}$

$f(t)$ = terhelés ábra

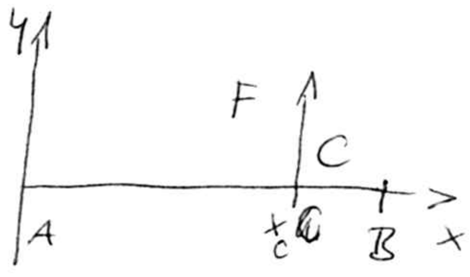
$$\vec{F} = \int_A^B \vec{f}\,dx = \underbrace{\int_A^B f(x)\,dx}_{F}\,\vec{j} = F\,\vec{j}$$

$$\vec{M}_A = \int \vec{r} \times d\vec{F} = \int (x\vec{i}) \times (f(x)\,\vec{j}\,dx) = \underbrace{\int x\,f(x)\,dx}_{M_z}\,\vec{k} =$$

$$= \underline{\underline{M_z\,\vec{k}}}$$

Eredő helye: $F \neq 0$ esetén

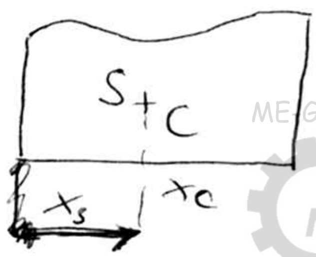
$x_c = ?$



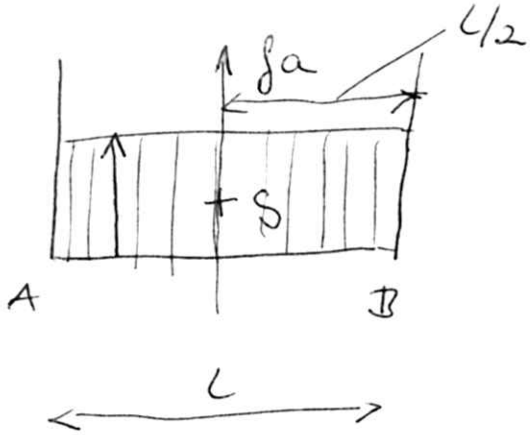
$$M_z = x_c F$$

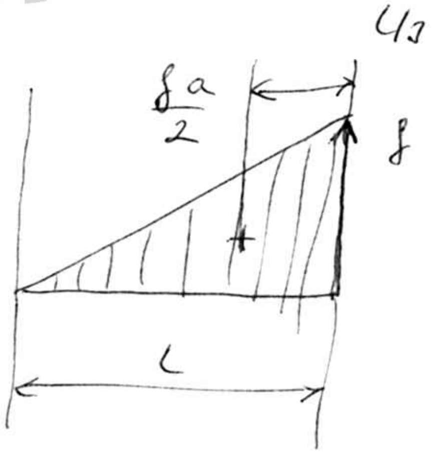
$$x_c = \frac{M_z}{F} = \frac{\int_A^B x\,f(x)\,dx}{\int_A^B f(x)\,dx}$$

$x_c = x_s$

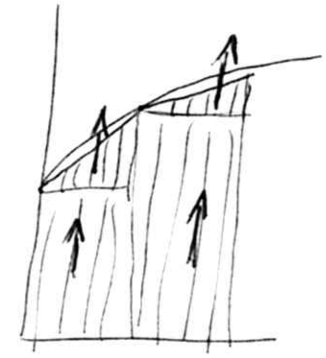


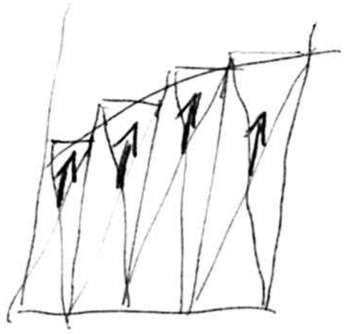
Nevezetes esetek:





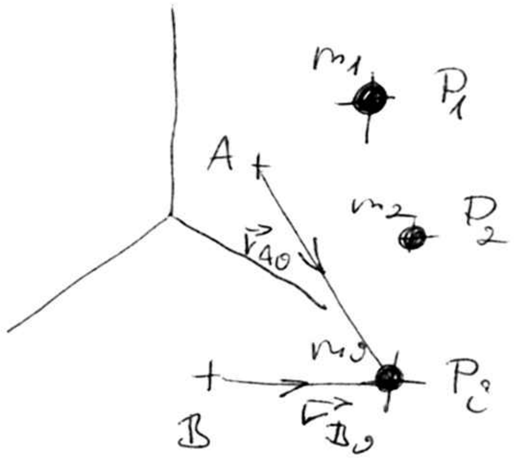
Általános eset.





Statikai nyomaték

- tömegpontrendszere



$$\vec{S}_A = \sum_{\vartheta=1}^{n} \vec{r}_{A\vartheta}\, m_\vartheta$$

$$\vec{S}_B = \sum_{\vartheta=1}^{n} \vec{r}_{B\vartheta}\, m_\vartheta =$$

$$= \sum \left(\vec{r}_{BA} + \vec{r}_{A\vartheta}\right) m_\vartheta$$

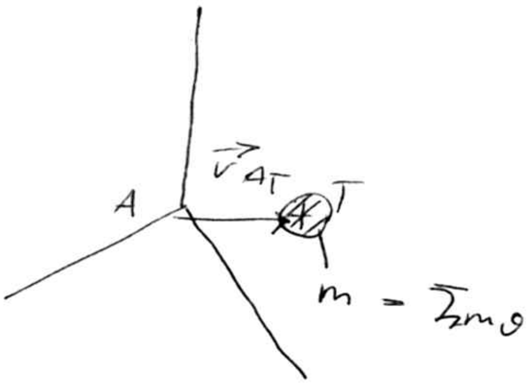
$$= \vec{r}_{BA} \underbrace{\sum m_\vartheta}_{m} + \underbrace{\sum \vec{r}_{A\vartheta}\, m_\vartheta}_{S_A}$$

$\vec{S}_B = \vec{r}_{BA}\, m + \vec{S}_A$

Tömegközéppont: T, $\vec{S}_T = \vec{0}$

$\vec{S}_T = \vec{r}_{TA}\, m + \vec{S}_A = 0$

$$\vec{r}_{AT} = \frac{\vec{S}_A}{m}$$



$\vec{S}_A = m\,\vec{r}_{AT}$

Statikai nyomaték számításánál a tömegpontok helyettesíthetők a tömegközéppontban az eredő tömeggel, ill. tömegével.

$$\vec{r}_{AT} = \frac{\sum \vec{r}_{Ai}\, m_i}{\sum m_i}$$

Általánosítás:

tömeg~~kö~~pont helyett:
- dm – elemi tömeg
- dV – elemi térfogat
- dA – elemi felület
- ds – elemi vonal

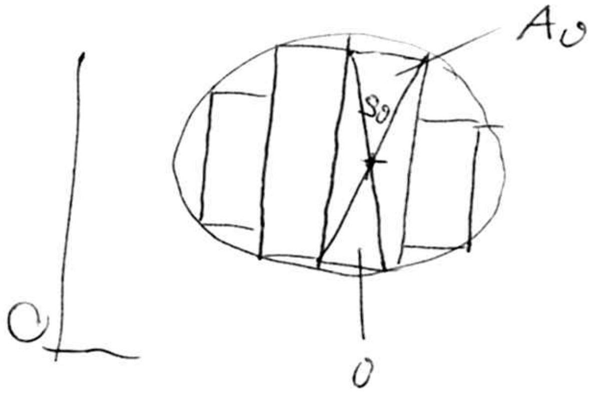
} súlypont

$T = S$

Fogalom	test, térfogat	felület	vonal
statikai nyomaték	$\int \vec{r}\,dm$, $\int \vec{r}\,dV$	$\int \vec{r}\,dA$	$\int \vec{r}\,ds$
osztó	m , V	A	L
súlypont	$\vec{S}_0/m$, $\vec{S}_0/V$	$\vec{S}_0/A$	$\vec{S}_0/L$

Homogén test, térfogat, felület, vonal } szimmetria { pontja, tengelye, síkja } egyben { súlypont, súlyvonal, súlysík }

Szabályos részek:



$$\vec{S}_0 = \sum \vec{r}_{S_i} A_i$$

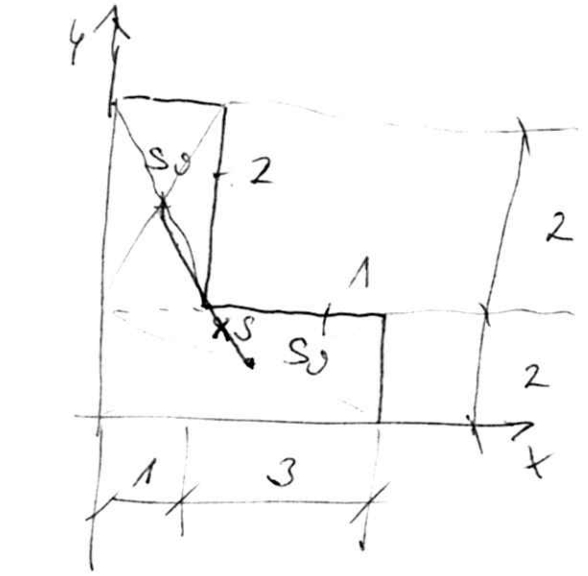
$$A = \sum A_i$$

$$\boxed{\vec{r}_S = \frac{\sum \vec{r}_{S_i} A_i}{\sum A_i}}$$

$A_i \rightarrow l_i$ vonal

V_i térfogat

Pl.:



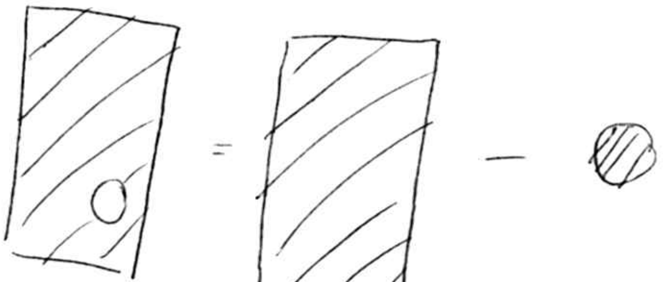
$\vec{r}_S = ?$

i	x_{Si}	y_{Si}	A_i
1	2	1	8
2	0,5	3	2

$$\vec{r}_S = \frac{\vec{r}_{S_1} \cdot A_1 + \vec{r}_{S_2} \cdot A_2}{A_1 + A_2}$$

$$x_S = \frac{1}{10}\left(2 \cdot 8 + 0{,}5 \cdot 2\right) = \frac{17}{10} = 1{,}7$$

$$y_S = \frac{1}{10}\left(1 \cdot 8 + 3 \cdot 2\right) = 1{,}4$$



$$\vec{r}_S = \frac{\vec{r}_{S_1} A_1 - \vec{r}_{S_2} A_2}{A_1 - A_2}$$

Összetett szerkezet statikája

Def: több test kapcsolódnak egymással, v. támaszkodnak egymáshoz.

Feladat: támasztó- és belső ER

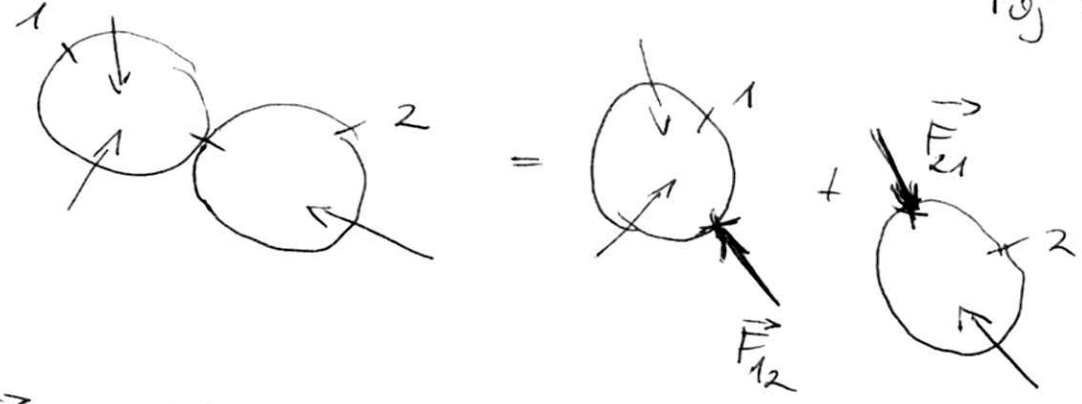
Módszer: - teljes szerkezet
- több test
- egy-egy test

} egyensúlya

n - test: $S_e = \begin{cases} 3n & \text{síkban} \\ 6n & \text{térben} \end{cases}$ (egyenlet)

Belső erők

F_{ij}: i-re, j-ről.

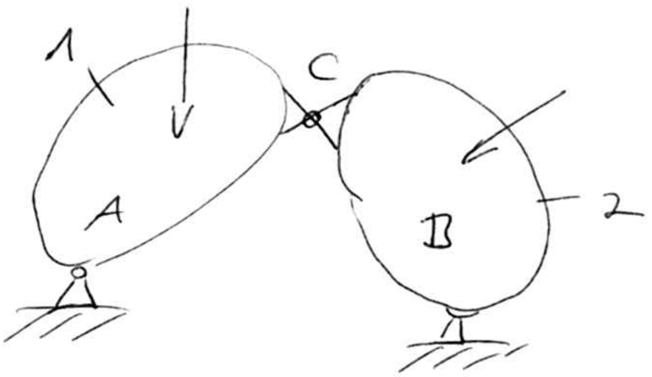


$\vec{F}_{ij} = -\vec{F}_{ji}$

Feladat típusok

- Csuklós szerkezet:

háromcsuklós ív:

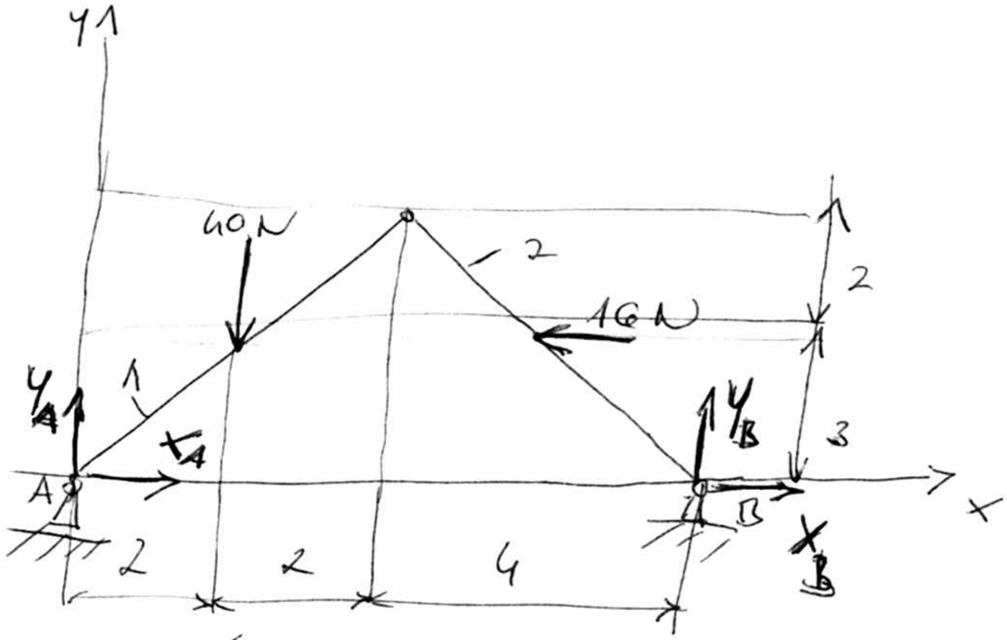


$S_e = 2 \cdot 3 = 6$

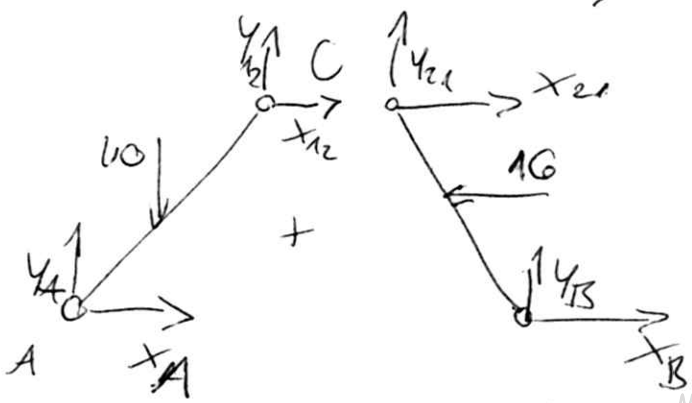
$S_j = 3 \cdot 2 = 6$

(ismeretlen)

Pl:



(4 ismeretlen)



(4 ism)

(4 ism)

(1+2): $M_a \overset{+}{=} -2 \cdot 40 + 3 \cdot 16 + 8 \cdot Y_B = 0$

$$Y_B = \frac{80 - 48}{8} = \frac{32}{8} = \underline{\underline{4\ N}}$$

(1+2): $\sum Y_i = Y = Y_A - 40 + Y_B = 0$

$$Y_A = \underline{\underline{36\ N}}$$

(1): $M_C \overset{+}{=} 2 \cdot 40 - 4 \cdot \overset{36}{Y_A} + 5 \cdot X_A = 0$

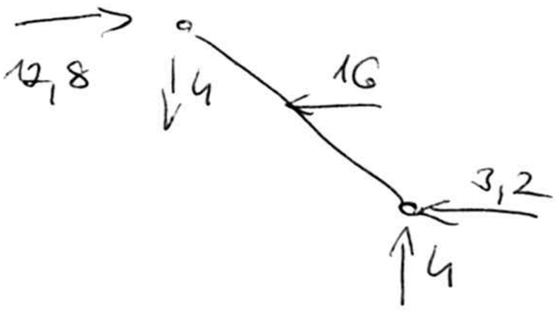
$$X_A = \frac{144 - 80}{5} = \underline{\underline{12,8\ N}}$$

(1): $\sum X_i = X = X_A + X_{12} = 0 \Rightarrow X_{12} = \underline{\underline{-12,8\ N}}$

(1): $\sum Y_i = Y = Y_A - 40 + Y_{12} = 0 \Rightarrow Y_{12} = \underline{\underline{4\ N}}$

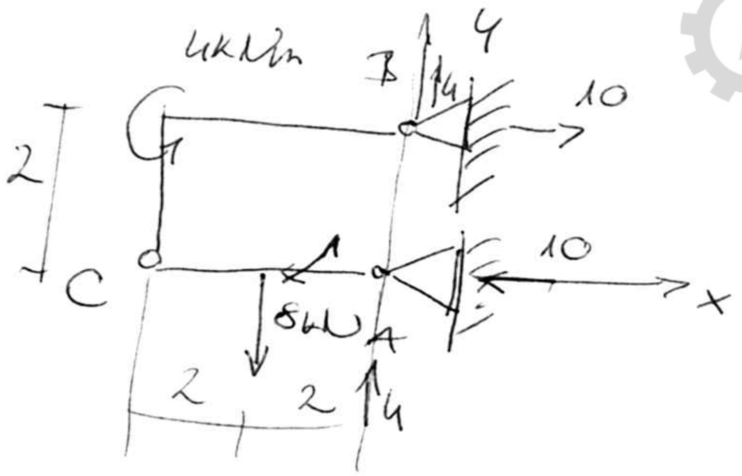
(1+2): $\sum X_i = X = X_A - 16 + X_B = 0 \Rightarrow X_B = \underline{\underline{3,2\ N}}$

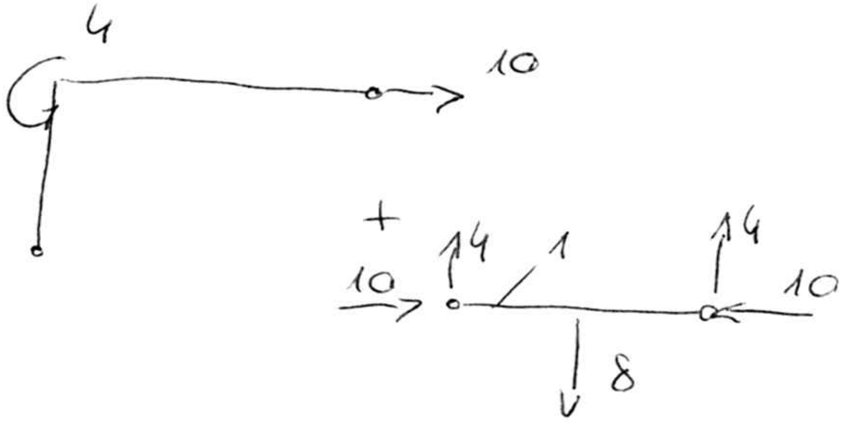
Ellenőrzés: (2-es test):



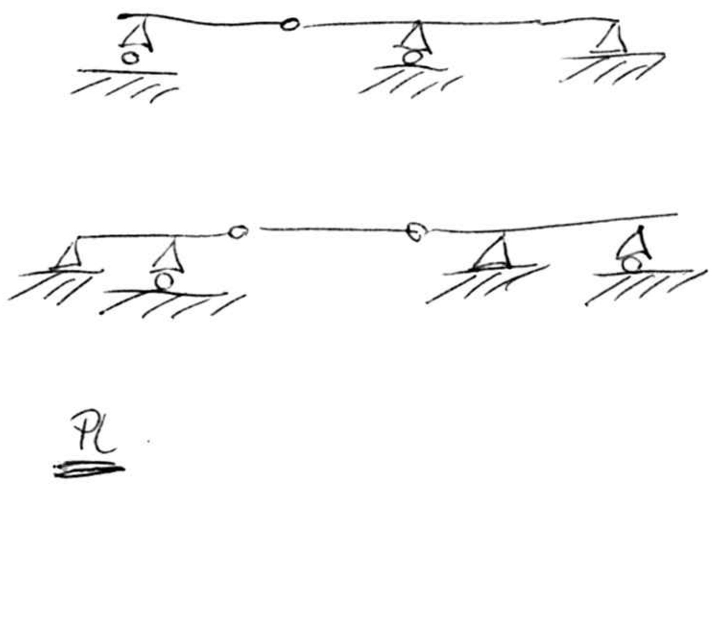
$$\left.\begin{aligned} X_{21} &= 12,8\ N \\ Y_{21} &= -4\ N \end{aligned}\right\}$$

Pl:

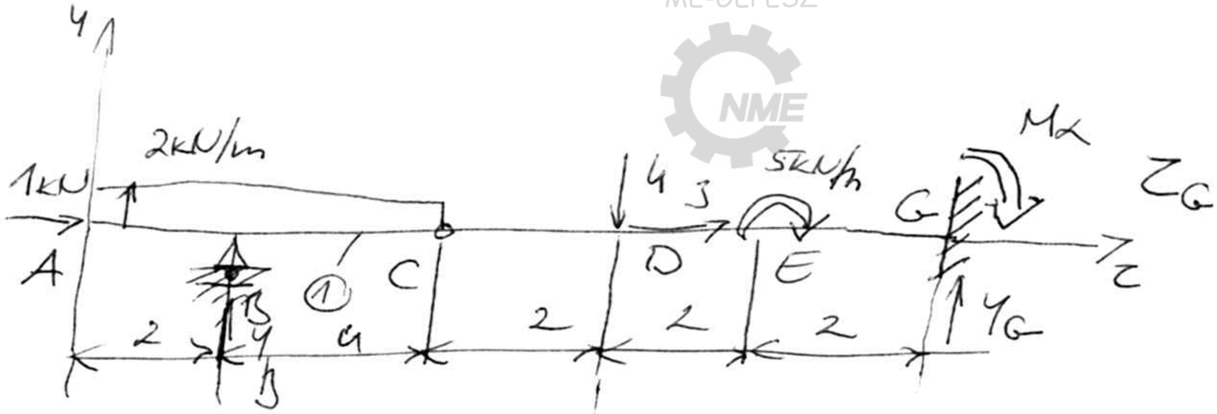


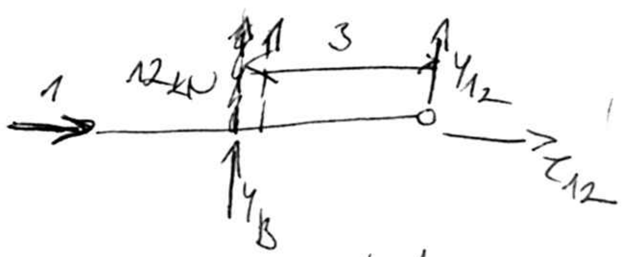


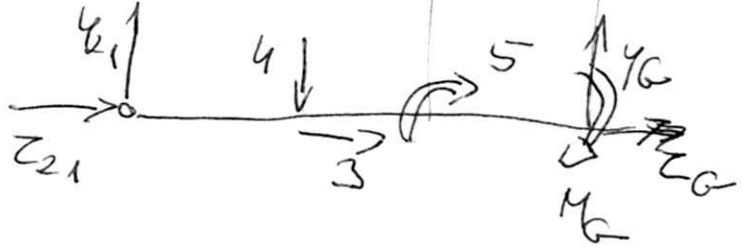
Gerber-tartók



Pl



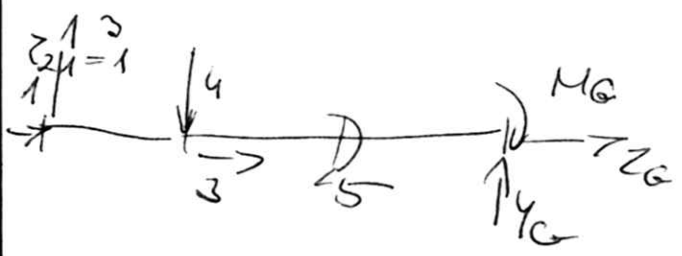




(1) : $M_C \overset{\curvearrowleft +}{=} -4Y_B - 3\cdot 12 = 0 \Rightarrow Y_B = -9\,kN$

(1) : $\sum Z_0 = 1 + Z_{12} = 0 \Rightarrow Z_{12} = -1\,kN$

(1) : $\sum Y_0 = \underset{-9}{Y_B} + 12 + Y_{12} = 0 \Rightarrow Y_{12} = -3\,kN$



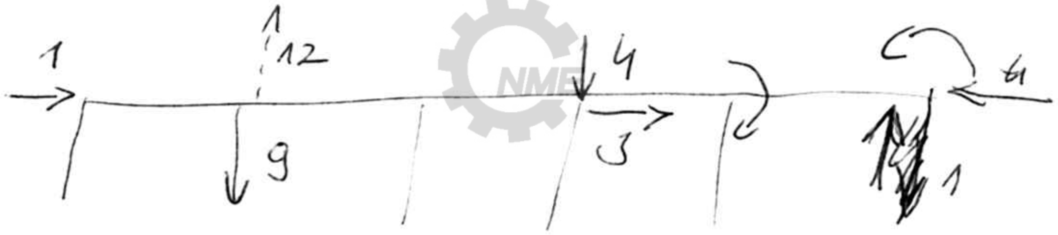
(2) $M_G \overset{\curvearrowleft +}{=} -6\cdot 3 + 4\cdot 4 - 5 - M_G = 0 \Rightarrow M_G = \underline{\underline{-7\,kNm}}$

(2) : $\sum Z_0 = 1+3+Z_G = 0 \Rightarrow Z_G = \underline{\underline{-4\,kN}}$

(2) : $\sum Y_0 = 3-4+Y_G = 0 \Rightarrow Y_G = \underline{1}$

Ellenőrzés:

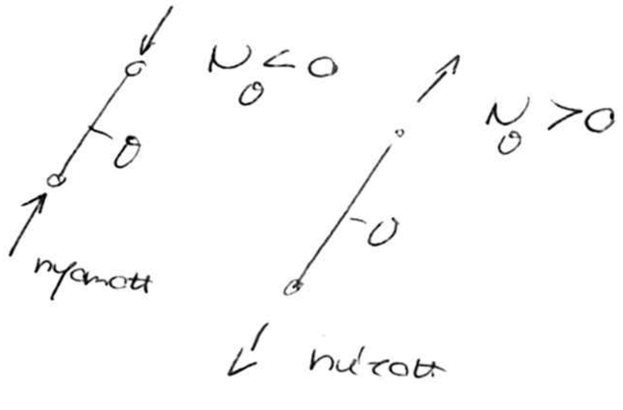
(1+2):



Rácsos tartók

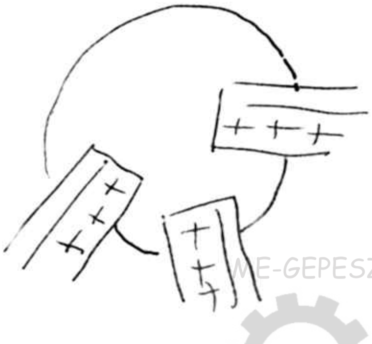
Feltételek: - rudak csuklóval kapcs. egymáshoz
- terhelés, támasztás csuklóban
- Önsúly = 0
- támasztó ER: teljes szerk. egyensúlya

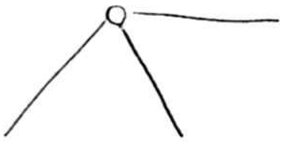
Rúderő:



$N_0 = 0$ vakrúd

rúdvég:

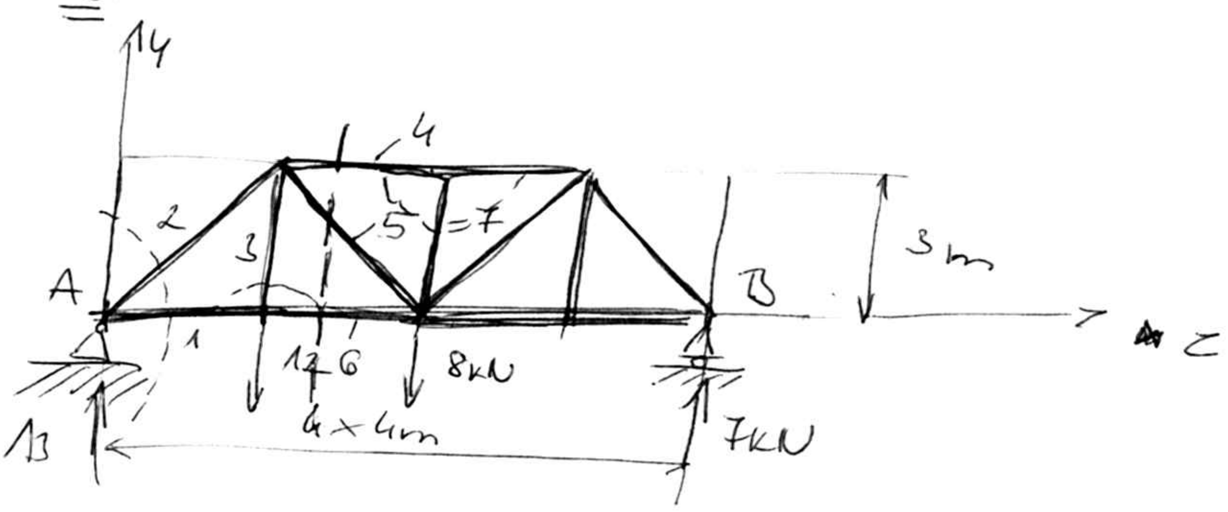




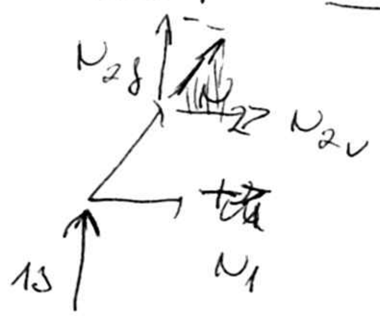
ME-GEPESZ
NME

Rúderők számítása:

Pl:



Csomóponti módszer



$$\Sigma y_j = 13 + N_{2y} = 0$$

$$N_{2y} = -13$$

$$\frac{N_{2v}}{N_{2y}} = \frac{4}{3}$$

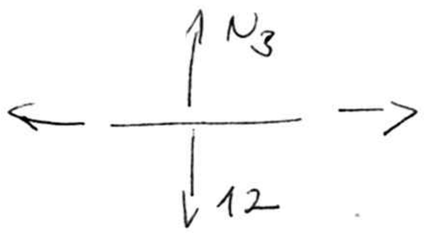
$$N_{2v} = \frac{4}{3} \cdot N_{2y} = \frac{4}{3} \cdot -13 =$$

$$N_2 = -13\sqrt{\left(\frac{4}{3}\right)^2 + 1^2} = -13\,\frac{5}{3}$$

$$N_2 = -\frac{65}{3} = -21{,}67$$

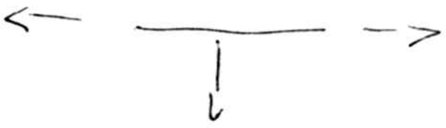
$\sum Z_0 = N_1 + N_{2v} = 0$

$N_1 = -N_{2v} = \frac{52}{3}$



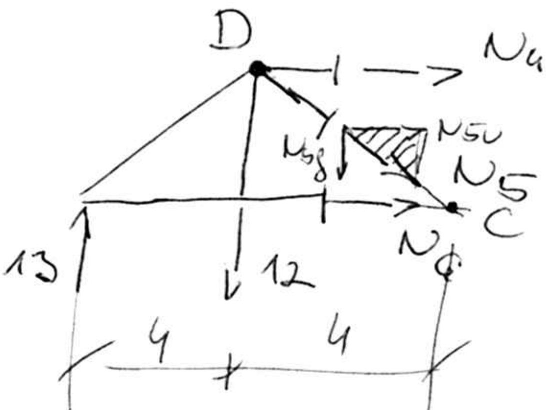
$\sum Y_0 = N_3 - 12 = 0 \Rightarrow N_3 = 12$

$N_3 = 12$



$N_7 = 0$

átmetsző módszerrel



$M_C = -8\cdot 13 + 4\cdot 12 - 3\cdot N_4 = 0$

$N_4 = \frac{48-104}{3} = -\frac{56}{3} =$

$= -18{,}67$

$\sum Y_0 = 13 - 12 - N_{5f} = 0$

$N_{5f} = 1$

$N_{5v} = N_{5f}\cdot\frac{4}{3} = \frac{4}{3}$

$$N_5 = \sqrt{N_{5x}^2 + N_{5y}^2} = \frac{5}{3}$$

$$M_a \overset{\curvearrowleft +}{=} -4 \cdot 13 + 3N_6 = 0$$

$$N_6 = \frac{52}{3}$$