2003.02.22

442-es terem
DR NÁNDORI FRIGYES

MECHANIKA

MM: a + gy

03.25	I. ZH	x I. hf.
04.11	2.3. ZH	
05.16	P. ZH	

3 x 17 = 51 pont
3 hf. = 3 x 3 pont

60 pont

0-21 = 1
22-29 = 2
30-37 = 3
38-44 = 4
~~37~~ 45 — 60 jeles

ÉGERT JÁNOS
STATIKA

A	f
2	2

MŰSZAKI MECHANIKA

- KINEMATIKA (mozgástan)
  - ANYAGI PONT KIN.
  - MEREV TEST KIN.
- DINAMIKA (erőtan)
  - STATIKA
    - MEREV TEST STATIKA — I.
    - ALAKVÁLT. KÉPES SZILÁRD TEST SZILÁRDSÁGTAN — II.
  - KINETIKA

TEST MODELL: KÉPZELT TEST AMELY A VALÓSÁGOS TEST TUL. TÜKRÖZI

MEREV TEST: BÁRMELY KÉT PONTJÁNAK TÁVOLSÁGA ÁLLANDÓ
(TERHELÉSI VISZONYOK)
(szivacs légy ez a hereim)

ANYAGI PONT: OLYAN TEST AMELYNEK A MOZGÁSÁT EGYETLEN PONTJÁBAN JELLEMEZHETJÜK.

SZILÁRD ~~PONT~~ TEST: ALAKVÁLTOZÁSKÉPES TEST

MENNYISÉGEK

- skaláris (szám és m.e.)
- vektor def. irányított fiz. mennyiség

jellemzői:

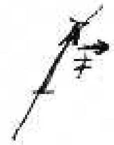
= hatásvonal

- nagyság
- irányítás
- mértékegység

pl. - helyvektor

$\vec{r}_{AB}$

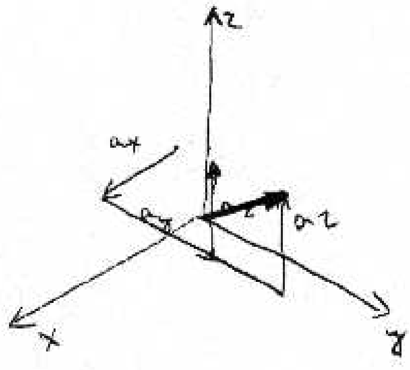
- Erővektor



- kötöttség (támadásponthoz v. hatásvonalhoz való kötöttség)

innen kiemelni szabad

X, Y, Z



megadása: - értelmezéssel $\vec{a}, \vec{b}$

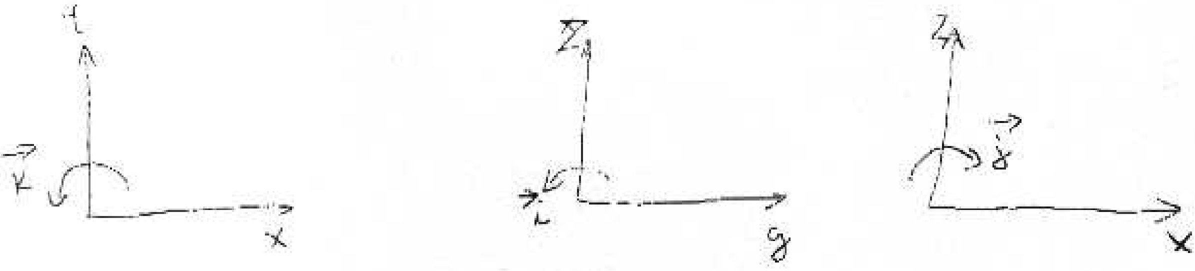
- koordináta rendszerben

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_x \vec{k}$$

$$\vec{i} = \vec{e}_x \quad \vec{j} = \vec{e}_y \quad \vec{k} = \vec{e}_z$$

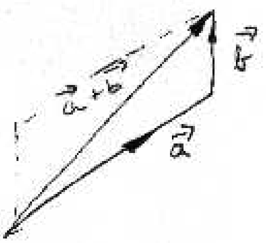
$a_x; a_y; a_z$ = koordináta

-3-



Műveletek:

$$\vec a \pm \vec b = (a_x \pm b_x)\,\vec i + (a_y \pm b_y)\,j + (a_z \pm b_z)\,\vec k$$



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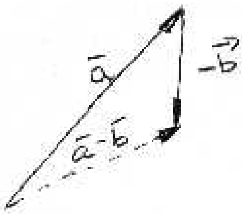
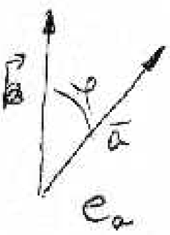


Szorzások:

- skaláris

$$c = \vec a \cdot \vec b = |\vec a| \cdot |\vec b| \cdot \cos\varphi$$

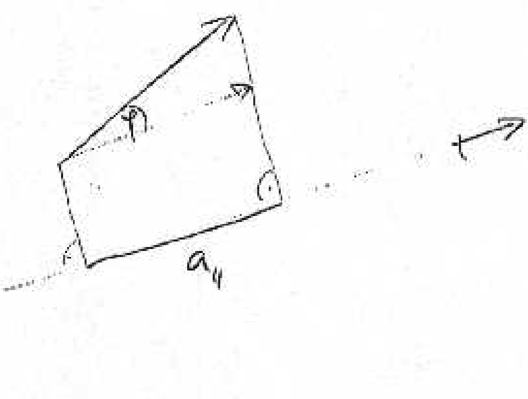
$$c = a_x b_x + a_y b_y + a_z b_z$$



alkalmazás: „nagyság"

$$\vec a \cdot \vec a = |\vec a|^2 = a^2 \rightarrow a = \sqrt{\vec a \cdot \vec a} = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\left.\begin{array}{l} |\vec e_a| = 1 \\ \vec a = a\,\vec e_a \end{array}\right\} \text{„irányegység vektor"}$$



$|\bar e| = 1$

$a_1 = a \cdot \cos\varphi = \bar a \cdot \bar e$

általában

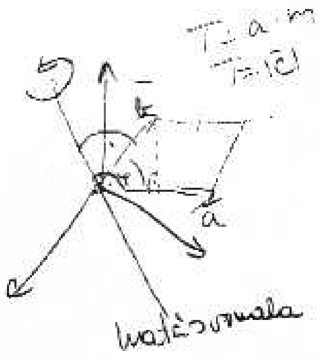
$\bar a \cdot i = a_x \qquad \bar i \cdot \bar i = 1$

$\bar a \cdot j = a_y \qquad \bar j \cdot \bar j = 1$

$\bar a \cdot k = a_z \qquad \bar k \cdot \bar k = 1$

$\bar i \bar j = \bar j \cdot \bar k = \bar i \bar k = 0$

Vektoriális szorzás:



$$\bar{c} = \bar{a} \times \bar{b}$$

$$|\vec{c}| = |\vec{a}|\,|\vec{b}| \cdot \sin\varphi$$

$$\bar{c} \perp \bar{a} ; \bar{b}$$

$\bar{a} ; \bar{b} ; \bar{c}$ = jobbsodrású

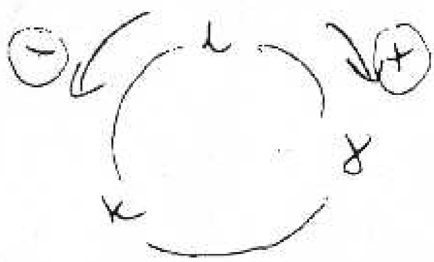
$$\vec{c} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a_x & a_y & a_x \\ b_x & b_y & b_z \end{vmatrix} = i \ldots$$

Spec eset

$\bar{i} \times \bar{j} = \vec{k}$ $\quad \bar{j} \times \bar{i} = -k$

$\bar{j} \times \bar{k} = \vec{i}$

$\bar{k} \times \bar{i} = \vec{j}$



Vegyes szorzás

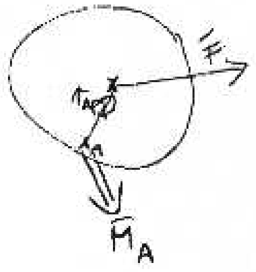
$$(\bar{a} \times \bar{b}) \cdot \bar{c} = [\bar{a} \cdot \bar{b} \cdot \bar{c}] = \pm V$$

ERŐRENDSZEREK

$$1N = 1 kg\, m/s^2$$

$$1KN = 10^3 N \approx 100 kg$$

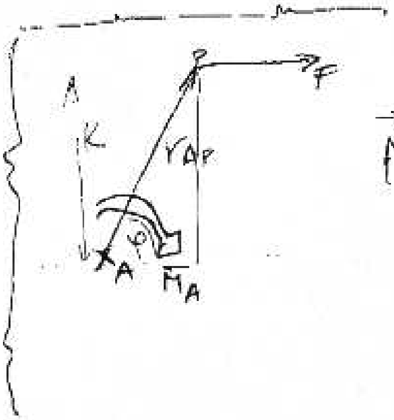
Egy erő nyomatéka



$$\boxed{\vec{M}_A \stackrel{def}{=} \vec{r}_{AP} \times \bar{F}} \quad (Nm)$$

Szemléltetés

$\bar{r}_{AP}$ $\bar{F}$ síkja:



$$|\vec{M}_A| = |\bar{r}_{AP}|\underbrace{|\bar{F}| \sin\varphi \ast f}_{\text{erő} \times \text{kar}}$$

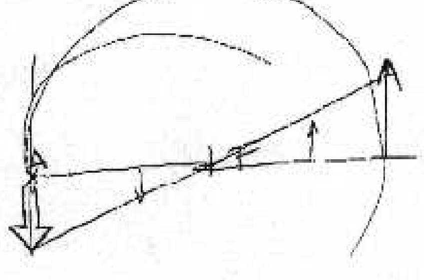
tulajdonsága

1. tul. a nyomaték nem változik ha az erőt a hatásvonala mentén elmozgatjuk

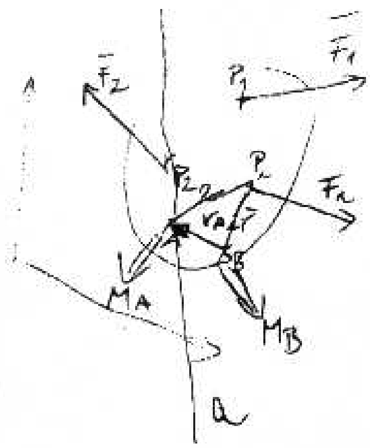
2. tul az erővel párh. egyenes pontjaiban a nyomaték állandó

3. tul az erő hatásvonalának pontjaiban a nyomaték zérus

F-re ⊥-os



Több erő nyomatéka



$$\vec{F}_i(P_i) \qquad i = 1 \dots n$$

$$\boxed{\vec{M}_A \overset{def}{=} \sum_{i=1}^{n} \vec{r}_{Ai} \times \vec{F}_i}$$

$$\vec{M}_B = \sum \vec{r}_{Bi} \times \vec{F}_i = \sum \left(\vec{r}_{BA} + \vec{r}_{Ai}\right) \times \vec{F}_i$$

$$= \vec{r}_{BA} \times \underbrace{\sum \vec{F}_i}_{F} + \underbrace{\sum \vec{r}_{Ai} \times \vec{F}_i}_{M_A}$$

$$\bar{F} = \sum_{i=1}^{n} \vec{F}_i \quad \text{- eredő}$$

$$\vec{M}_B = \vec{r}_{BA} \times \vec{F} + \vec{M}_A$$

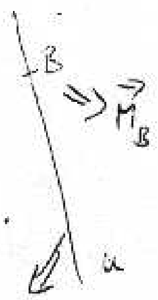
$$\vec{M}_B = \vec{M}_A + \vec{F} \times \vec{r}_{AB}$$

Tengelyre számított nyomaték

tengely: irányított egyenes

a-tengely: $\vec{r}_A$ - egy pont; $\bar{a}$ - irány

$$\boxed{m_a \overset{def}{=} \vec{M}_A \cdot \vec{e}_a} = M_A \cdot \frac{\bar{a}}{a}$$

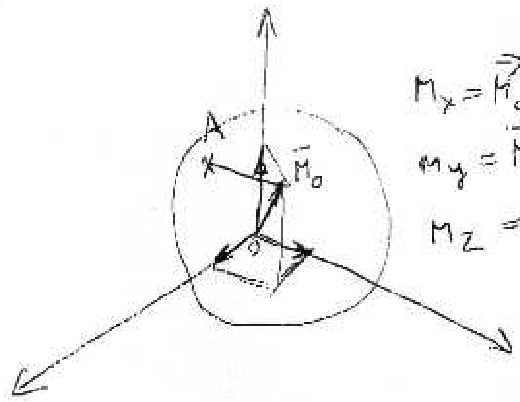


$$M_a = \vec{M}_A \vec{e}_a$$

$$M_a = \vec{M}_B \vec{e}_a$$

$$M_B \cdot \bar{e}_a = \left(\vec{M}_a + \vec{F} \times \vec{r}_{AB}\right) \cdot \bar{e}_a = M_A \cdot \vec{e}_a + \underbrace{\left(F \times r_{AB}\right) \cdot \bar{e}_a}_{0}$$

Speciális eset



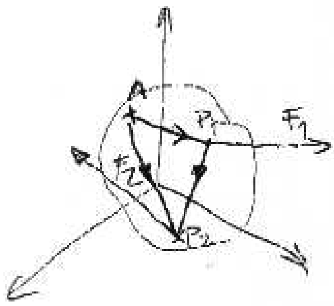
$M_x = \vec{M}_0 \cdot \vec{i}$

$M_y = \vec{M}_0 \cdot \vec{j}$

$M_z = \bar{M}_0 \cdot \bar{k}$

$M_0 = M_x \cdot \bar{i} + M_y \cdot \bar{j} + M_z \cdot \bar{k}$

Spec. eset: erőpár



$\bar{F}_1 = -\bar{F}_2$

$$M_A = \bar{r}_{A1} \times \underbrace{\bar{F}_1}_{-F_2} + \bar{r}_{A2} \times \bar{F}_2$$

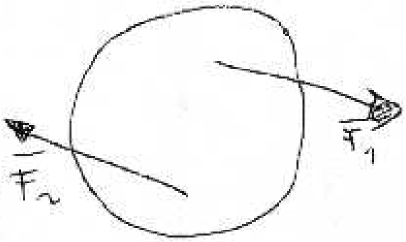
$$= (\bar{r}_{A2} - \bar{r}_{A1}) \times \bar{F}_2$$

kiesik

mert →

$$\boxed{\bar{M}_A = \bar{r}_{12} \times \bar{F}_2} = \bar{M}_B$$

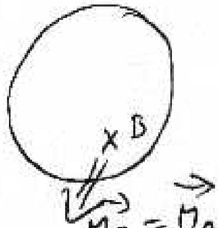
homogén nyom. tér



$\bar{F}_1 = -\bar{F}_2$

$=$

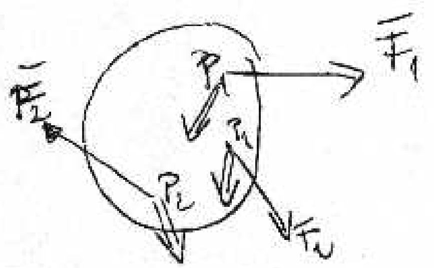
$=$



erőpárr ≡ nyomaték

$\vec{M}_B = \vec{M}_A$

Ált. erőrendszer:



Erőrendszer áll

$F_i\ (P_i)$ és $M_i\ (P_i)$ $i = 1 \dots n$

E.R. által létrehozott nyomaték

$$\boxed{\bar{M}_A \overset{\text{def}}{=} \sum_{i=1} \vec{r}_{Ai} \times F_i + \sum_{i=1} \vec{M}_i}$$

Érvényes $F = \sum F_i$ - eredő

$$\vec{M}_B = \vec{M}_A + \vec{F} \times \vec{r}_{AB}$$

Erőrendszer egyenértékűsége és egyensúlya:

Def: két erőrendszer egyenértékű ha a tér bármely pontjában azonos a nyomatéka.

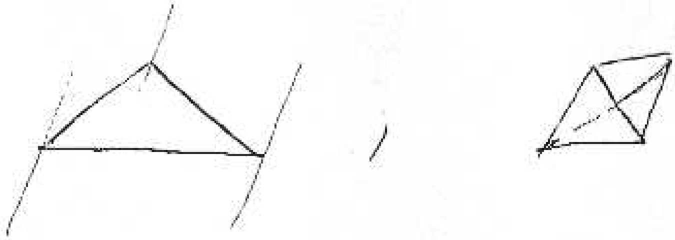
jel $E' \stackrel{M}{=} E''$

egy erőrendszer egyensúlyi ha a tér bármely pontjában $\emptyset$ a nyomatéka.

$$E \stackrel{M}{=} \vec{0}$$

Feltételek:

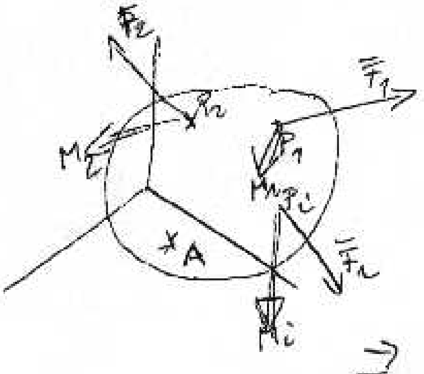
tétel	$E' \stackrel{M}{=} E''$	$E \stackrel{M}{=} \bar{0}$	
a,	$F' = F''$, $\vec{M}'_A = \vec{M}''_A$	$\bar{F} = \bar{0}$, $\bar{M}_A = \bar{0}$	A = egy pont
b,	$\bar{M}'_A = \bar{M}''_A$ $\bar{M}'_B = \bar{M}''_B$ $\bar{M}'_C = \bar{M}''_C$	$M_A = \bar{0}$ $M_B = \bar{0}$ $M_C = \bar{0}$	A, B, C ... nem kolineáris (nem esik 1 egyenesbe)
c,	$M'_i = M''_i$	$M_i = 0$	$i = 1, \dots 6$ 6 lineárisan független tengely



Redukált vektor kettős:

Def: bármely erőrendszer egyenértékű a redukált vektor kettősével, amely 1 erőből és ~~1 vektor~~ egy erőpár nyomatékából áll.

$$\text{Def} \quad E \overset{M}{=} \underbrace{\left[\vec{F}; \vec{M}_A\right]_A}_{\text{Redukált vektor kettős}}$$

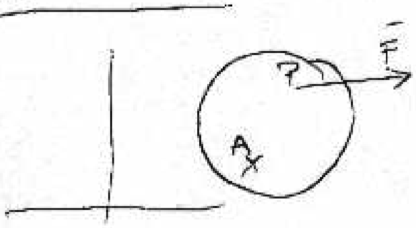


$\overset{M}{=}$

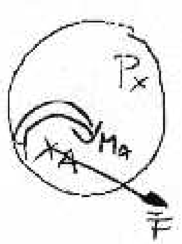
$$\vec{F} = \sum \vec{F}_i$$

$$\vec{M}_A = \sum \vec{r}_{A_i}^{\,i} \times \vec{F}_i + \sum \vec{M}_i$$

Alkalmazás:

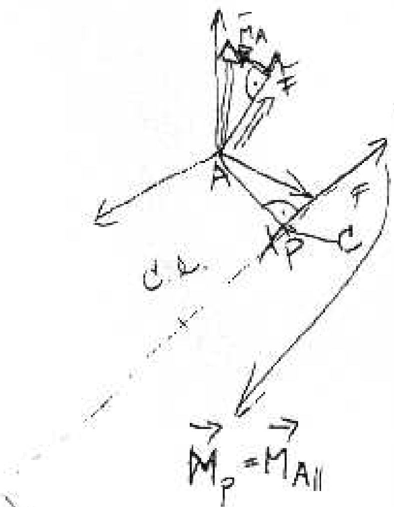


$\overset{M}{=}$



Osztályozás - [illegible]

$[\bar{F}; \bar{M}_A]_A$



1.) $\bar{F} = \bar{0}$

a.) $\bar{M}_A = \bar{0}$ - egyensúlyi

b.) $M_B \neq \bar{0}$ - erőpár

2.) $\bar{F} \neq \vec{0}$

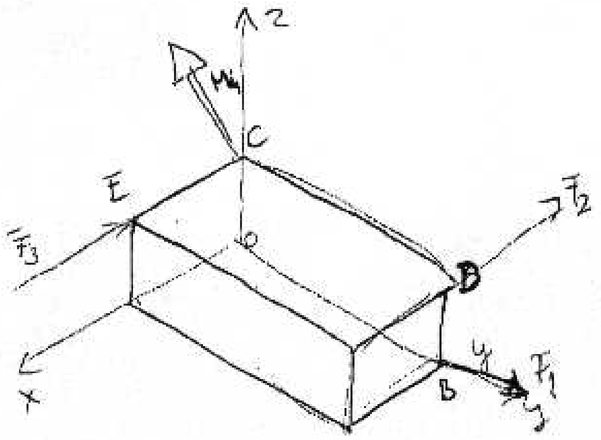
a.) $\bar{F} \cdot \vec{M}_A = 0$ - $[\vec{F}; \vec{0}]_P$ eredő erő

b.) $F \cdot M_A \neq 0$ - $[\vec{F}; \vec{M}_{A\parallel}]_P$ erőcsavar

centrális egyenes

$$\vec{r}_{AC} = \frac{\bar{F} \times \bar{M}_A}{F^2}$$

Pl:



Adatok: $\bar{OA} = 2m$ $\bar{OB} = 4m$ $OC = 1m$

$F_1 = 4j$

$F_2 = 2j + 2k$

$F_3 = -6i$

$\bar{M}_4 = 4i + 2k$

Feladat redukáljuk az origóba

$[\vec{F}, \vec{M_o}]_o = ?$

$\vec{F} = \Sigma \vec{F_i} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$\vec{F} = -6\vec{i} + 6\vec{j} + 2\vec{k}$ (KN)

$M_o = \sum_1 \vec{r_{oi}} \times \vec{F_i} + M_4 = \underbrace{4\bar{j} \times 4\bar{j}}_{0} + (4j+k) \times (2\bar{j}+2\bar{k}) + (2\bar{i}+\bar{k}) \times (-6i) + M_4$

Mholdal = Mjobb

$$\begin{vmatrix} i & j & k \\ 0 & 4 & 1 \\ 0 & 2 & 2 \end{vmatrix} = i(8-2) - j(0) + k(0) = 6i$$

$$\begin{vmatrix} i & j & k \\ 2 & 0 & 1 \\ -6 & 0 & 0 \end{vmatrix} = i(0) - j(0+6) + k(0) = -6j$$

$M_o = 6\bar{i} - 6\bar{j} + 4\bar{i} + 2\bar{k} = \underline{10i - 6j + 2k \text{ KNm}}$

Osztályozás:

1, $\bar{F} \neq 0$

2, $\bar{F} \cdot \bar{M_o} = -6 \cdot 10 - 6 \cdot 6 + 2 \cdot 2 = -92 \text{KN}^2\text{m} \neq 0$ 2b

erőcsavar

MECHANIKA

Alapfeladatok

- redukálás $[\vec{F}, \vec{M}_0]_O$
- helyettesítés $E' \overset{M}{=} E''$ (felbontás, eredő)
- egyensúlyozás $E' + E'' \overset{M}{=} \vec{0}$

E': adott

E'': ismeretlen

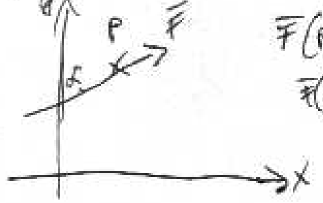
A helyettesítés és egyensúlyozás megoldása csak előjelben tér el egymástól.

Szerkesztési feladatok

Szerkesztések:

- helyzet (szerkezeti) ábra: erők elhelyezkedését, esetleg irányát is mutatja méretarányosan (méter)
- Erő (vektor) ábra: erőt nagyság és ir. szerint is tartalmazza. m.e.: N (Newton)

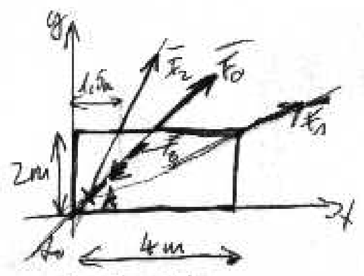
Egy erő:



$\vec{F}(P): \vec{F}, \vec{r}_P : 2+2=4$ adat

$\vec{F}(\alpha): \vec{F}, \alpha : 2+1=3$ adat

- Két erő eredője:



helyzetábra

$\vec{F}_1 = 3\vec{i} + \vec{j}$ N $\quad \vec{F}_0 = 5\vec{i} + 5\vec{j}$ N

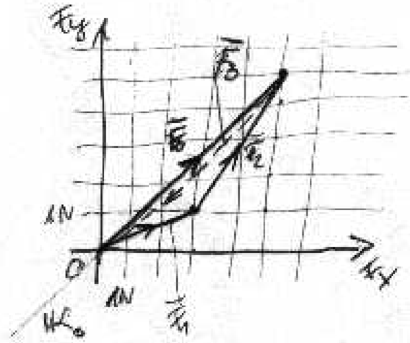
$\vec{F}_2 = 2\vec{i} + 4\vec{j}$ N

$(\vec{F}_1(\alpha_1), \vec{F}_2(\alpha_2)) \overset{M}{=} (\vec{F}_0(\alpha_0))$

$\vec{F}_1 + \vec{F}_2 = \vec{F}_0 \quad (\vec{F}' = \vec{F}'')$

$\vec{M}_A' = \vec{M}_A''$

$\vec{0} = \vec{M}_A'' \quad \alpha_0 \to A$



Két metsződő erő mindig helyettesíthető az eredőjével a hatásvonalaik metszéspontjában.

– Három erő egyensúlyban

$$\underbrace{\left(\bar F_1(\alpha_1),\ \bar F_2(\alpha_2)\right.}\left.,\ \bar F_3(\alpha_3)\right) \overset{M}{=} (\bar 0)$$

egyensúly feltételei: $\bar F_{12} \overset{?}{=} \bar F_1 + \bar F_2 \to A$

$(\bar F = \bar 0)\quad \bar F_{12} + \bar F_3 = \bar 0 \Rightarrow \bar F_3 = -\bar F_1 - \bar F_2$

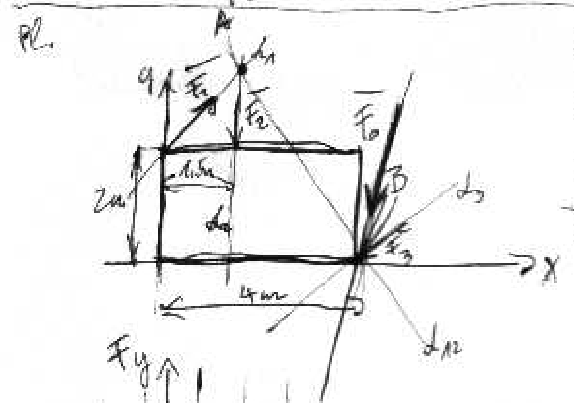
$\bar M_A = \bar 0 \quad \bar F_3 \to A$

Három metsződő erő csak akkor lehet egyensúlyban, ha eredőjük zérus és ~~...~~ hatásvonalaik 1 pontban metsződnek.

2 erő akkor lehet egyensúlyban, ha nagyságuk azonos, ir.-uk ellentétes, hatásvonaluk közös (azonos).

– 3 erő eredője – részeredő sorozatos szerkesztés

Pl.



$\bar F_1 = 2\bar i + 2\bar j\ N$

$\bar F_2 = \quad -6\bar j\ N$

$\bar F_3 = -4\bar i - 2\bar j\ N$

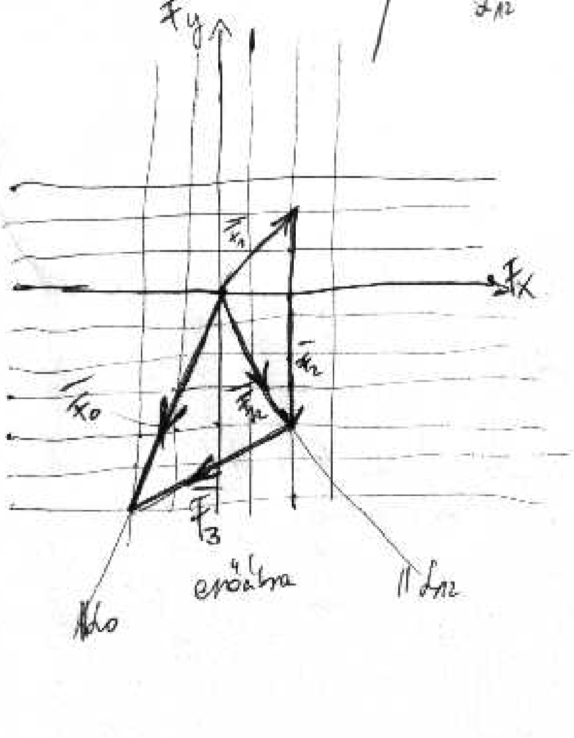
$\bar F_0 = 2\bar i - 6\bar j\ N$

$$\left(\bar F_1(\alpha_1),\ \bar F_2(\alpha_2),\ \bar F_3(\alpha_3)\right) \overset{M}{=} \left(\bar F_0^{?}(\alpha_0)\right)$$

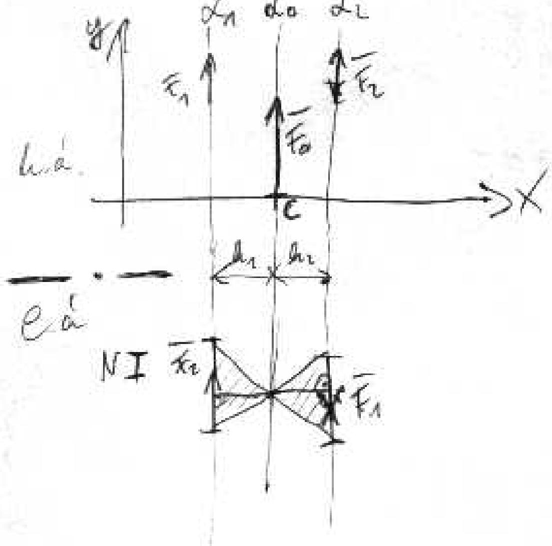
$\bar F_{12} = \bar F_1 + \bar F_2 \to A$

$$\left(\bar F_{12}(\alpha_{12}),\ \bar F_3(\alpha_3)\right) \overset{M}{=} \left(\bar F_0(\alpha_0)\right)$$

$\bar F_{12} + \bar F_3 = \bar F_0$



– Párhuzamos erők eredője



$$\left(\bar F_1(\alpha_1), \bar F_2(\alpha_2)\right) \overset{M}{=} \left(\bar F_0(\alpha_0)\right)$$

$$\bar F_1 + \bar F_2 = \bar F_0$$

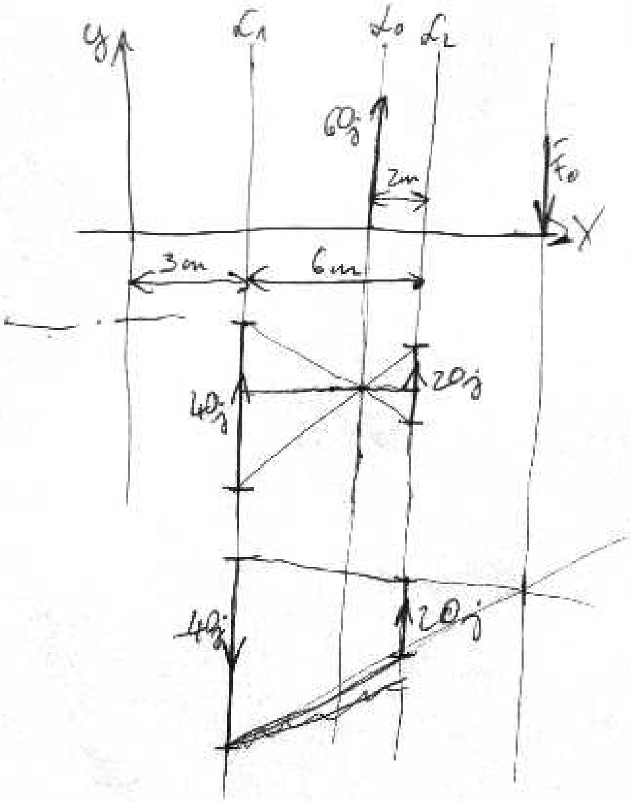
$$\bar M_C' = \bar M_C'' = \bar 0$$

$$\bar M_C' = \bar 0$$

$$l_1 F_1 - l_2 F_2 = 0$$

$$\frac{F_1}{l_2} = \frac{F_2}{l_1}$$

Pl.



$\bar F_1 = 20\bar j$ N

$\bar F_2 = 40\bar j$ N

$\bar F_0 = 60\bar j$ N

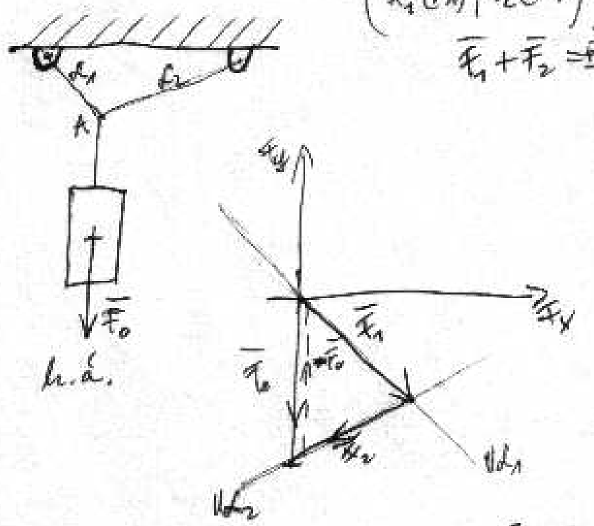
ha $\bar F_2 = -40\bar j$

$\bar F_0 = -20\bar j$

– Erő felbontása 2 erőre

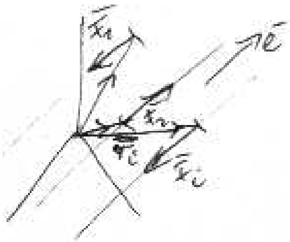
$$\left(\bar F_1(\alpha_1), \bar F_2(\alpha_2)\right) \overset{M}{\simeq} \left(\bar F_0(\alpha_0)\right)$$

$$\bar F_1 + \bar F_2 = \bar F_0$$



Speciális erőrendszer

1, Párhuzamos erőrendszer



$\bar{F}_i = F_i \bar{e} \quad i = 1, \dots, n$

O-ba redukálva

$$\bar{F} = \sum_{i=1}^{n} \bar{F}_i = \underbrace{\left(\sum_{i=1}^{n} F_i\right)}_{F} \bar{e} = F\bar{e}$$

$$\bar{M}_o = \sum \bar{r}_i \times \bar{F}_i = \left(\sum \bar{r}_i F_i\right) \times \bar{e}$$

$\bar{F} \cdot \bar{M}_o = 0 \quad$ 1a, b, 2a, ~~3b~~ osztályok

ha $F \neq 0$

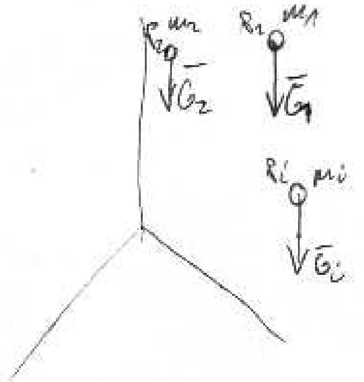
$$\bar{M}_o = \left(\sum \bar{r}_i F_i\right) \times \bar{e}\, \frac{F}{F} = \underbrace{\frac{\sum \bar{r}_i F_i}{F}}_{\text{helyvektor}} \times \underbrace{(\bar{e} F)}_{\bar{F}}$$

$\bar{r}_K$

erőközéppont

$$\bar{r}_K = \frac{\sum \bar{r}_i F_i}{\sum F_i}$$ K → centrális egyenes

Speciális eset:

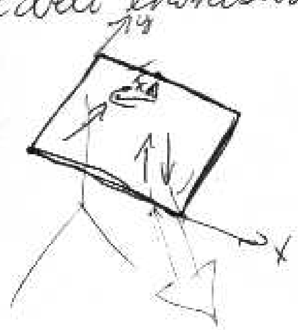


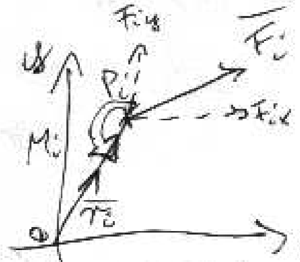
$\bar{e} = \bar{g} = \text{áll}$

$\bar{F}_i = G_i = m_i g$

$$\bar{r}_K = \underbrace{\frac{\sum r_i m_i g}{\sum m_i g}}_{\text{súlypont (S)}} = \underbrace{\frac{\sum \bar{r}_i m_i}{\sum m_i}}_{\text{tömegközéppont (T)}} = \bar{r}_S = \bar{r}_T$$

2, Síkbeli erőrendszer





$\bar F_i(P_i),\ \bar M_i(P_i)$

$\bar r_i = x_i \bar i + y_i \bar j$

$\bar F_i = F_{ix}\bar i + F_{iy}\bar j$

$\bar M_i = M_i \bar k$

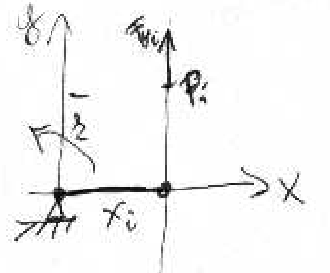
O-ba redukálva

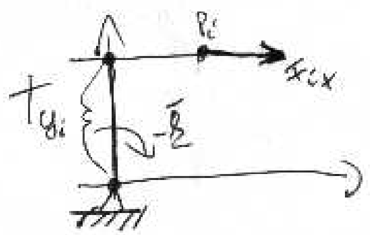
$\bar F = \sum \bar F_i = \sum F_{ix}\,\bar i + \sum F_{iy}\,\bar j$

$\bar F = F_x \bar i + F_y \bar j$

$\bar M_O = \sum \bar r_i \times \bar F_i + \sum \bar M_i = M_z \cdot \bar k$

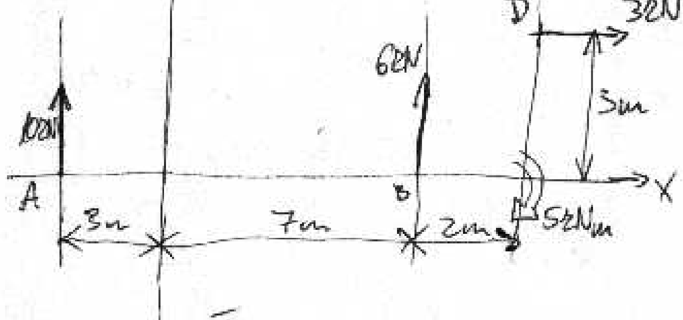
$$\bar r_i \times \bar F_i = \begin{vmatrix} \bar i & \bar j & \bar k \\ x_i & y_i & 0 \\ F_{ix} & F_{iy} & 0 \end{vmatrix} = \underbrace{\bar k\left(F_{iy}x_i - F_{ix}y_i\right)}_{\text{(előjel)}*\text{erő}*\text{kar}}$$





$\bar F \cdot \bar M_O = 0$ 1a, b, 2a, $\cancel{2b}$

Pl.



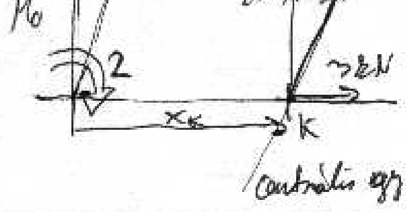
a) $[\bar F, \bar M_O]_O = ?$

$\bar F = \sum \bar F_i = 10\bar j + 6\bar j + 3\bar i = 3\bar i + 16\bar j$ (kN)

$\bar M_O = \bar k(-3\cdot 10 + 7\cdot 6 - 3\cdot 3 - 5) = -2\bar k$ (kNm)

$\bar M_B = \bar k(-10\cdot 10 - 3\cdot 3 - 5) = -114\bar k$ (kNm)

$\bar M_B = \bar M_O + \bar F \times \bar r_{OA}$



$\bar M_O \overset{(+)}{=} -2\bar k = x_K 16 \bar k \rightarrow x_K = -\frac{2}{16}$ m

centrális egy.

Számítási feladatok

– feltételi egyenletek (x, y) síkban

a) $\bar F' = \pm \bar F''$ / $\bar i, \bar j$ – vetületi egyenlet (2db) } 3db

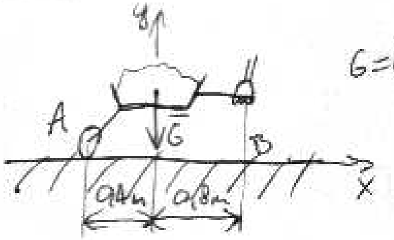
$\bar M_A' = \pm \bar M_A''$ / $\bar k$ – nyomatéki egyenlet (1db)

b) $\left.\begin{array}{l} \bar M_A' = \pm \bar M_A'' \ / \bar k \\ \bar M_B' = \pm \bar M_B'' \ / \bar k \\ \bar M_C' = \pm \bar M_C'' \ / \bar k \end{array}\right\}$ 3db nyomatéki egyenlet

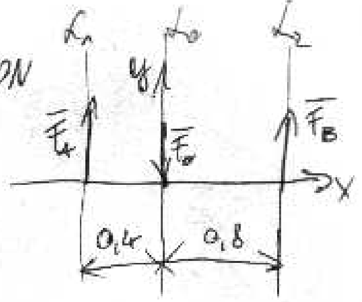
c) $M_i' = \pm M_i''$ $i = 1, \dots, 6$ 3db – " –

– Erő egyensúlyozása

1, párhuzamos erőkkel



$G = 600N$



$\bar F_0 = -G\bar j = -600\bar j$

$\left(\bar F_A(l_1), \bar F_B(l_2) + \bar F_0(l_0)\right) \overset{M}{=} 0$

$\bar F_A = F_A \bar j = 400 \bar j$ N

$\bar F_B = F_B \bar j = 200 \bar j$ N

$\left.\begin{array}{l} \bar F_A + \bar F_B + \bar F_0 = \bar 0 \\ \bar M_0 = \bar 0 \end{array}\right\}$

$F_A \bar j + F_B \bar j \cancel{+} - 600 \bar j = \bar 0$

$F_A + F_B - 600 = 0$

$M_z \overset{\curvearrowleft +}{=} -0,4F_A + 0,8F_B$

$F_A = 2F_B$

$2F_B + F_B - 600 = 0$

$3F_B = 600$

$F_B = 200$

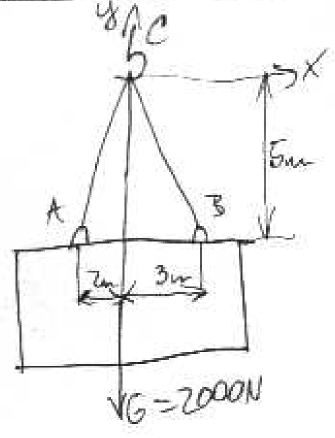
$F_A = 400$

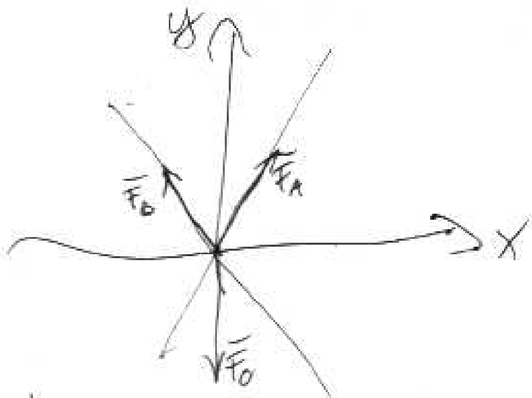
Más út:

$M_B \overset{\curvearrowleft +}{=} 600 \cdot 0,8 - 1,2 F_A = 0$

$F_A = 400$

2, Metsződő erővel





$\bar{F}_A = \lambda_A \cdot \bar{r}_{AC} = \lambda_A (2\bar{i} + 5\bar{j})$

$\bar{F}_B = \lambda_B \bar{r}_{BC} = \lambda_B (-3\bar{i} + 5\bar{j})$

$\bar{F}_A + \bar{F}_B + \bar{F}_0 = \bar{0}$

$\bar{M}_C = \bar{0}$ ✓

$$\left.\begin{aligned} 2\lambda_A - 3\lambda_B &= 0 \\ 5\lambda_A + 5\lambda_B - 2000 &= 0 \end{aligned}\right\}$$

$\lambda_A = \frac{3}{2}\lambda_B$

$\frac{15}{2}\lambda_B + 5\lambda_B - 2000 = 0$

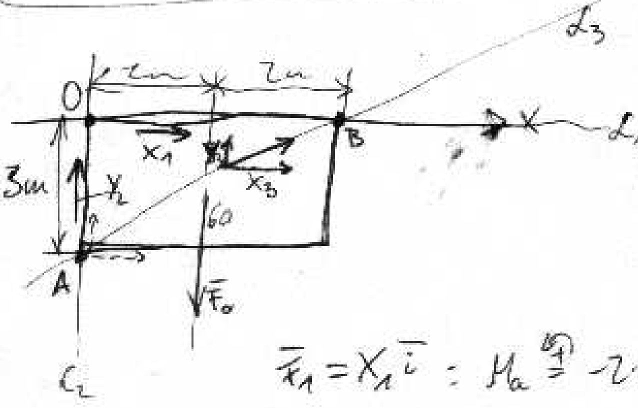
$\frac{25}{2}\lambda_B = 2000 \rightarrow \lambda_B = \frac{4000}{25} = 160$ N/m

$\lambda_A = 240$ N/m

$\bar{F}_A = 480\bar{i} + 1200\bar{j}$ (N)

$\bar{F}_B = -480\bar{i} + 800\bar{j}$ (N)

3, 3 adott hatásvonalú erővel



$\bar{F}_0 = -60\bar{j}$ (kN)

$(\bar{F}_1(\alpha_1), \bar{F}_2(\alpha_2), \bar{F}_3(\alpha_3), \bar{F}_0) \doteq (\bar{0})$

Ritter-féle módszer: a hatásvonalak metszéspontjaira írunk fel nyomatéki egy.-et

$\bar{F}_1 = X_1\bar{i}$: $M_A \overset{!}{=} -2\cdot 60 - 3X_1 = 0$

$X_1 = -40$ kN $\rightarrow \bar{F}_1 = -40\bar{i}$ kN

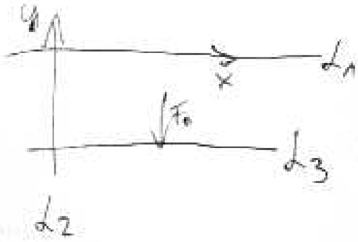
$\bar{F}_2 = Y_2\bar{j}$: $M_B \overset{!}{=} 2\cdot 60 - 4Y_2 = 0$

$Y_2 = 30$ kN $\rightarrow \bar{F}_2 = 30\bar{j}$ kN

$\bar{F}_3 = X_3\bar{i} + Y_3\bar{j}$; $\frac{Y_3}{X_3} = \frac{3}{4}$: $M_O \overset{!}{=} -2\cdot 60 + 3X_3 = 0$
(4Y3)

$\left.\begin{aligned} X_3 &= 40 \text{ kN} \\ Y_3 &= 30 \text{ kN} \end{aligned}\right\} \rightarrow \bar{F}_3 = 40\bar{i} + 30\bar{j}$ kN

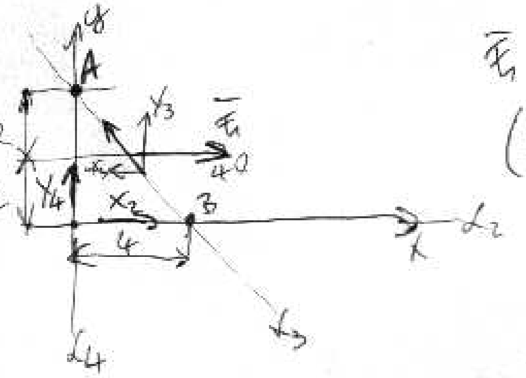
Spec. eset $\alpha_1 \parallel \alpha_3$



$\bar{F}_2 = ?$

$\sum Y_i = 0$

(Pl)



$\bar{F}_1 = 40 \bar{i}$ [N]

$\left(\bar{F}_1, \bar{F}_2(\alpha_2), \bar{F}_3(\alpha_3), \bar{F}_4(\alpha_4)\right) \overset{M}{=} (\bar{0})$

$\bar{F}_2 = X_2 \bar{i}$: $M_a \overset{+}{=} 4X_2 + 40 \cdot 2 = 0$

$X_2 = -20 \rightarrow \bar{F}_2 = -20\bar{i}$ [N]

$\bar{F}_4 = Y_4 \bar{j}$: $M_b \overset{+}{=} -4Y_4 - 2 \cdot 40 = 0$

$Y_4 = -20 \rightarrow \bar{F}_4 = -20\bar{j}$ [N]

$\bar{F}_3 = X_3 \bar{i} + Y_3 \bar{j}$: $M_z \overset{+}{=} 4Y_3 - 2 \cdot 40 = 0$

$\frac{Y_3}{-X_3} = 1$

$\left.\begin{array}{l} Y_3 = 20 \\ X_3 = -20 \end{array}\right\} \rightarrow \bar{F}_3 = -20\bar{i} + 20\bar{j}$ [N]

MECHANIKA

03.03.29.

Mai anyag + jövő pénteki a köv. zh.

117. o. gyak. (m.o. 118)

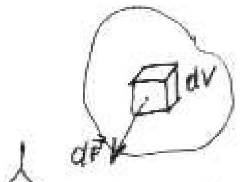
Megoszló erőrendszerek

Kötött megoszló erőrendszer

Def.: a támadáspontok folytonos pontsokaságot alkotnak.

Térfogaton megoszló }
Felületen —"— } ER
Vonalon —"— }

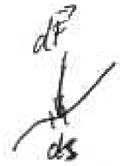
Megadás: sűrűség-vektorral



(tömeg: $dm = \rho dV$)

ER: $d\vec{F} = \underbrace{\vec{q}}_{\text{ER sűrűsége}} dV$

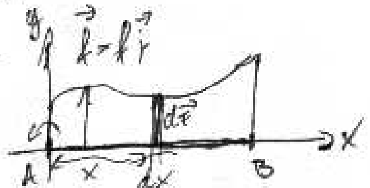
$d\vec{F} = \underbrace{\vec{p}}_{\text{nyomás}} dA$



$d\vec{F} = \vec{f} ds$

– Egyenes vonalon megoszló, párh. ER

pl.



$f = f(x)$

terhelési ábra

Redukálás:

$$\vec{F} = \int d\vec{F} = \int_A^B \vec{f} dx = \underbrace{\int_A^B f dx}_{F} \vec{j} = F\vec{j}$$

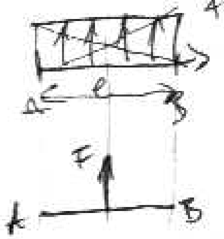
$$\vec{M}_A = \underbrace{\int_A^B x f dx}_{M_z} \vec{k}$$

Eredő: $M_z = x_c F \rightarrow x_c = \frac{M_z}{F}$

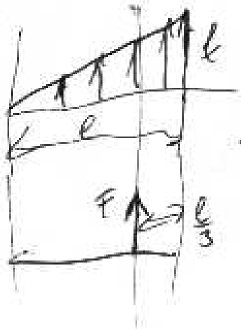
$x_c = \frac{\int x f dx}{\int f dx}$ az eredő mindig áthalad a terhelési ábra súlypontján.

RL

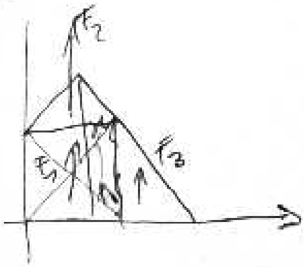
f = áll.



$F = \ell \cdot f$

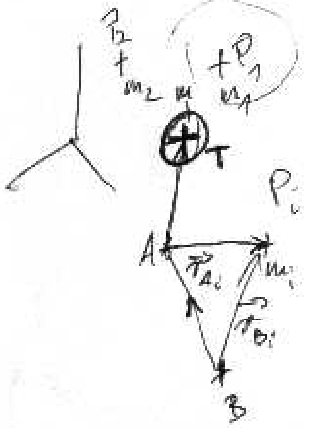


$F = \frac{\ell f}{3}$



Statikai nyomaték, súlypont

– tömegpontrendszer



m_i ; $i = 1, \dots, n$

$\boxed{\vec S_A \overset{def}{=} \sum_{i=1}^{n} \vec r_{Ai}\, m_i}$

$\vec S_B = \sum_{i=1}^{n} \vec r_{Bi}\, m_i$

$= \sum (\vec r_{BA} + \vec r_{Ai})\, m_i$

$= \vec r_{BA} \underbrace{(\sum m_i)}_{m} + \underbrace{\sum \vec r_{Ai}\, m_i}_{S_A}$

$m = \sum_{i=1}^{n} m_i$ – eredő tömeg

$\boxed{\vec S_B = \vec S_A + m \vec r_{BA}}$

tömegközéppont: T ; $\vec S_T = \vec 0$

statikai nyomaték zérus

$\vec S_T = \vec S_A + m \cdot \vec r_{TA} = \vec 0$

$\boxed{\vec S_A = m \vec r_{AT}}$

statikai nyomaték szempontjából a tömegpontrendszer egyenlő a tömegközép-ba képzelt eredőjével.

$\boxed{\vec r_{AT} = \frac{\vec S_A}{m}} = \frac{\sum \vec r_{Ai} \cdot m_i}{\sum m_i} = \vec r_{AS}$

T = S súlypont

– Geometriai alakzatok súlypontja

test (térfogat) felület vonal

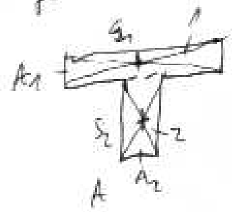
<u>def.:</u> $\vec{r}_{AS} = \frac{\vec{S}_A}{E}$

E: eredő (V, A, l)

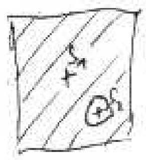
<u>Tétel:</u> szimmetria $\left\{\begin{matrix}\text{pont}\\ \text{tengely}\\ \text{sík}\end{matrix}\right\}$ egyetlen $\left\{\begin{matrix}\text{súlypont}\\ \text{súlyvonal}\\ \text{súlysík}\end{matrix}\right\}$

Alkalmazás

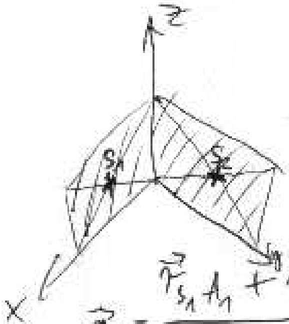
Pl. felület



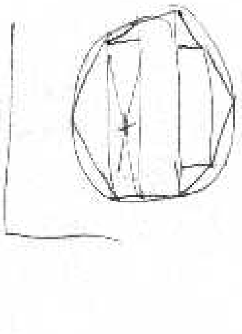
$$\vec{r}_{AS} = \frac{\sum \vec{r}_{S_i} \cdot A_i}{\sum A_i}$$

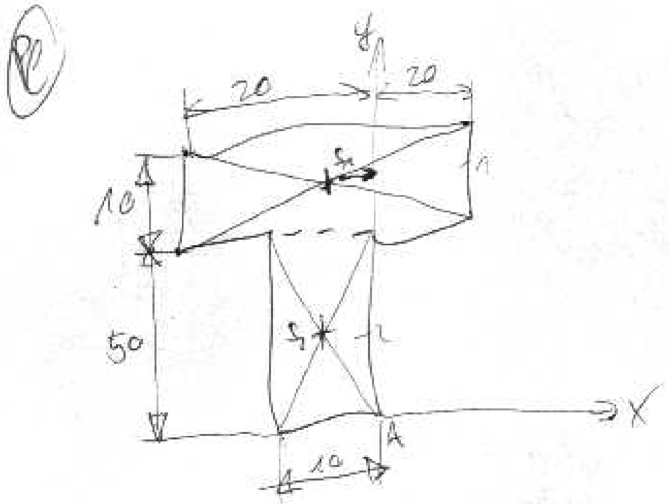


$$\vec{r}_{AS} = \frac{\vec{r}_{S_1}A_1 - \vec{r}_{S_2}A_2}{A_1 - A_2}$$



$$\vec{r}_S = \frac{\vec{r}_{S_1}A_1 + \vec{r}_{S_2}A_2}{A_1 + A_2}$$





i	x_{Si}	y_{Si}	A_i
1	0	55	400
2	-5	25	500

$$\vec r_{AS} = \frac{\vec r_{S1} A_1 + \vec r_{S2} A_2}{A_1 + A_2} = \frac{1}{900}\left(55\cdot 400\,\vec j - 5\cdot 500\,\vec i + 25\cdot 500\,\vec j\right)$$

$$= -\frac{2500}{900}\vec i + \frac{34500}{900}\vec j = -2{,}77\vec i + 38{,}33\vec j \ (\mathrm{mm})$$

Merev test statikája

Stat. Alaptétel: egy merev test csak akkor lehet tartós nyugalomban, ha a rá ható erőrendszer egyensúlyi.

Elégséges felt.: megfelelő támasz biztosítja a nyugalmat.

ER: – terhelés (ismert)
– támasztó ER (ismeretlen)

Egyensúlyi egyenlet

$$s_e = \begin{cases} 6 & \text{térben} \\ 3 & \text{síkban} \end{cases}$$

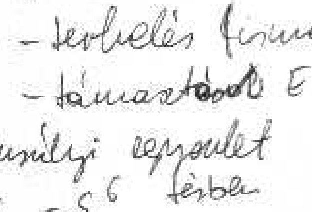
s_i: ismeretlenek száma

$s_e = s_i$ statikai szempontból határozott

$s_e < s_i$ – " – határozatlan

$s_e > s_i$ – " – túlhatározott (labilis)

– Támasztások
érintkezés:



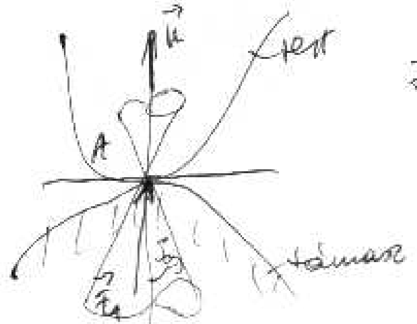
támasztó ER: koncentrált erő

vonalon megoszló

felületen megoszló

Statikai módszerrel csak a támasztó ER eredőjét lehet meghatározni.

Pontbeli érintkezés, Coulomb-féle súrlódás



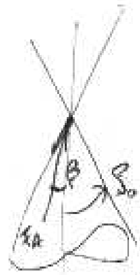
$\vec{F}_A \cdot \vec{n} > 0$

$\vec{F}_t = \mu_0 F_n$

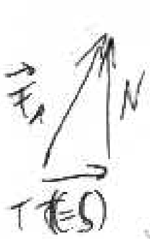
- nyugvó súrlódási kúp ρ_0
- — " — félszög: $\mu_0 = tg\,\rho_0$

$F_t \le \mu_0 \vec{F}_n$

$\beta \le \rho_0$



A tapad. nyugvó súrlódás mindaddig, amíg a támasztó erő súrl.-i kúpon belül v. annak határán van.



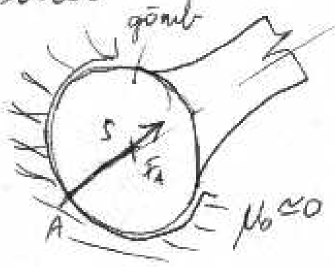
$tg\,\beta = tg\,\rho_0$

$\frac{|T|}{N} \le \mu_0$

($F_t = \mu F_n$) csúszás

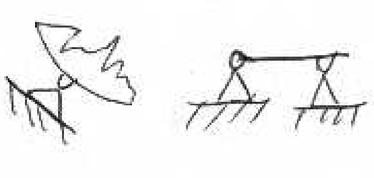
Kényszerek v. megtámasztási módok

- adott ponton gömbcsukló

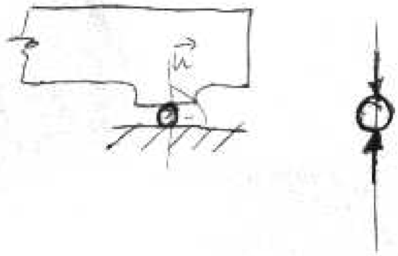


$F_A \to S$

jele:



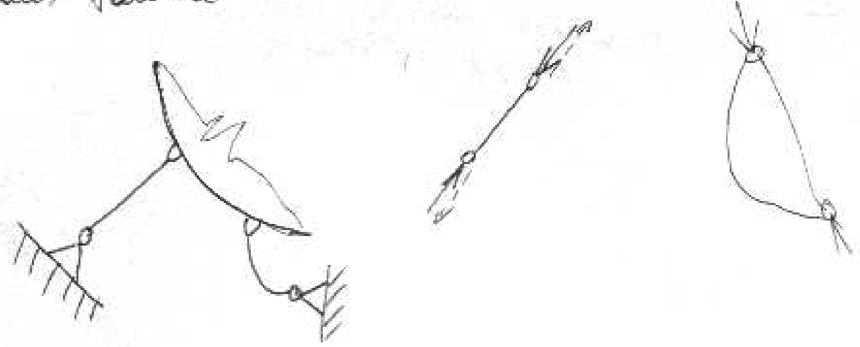
– adott hatásvonalú görgő



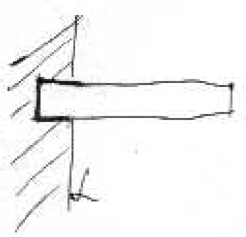
jele:

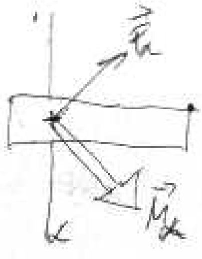
$\vec{F}_A \parallel \vec{n}$

rudas támasz



– befogás, befalazás

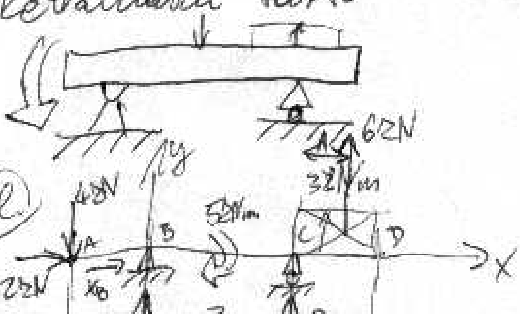




$\vec{F}_A$	$\vec{M}_A$		S_i
3	3	térben	6
2	1	síkban	3

Típus feladatok

– kéttámaszú tartó



$\vec{F}_B = X_B \vec{i} + Y_B \vec{j}$

$\vec{F}_C = \qquad Y_C \vec{j}$

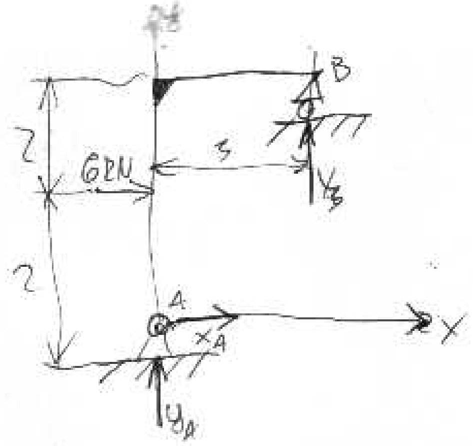
$M_B \overset{(B)}{=} 4 \cdot 1 - 5 + 3Y_C + 4 \cdot 6 = 0$

$Y_C = -\frac{23}{3}$

$X = \sum X_i = 2 + X_B = 0 \rightarrow X_B = -2$

$Y = \sum Y_i = -4 + Y_B - \frac{23}{3} + 6 = 0 \rightarrow Y_B =$

– Törtvonalú tartó



$M_A \overset{\curvearrowleft}{=} -2\cdot 6 + 3Y_B = 0$

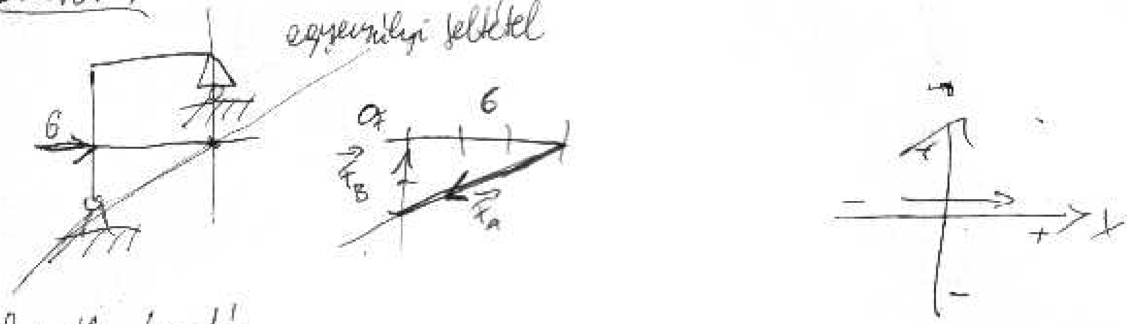
$Y_B = 4\,kN$

$X = 6 + X_A = 0$

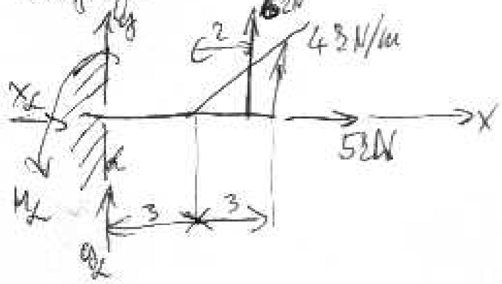
$x_A = -6$

$Y = Y_A + Y_B = 0 \to Y_A = -4$

Szerkesztés



– befogott tartó

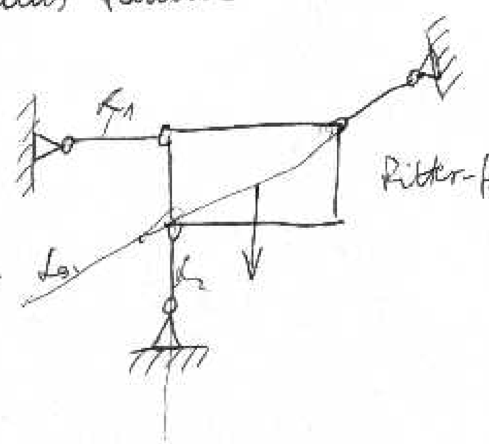


$M_\alpha \overset{\curvearrowleft}{=} M_\alpha + 5\cdot 6 = 0 \to M_\alpha = -30\,kNm$

$X = X_\alpha + 5 = 0 \to X_\alpha = -5$

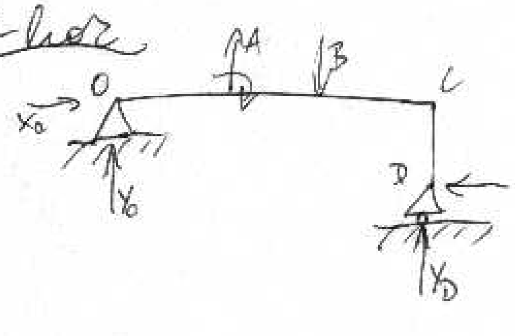
$Y = Y_\alpha + 6 = 0 \to Y_\alpha = -6$

– rudas támasz



Ritter-féle számítás

HF-híd



$\vec{F} = \dots$

$\vec{M}_{O1} = \dots$

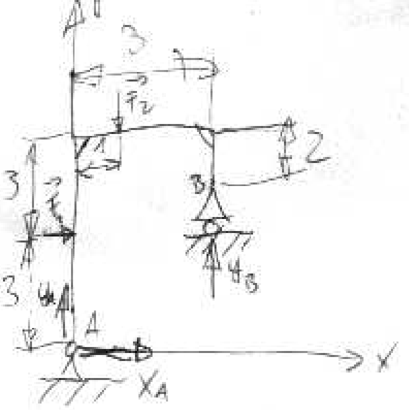
$M_O \overset{\curvearrowleft}{=} \dots + \dots Y_D = 0 \to Y_D =$

$X =$

$Y =$

$\vec{T}$ – . .

Pl.



$\vec F_1 = 2\vec i$ kN

$\vec F_2 = -6\vec j$ kN

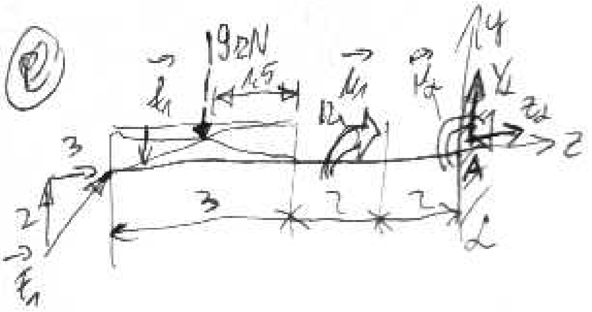
$$\boxed{\begin{aligned}\vec F_A &= X_A\vec i + Y_A\vec j = -2\vec i + 2\vec j \text{ kN}\\ \vec F_B &= Y_B\vec j = 4\vec j \text{ kN}\end{aligned}}$$ uložení

$M_a = 3Y_B - 3\cdot 2 - 1\cdot 6 = 0 \Rightarrow Y_B = 4$ kN

$X = X_A + 2 = 0 \Rightarrow X_A = -2$ kN

$Y = Y_A - 6 + \underbrace{Y_B}_{4} = 0 \Rightarrow Y_A = 2$ kN

Pl.



$\vec F_1 = 2\vec j + 3\vec k$ kN

$\vec q_1 = -3\vec j$ kN/m

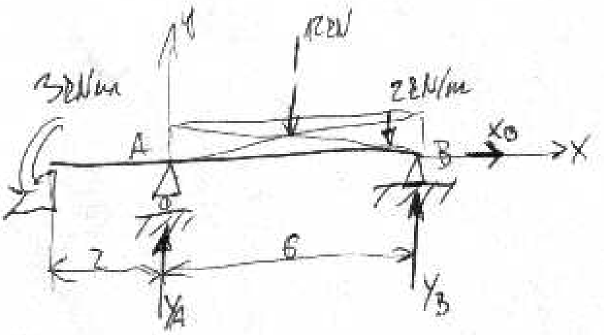
$\vec M_1 = 12\vec i$ kNm

$M_a = 7\cdot 2 - 9\cdot 5{,}5 + 12 + M_L = 0 \Rightarrow M_L = 23{,}5$

$Y = 2 - 9 + Y_L = 0 \rightarrow Y_L = 7$ kN

$Z = 3 + Z_L = 0 \rightarrow Z_L = -3$

Pl.

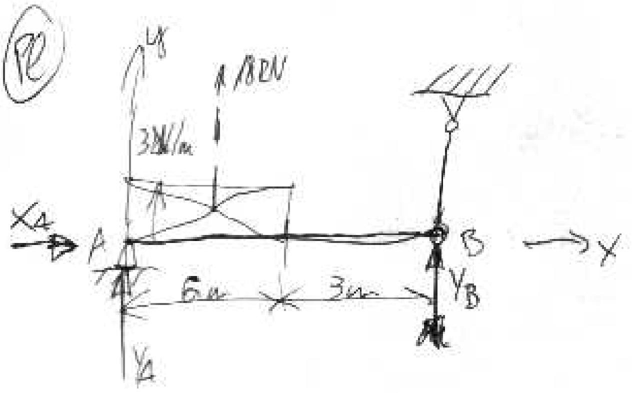


$X = X_B = 0$

$M_a = 3 - 3\cdot 12 + 6Y_B = 0 \rightarrow Y_B = 5{,}5$

$Y = Y_A - 12 + Y_B = 0 \rightarrow Y_A = 6{,}5$

PC

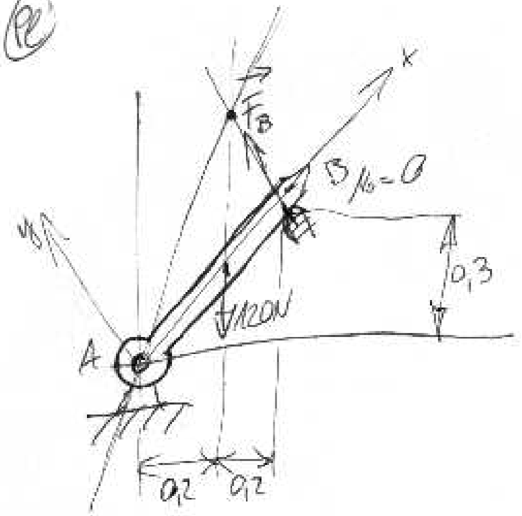


$M_a \overset{\circlearrowleft}{=} 3\cdot 18 + 9Y_B = 0 \rightarrow Y_B = -6$

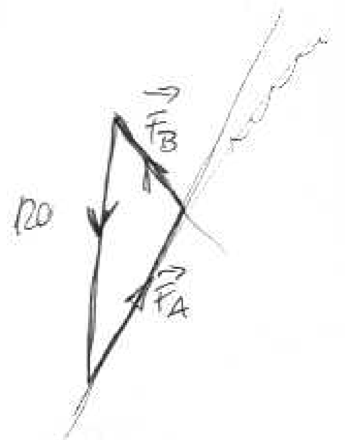
$X = X_A = 0$

$Y = Y_A + 18 - 6 = 0 \rightarrow Y_A = -12$

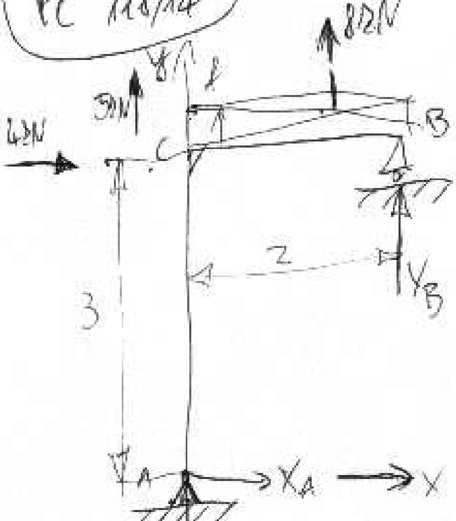
PC



Tnižž: (x,y) teng. jelv. (lia námolnéz)



PC 118/14



1. tech. $f = 4$ kN/m

$M_a \overset{\circlearrowleft}{=} 8 + 2Y_B = 0 \rightarrow Y_B = -\cancel{4}$

$Y = Y_A + \underbrace{Y_B}_{-4} \overset{+8\,!}{=} 0 \rightarrow Y_A = -\cancel{4}$

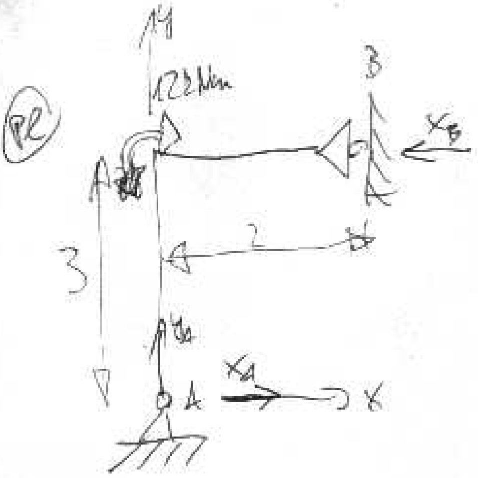
$X = X_A = 0$

2. tech.

$M_a \overset{\circlearrowleft}{=} -3\cdot 4 + 8 + 2Y_B = 0 \rightarrow Y_B = 2$

$X = 4 + X_A = 0 \rightarrow X_A = -4$

$Y = Y_A + 5 + 8 + \underbrace{Y_B}_{2} = 0 \rightarrow Y_A = -15$



$$M_A^{(+)} = -12 - 3X_B = 0 \Rightarrow X_B = -4$$

$$X = X_A + \underset{-4}{X_B} = 0 \Rightarrow X_A = 4$$

$$Y = Y_A = 0$$

Összetett szerkezet statikája

több egymáshoz kapcsolódó v. egymásra támaszkodó test együttese.

Célja: - támasztó ER
- belső ER

$$\vec{F}_{21} = -\vec{F}_{12}$$

-ról -re

Módszere: - egy-egy testre bontjuk
- több testre
- teljes szerkezetre

} egyensúlyi ER-ek vizsg.

n-test

$$S_e = \begin{cases} 6n & \text{térben} \\ 3n & \text{síkban} \end{cases}$$

statikai egyenlet.

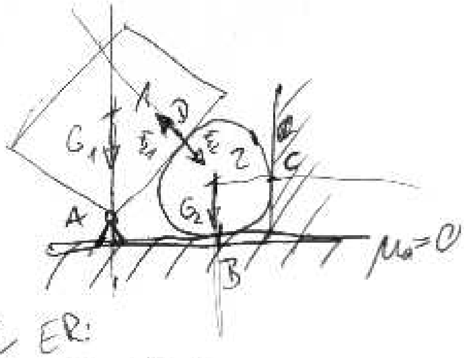
ismeretlen: $S_i = S_e$ - statikailag határozott feladatok

$S_i > S_e$ — " — határozatlan

$S_i < S_e$ — " — túlhatározott

Típus feladatok:

– Nyugalomban maradó testek



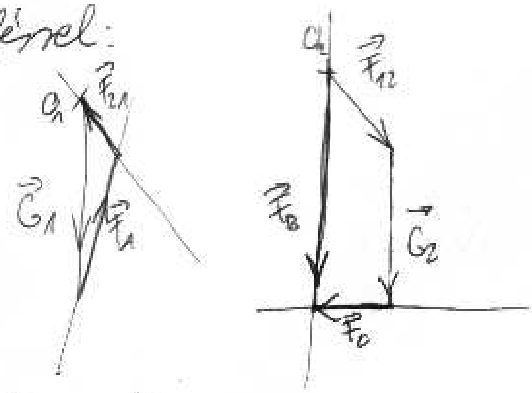
Egyensúlyi ER:

$(1): (\vec{F}_A, \vec{G}_1, \vec{F}_{21}) \overset{M}{=} \vec{O}$

$(2): (\vec{G}_2, \vec{F}_B, \vec{F}_C, \vec{F}_{12}) \overset{M}{=} \vec{O}$

$(1+2): (\vec{G}_1, \vec{G}_2, \vec{F}_A, \vec{F}_B, \vec{F}_C) \overset{M}{=} \vec{O}$

Szerkesztéssel:

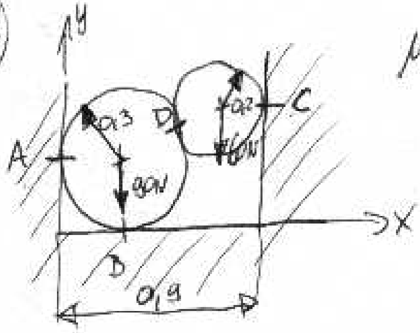


$S_i=5$

$S_e=2\cdot3=6$

} csak spec. terh. marad egyensúlyban.

②



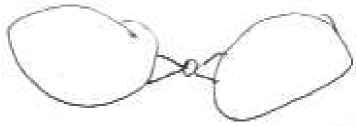
$\mu_0=0$

$\vec{F}_A=80\vec{i}$

$\vec{F}_B=150\vec{j}$

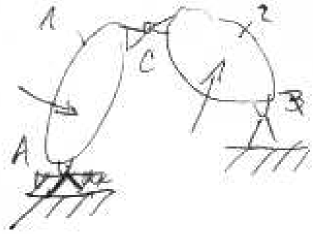
$\vec{F}_C=-80\vec{i}$

- Csuklós szerkezetek



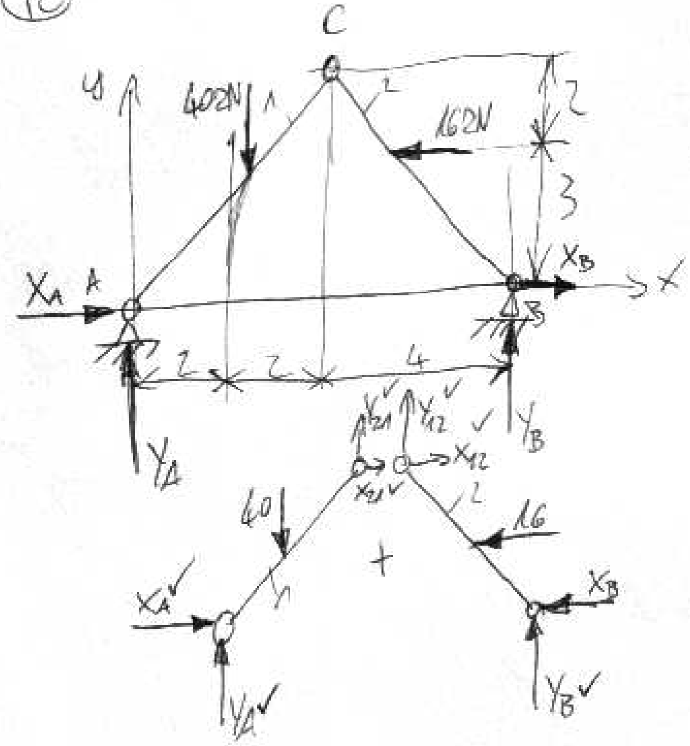
- sima csukló
- test helyett vonal

- Háromcsuklós ív



$s_e = 2\cdot 3 = 6$
$s_i = 3\cdot 2 = 6$

Pl



(1+2): $M_A = -2\cdot 40 + 3\cdot 16 + 8Y_B = 0 \Rightarrow$

$\Rightarrow Y_B = 4\ kN$

$Y = Y_A - 40 + \underbrace{Y_B}_{4} = 0 \Rightarrow Y_A = 36\ kN$

(1): $M_C = 2\cdot 40 + 5X_A - 4\underbrace{Y_A}_{36} = 0$

$X_A = 12{,}8\ kN$

$X = X_A + X_{21} = 0 \Rightarrow X_{21} = -12{,}8\ kN \Rightarrow$

$\Rightarrow X_{12} = 12{,}8\ kN$

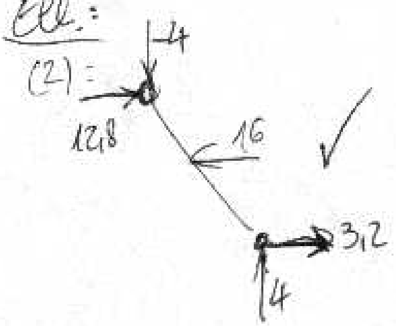
$Y = \underbrace{Y_A}_{36} - 40 + Y_{21} = 0 \Rightarrow Y_{21} = 4\ kN$

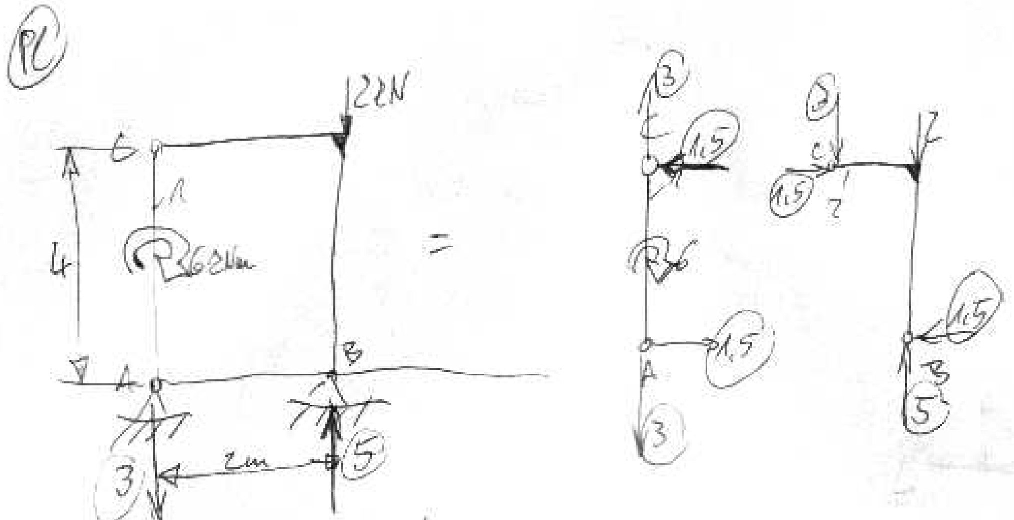
$Y_{12} = -4\ kN$

(1+2): $X = \underbrace{X_A}_{12,8} + X_B - 16 = 0 \Rightarrow X_B = 3{,}2\ kN$

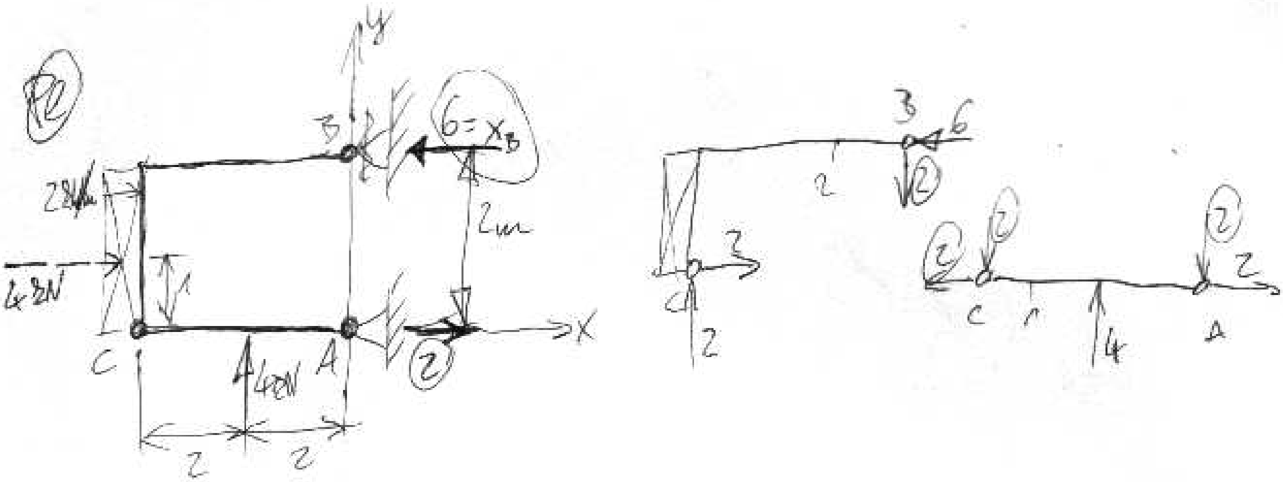
Ell.:

(2):

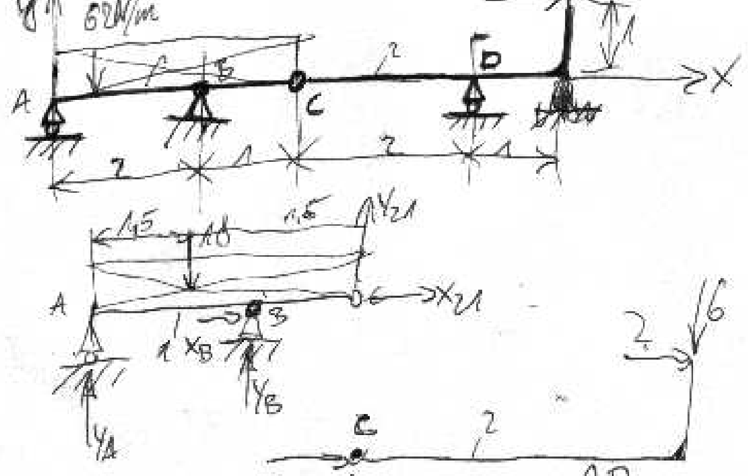




kell egyenlet!



Gerber-tartó



(2): $M_C^{(4)} = 2Y_D - 1\cdot 2 - 3\cdot 6 = 0 \to Y_D = 10$

$Y = Y_{12} + 10 - 6 = 0 \to Y_{12} = -4$

$X = X_{12} + 2 = 0 \to X_{12} = -2$

(1): $M_A^{(4)} = -1{,}5\cdot 18 + 2Y_B + 3\underbrace{Y_{21}}_{4} = 0 \to$

$\to Y_B = \cancel{\frac{39}{12}}\ \frac{15}{2}$

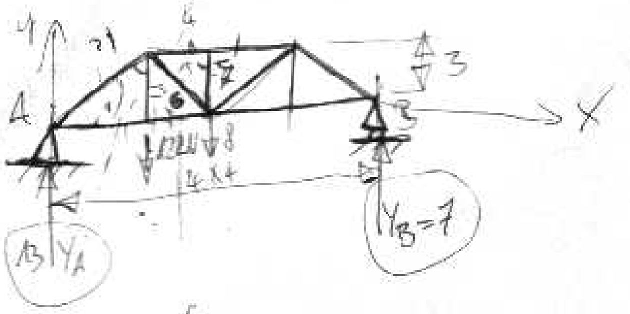
$X = X_B + X_{21} = 0 \to X_B = -X_{21} = -2$

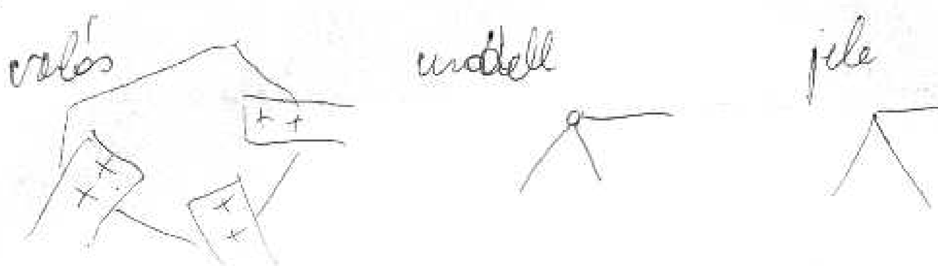
$Y = Y_A + \underbrace{Y_B}_{\frac{15}{2}} - 18 + \underbrace{Y_{21}}_{4} = 0 \to Y_A = \dots$

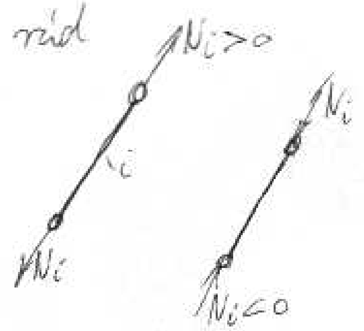
Rácsos tartó

- rudakból áll

feltételezés:
- rudak csuklóval kapcsolódnak
- terhelés és támasztás is csak csuklóban van
- rudak súlyát elhanyagoljuk





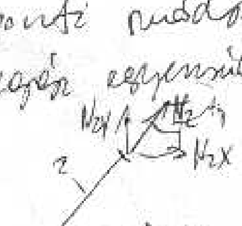


$N_i=0$ vakrúd

Rúderők számítása

- csomóponti módszer

ha egyben egyensúly, akkor kettő határozható

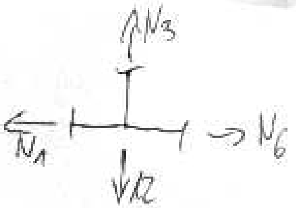


$$Y = 13 + N_{2Y} = 0 \rightarrow N_{2Y} = -13$$

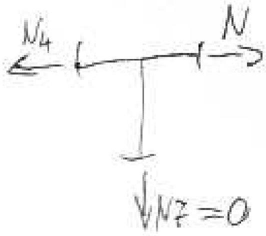
$$\frac{N_{2x}}{N_{2Y}} = \frac{4}{3} \rightarrow N_{2x} = \frac{4}{3} N_{2Y} = -\frac{52}{3}$$

$$N_2 = -\sqrt{N_{2x}^2 + N_{2Y}^2} = -13\sqrt{\left(\frac{4}{3}\right)^2 + 1^2} = -13\cdot\frac{5}{3} = -\frac{65}{3}\ kN$$

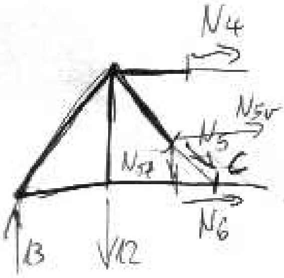
$$X = N_1 + N_{2x} = 0 \rightarrow N_1 = -N_{2x} = \frac{52}{3}$$



$Y = N_3 - 12 = 0 \rightarrow N_3 = 12$ (I)



– átmetsző módszer



N_4 – ?

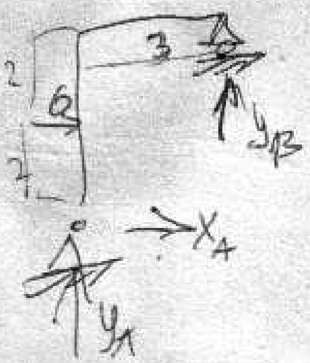
$M_C \overset{\circlearrowleft +}{=} 4\cdot 12 - 8\cdot 13 + 3\cdot N_4 = 0 \rightarrow N_4 = -\frac{53}{3}$ (→|—|←)

$Y = 13 - 12 - N_{5f} = 0 \rightarrow N_{5f} = 1$

$\frac{N_{5v}}{N_{5f}} = \frac{4}{3} \rightarrow N_{5v} = \frac{4}{3}$

$N_5 = \sqrt{1^2 + \left(\frac{4}{3}\right)^2} = \frac{5}{3}$

$M_d \overset{\circlearrowleft +}{=} -4\cdot 13 + 3 N_6 = 0 \Rightarrow N_6 = \frac{52}{3}$ (←|—|→)



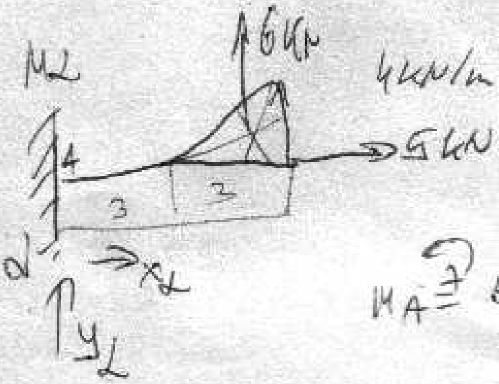
$M_A \overset{+}{=} 1\cdot 6 + 3\cdot y_B$

$2 = y_B$

$y = y_A + 2$

$y_A = y$

$X = 6$



$M_A \overset{+}{=} 5\cdot 6 + M_2$

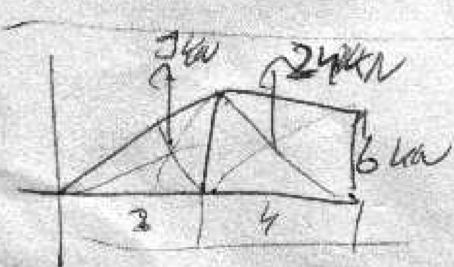
$M_2 = -30$ kN

$Y = Y_2 + 6$

$Y_2 = -6$ kN

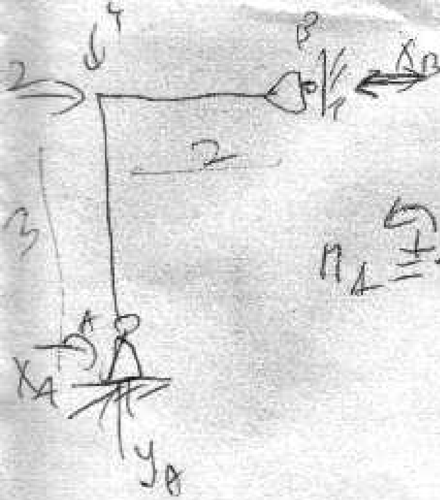
$X = X_2 + 5$ kN

$X_2 = -5$ kN



$\frac{3\cdot 6}{2} = 9$

$M_0 \overset{+}{=} 2\cdot 9 + 5\cdot 24 =$ [illegible] kN



$M_A \overset{+}{=} -3\cdot 2 = 3\cdot X_B$

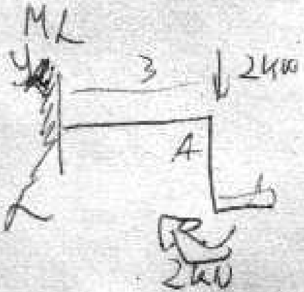
$\underline{-2 = X_B}$

$X = X_A + 2 - 2$

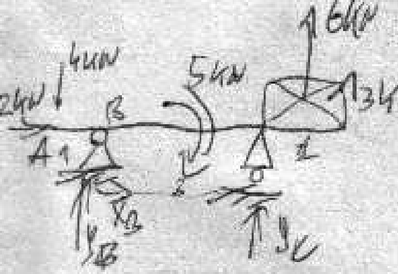
$X_A = 0$

$M_A \overset{+}{=}$

$\underline{y = y_A - 4}$



x y z
i j k



$\cancel{M_A \overset{+}{=} 1 y_A + 5 + 5\cdot 6 + 4 y_C}$

$2\cdot 8 - 5 = 2\cdot 3 + 3 y_C$

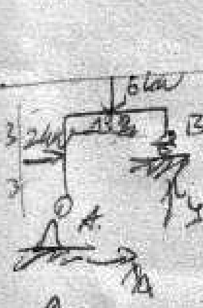
$M_B \overset{+}{=} -5 + 3y_C + 4\cdot 6 + 4\cdot 1$

$\underline{y_C = \frac{-23}{3}}$

$X = X_B + 2 \Rightarrow \underline{X_B = -2}$

$y = y_C \cancel{y_A} - 4 + 6 + y_B$

$\underline{y_B = \frac{-17}{3}}$



$\cancel{M_A = 3\cdot 2 + 4\cdot 6 - 3y_B}$

$\cancel{y_B = 4\,kN}$

$\cancel{y_A = y_A + 4 - 6}$

$\cancel{y_A = 2\,kN}$

$M_A \overset{+}{=} -3\cdot 2 + 1\cdot 6 + 3y_B$

$y_B = 4$

$y = 4 + y_A - 6$

$\underline{y_A = 2}$

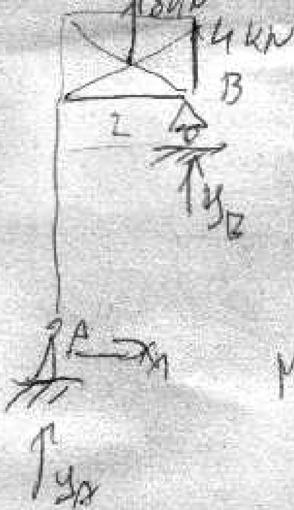
$X = X_A + 2$

$X_A = -2$

$X = X_A + 2$

$F_A = 2i + 2j \qquad F_A = -2i + 2j$

$F_B = 4j \qquad F_B = 4j$



$M_A = 1\cdot 8 + 2y_B$

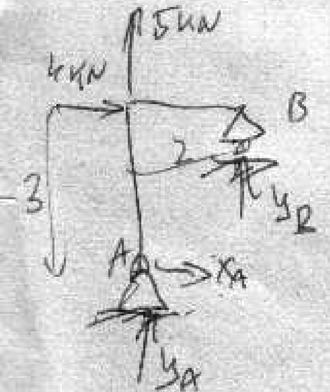
$y_B = -4$

$Y = y_B + 8 + y_A$

$-4 = y_A$

$X = x_A = 0$

$x_A = 0$



$M_A = -3\cdot 4 + 2y_B = 0$ $\quad y_B = 6$

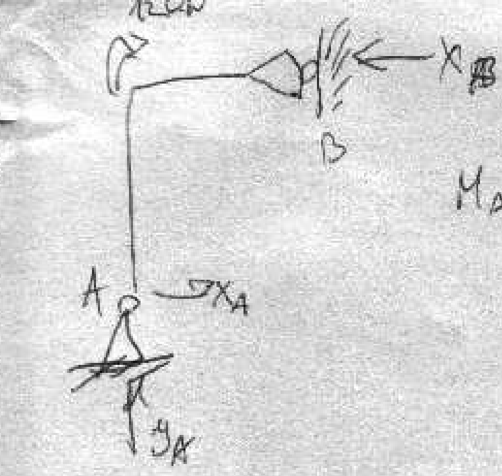
$+12 = 2y_B$

$X = X_A + 4$

$X_A = -4$

$Y = 6 + 5 + y_A$

$y_A = -11$



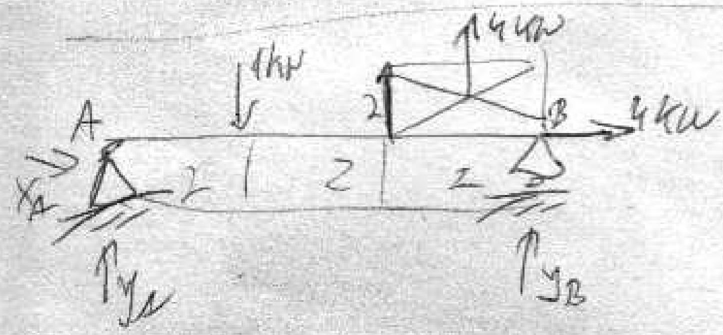
$M_A \stackrel{+}{=} -12 + 3 \cdot X_B$

$4 = X_B$

$X = 4 + X_A$

$X_A = -4$

$Y = Y_A = 0$



$M_A \stackrel{+}{=} -2 \cdot 1 + 5 \cdot 4 + 6 Y_B = 0$

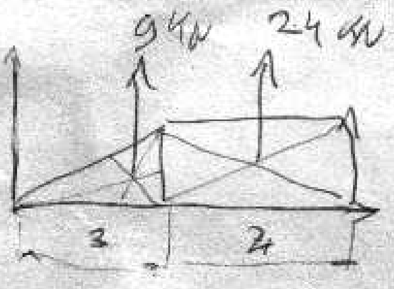
$Y_B = -3 \text{ kN}$

$Y = Y_A - 1 + 4 + \underset{(-3)}{Y_B}$

$Y_A = 0$

$X = X_A - 4$

$X_A = 4$



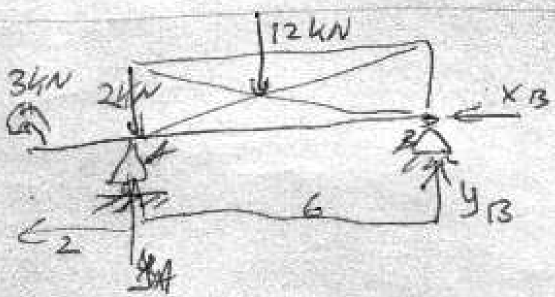
$\frac{9\cdot m}{2} = \frac{3\cdot 6}{2}$

$f = 6\,kN/m$

$[F, \vec{M}_0]$

$F = 9j + 24j = 33j$

$M_0 = 2\cdot 9 + 5\cdot 24 = 138\,k$



$M_A = -3\cdot 12 + 3 + 6y_B$

$M_B = 3\cdot 12 - 6\cdot y_A + 3 = 0$

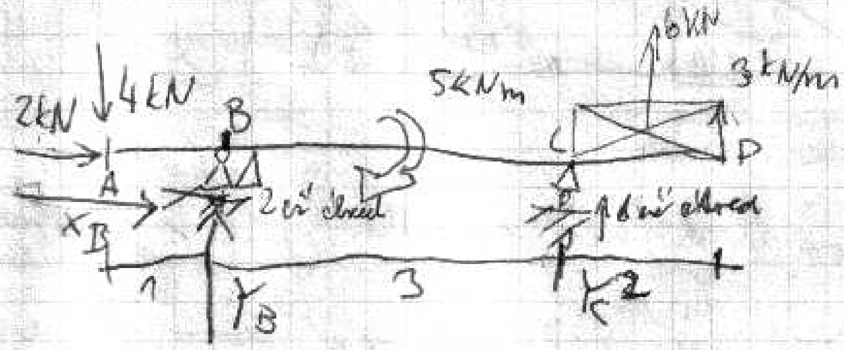
$39 = 6y_A$

$6{,}5 = y_A$

$y_A\ y_B + 6{,}5 - 12$

$y_B = 5{,}5$

MECHANIKA



falra merőleges

$F_A = 4j$

$F_B = -2i + \frac{17}{3} j$

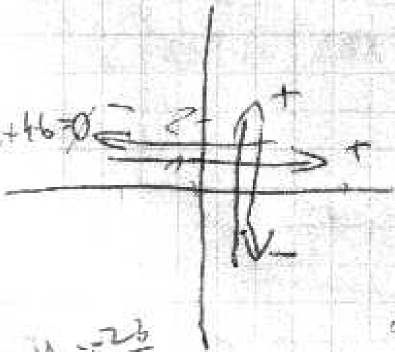
$F_C = -\frac{23}{3} j$

$M_B = 1\cdot(+4) + 5 + 3y_C + 4\cdot 6 = 0$

$+4 - 5 + 3y_C + 24$

$+23 = 3y$

$y_C = -\frac{23}{3}$



$X = +2 + X_B = 0$

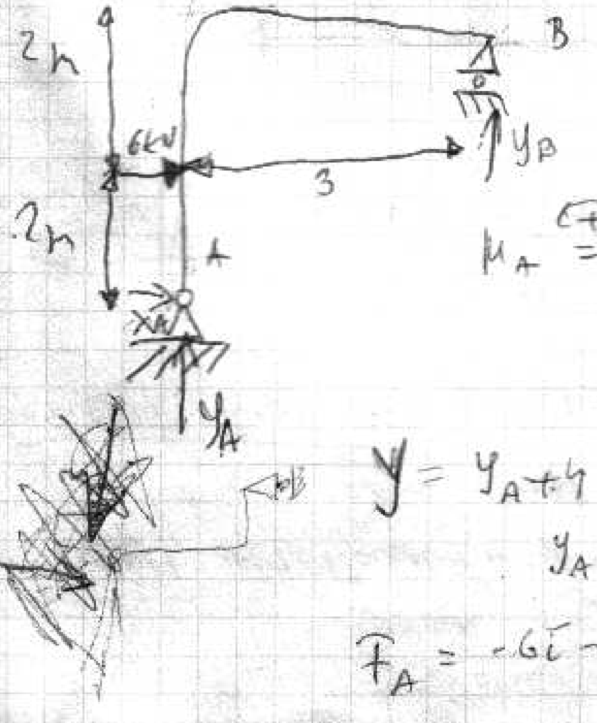
$X_B = -2$

$Y = +4 + y_B - \frac{23}{3} + 6 = 0$

$-\frac{23}{3} + 2 + y_B = 0$

$-\frac{17}{3} + y_B = 0$

$\frac{17}{3} = y_B$



$M_A = 2(6) + 3y_B = 0$

$-12 = -3y_B$

$4 = y_B$

← ordnál ort vonalon

$i \quad j \quad k$

$x \quad y \quad z$

$Y = y_A + 4$

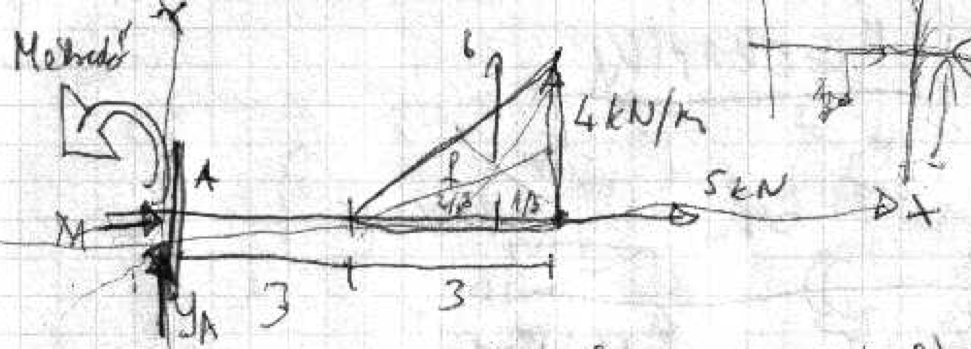
$y_A = -4$

$X = X_A + 6$

$X_A = -6$

$F_A = -6i - 4j$ [kN]

$F_B = +4j$ [kN]



f = tégla kN/m

M = kN·m

$\frac{a \cdot ma}{2} = \frac{3 \cdot 4}{2} = 6$

felcserével 2 erő ébred (mint a csuklóval) + 1 forg. nyom.

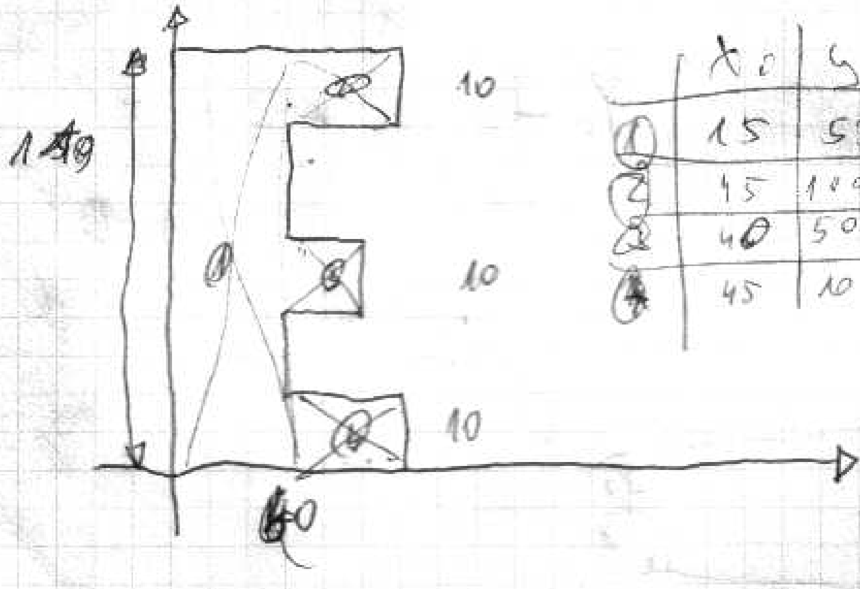
$M_A = +M_c + 6[kN] \cdot 5 = 0$

$M_c = -30 kN \cdot m$

$Y = Y_A + 6$ $\quad X = X_A + 5$

$Y_A = -6 \,N$ $\quad X_A = -5 \,kN$

$F_A = -5i + 6j - 30k$

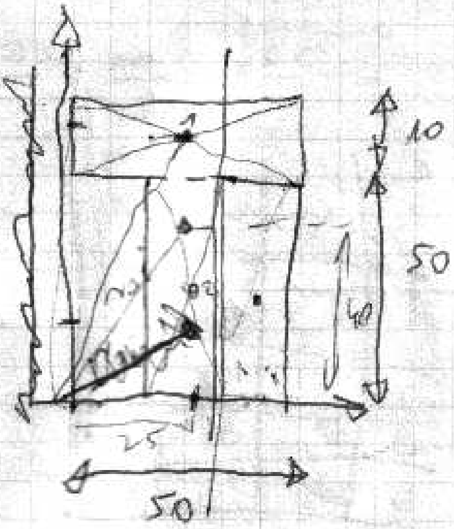


	x_i	y_i	A
1	15	55	30×10=300
2	45	100	30×20=600
3	40	50	20×20=400
4	45	10	30×10=300

$$r_{si} = \frac{r_{s1} \cdot A_1 + r_{s2} A_2 + r_{s3} A_3 + r_{s4} A_4}{A_1 + A_2 + A_2 + A_4} \Rightarrow$$

$$\Rightarrow \frac{(15;55)\ 300 + (45;100) \cdot 600 + (40;50) \cdot 400 + (45;10) \cdot 300}{16000} =$$

$$= \frac{4500i + 16500j + 27000i + 60000j + 16000i + 20000j + 1350 + 30}{16000}$$



$r_{si} =$

	x_i	y_i	A_i
1	25	55	$10 \times 50 = 500$
2	25	25	$10 \times 50 = 500$

$$r_{si} = \frac{\bar{r}_{01} A_1 + \bar{r}_{02} \cdot A_2 \ \dots}{A_1 + A_2} = \frac{(25\bar{i}, 55\bar{j}) \cdot 500 + (25\bar{i}, 25\bar{j}) \cdot 500}{1000}$$

$r_{01} = (25\bar{i}, 55\bar{j})$

$r_{02} = (25\bar{i}, 25\bar{j})$

$$= \frac{12500i + 27500j + 12500i + 12500j}{1000}$$

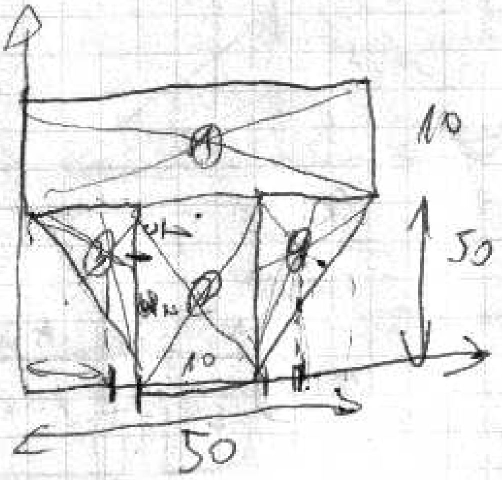
$$\frac{25000i + 40000j}{1000} = 25i + 40j$$

i	j
4500	16500
27000	60000
16000	10000
13500	3000

$$\Rightarrow \frac{61000i + 89500j}{16000} \quad (3{,}8125i, 6{,}21575j)$$

$36000j$

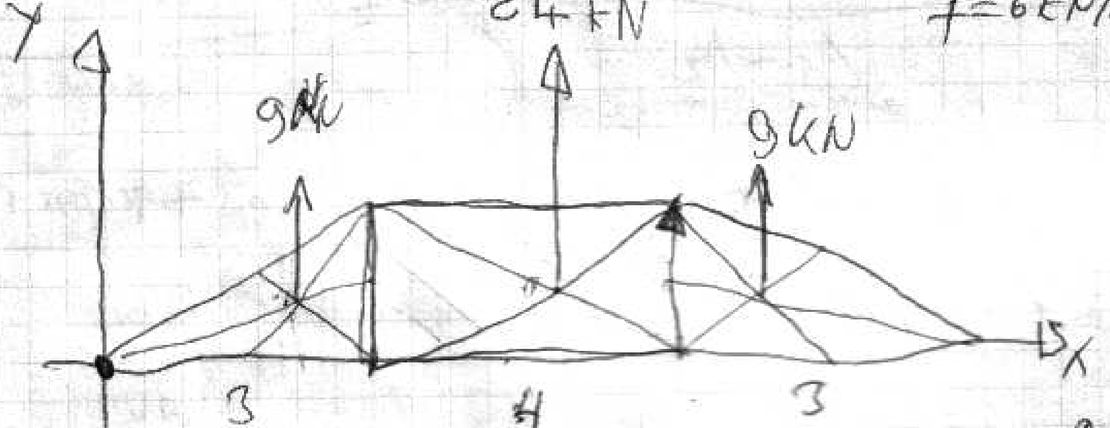
$$\frac{a \cdot ma}{2} = \frac{50 \cdot 20}{2}$$



	X_i	Y_i	A
1	25	55	500
2	25	25	500
3	$2 \cdot \frac{20}{3}$	$\frac{50}{3} \cdot 2$	500
4	$30 + \frac{20}{3}$	$\frac{100}{3}$	500

$$\frac{(25i, 55j) \cdot 500 + (25i, 25j) \cdot 500 + (2 \cdot \frac{20}{3} ; \frac{100}{3}) \cdot 500 + 30 + \frac{20}{3} ; \frac{100}{3}) \cdot 5}{2000}$$

$f = 6 kN/m$



① $\frac{a \cdot ma}{2} = \frac{3 \cdot 6}{2} = 9$

② $\frac{a \cdot ma}{2} = \frac{3 \cdot 6}{2} = 9$

$[\bar{F}, \bar{M}_0] = ?$

$\bar{F} = 42\bar{j}\ kN$

$\bar{M}_0 \overset{+}{=} 18 + 120 + \cancel{8}72 = 210\ kN \cdot m$