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Nincs ZH!!!!!   nov. 9. feladatkiadás

Pl. 4.1. a.

$$\vec{u} = u\,\vec{i} + v\,\vec{j} + w\,\vec{k}$$

$$u = -\frac{\nu}{R}\,x\,y$$

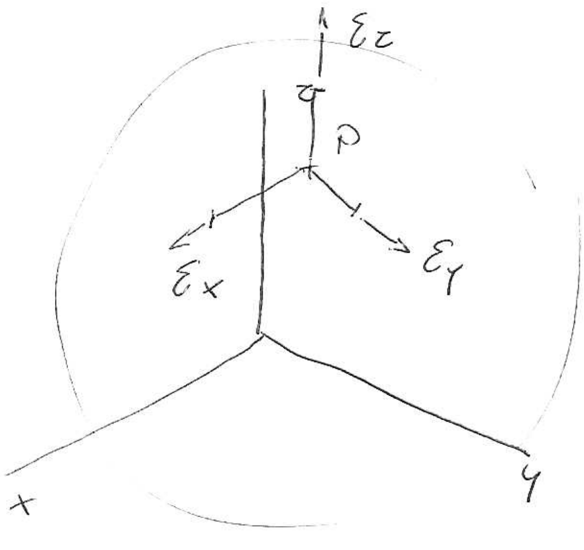
$$v = (\nu x^2 - \nu y^2 - z^2)/(2R)$$

$$w = yz/R$$

$R = 10$ m

$\nu = 0{,}25$

$P = (4, -2, 5)$ mm



a.) $\underline{\underline{A}}, \underline{\underline{A}}_P$?

$$\varepsilon_x = \frac{\partial u}{\partial x} = -\frac{\nu}{R}\,y$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = \frac{\partial}{\partial y}\left(-\nu y^2/(2R)\right) = -\frac{\nu}{R}\,y$$

$$\varepsilon_z = \frac{\partial w}{\partial z} = \frac{y}{R}$$

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$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -\frac{\nu}{R}x + \frac{\nu}{R}x = 0$$

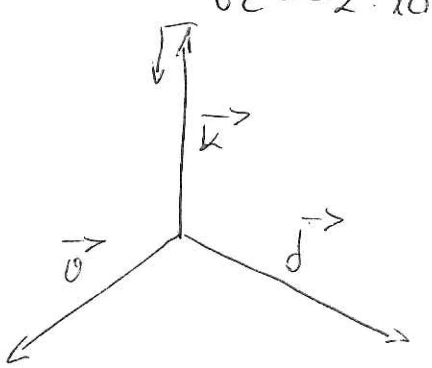
$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0 + 0 = 0$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = -\frac{z}{R} + \frac{z}{R} = 0$$

$$A(x, y, z) = \begin{bmatrix} -\frac{\nu}{R}y & 0 & 0 \\ 0 & -\frac{\nu}{R}y & 0 \\ 0 & 0 & \frac{y}{R} \end{bmatrix}$$

$$\underline{\underline{A}}_p = \underline{\underline{A}}(x_p, y_p, z_p) = \begin{bmatrix} 0{,}5 & & \\ & 0{,}5 & \\ & & -2 \end{bmatrix} 10^{-4}$$

$\varepsilon_z = -2 \cdot 10^{-4}$



$\varepsilon_x = 0{,}5 \cdot 10^{-4} = \varepsilon_y$

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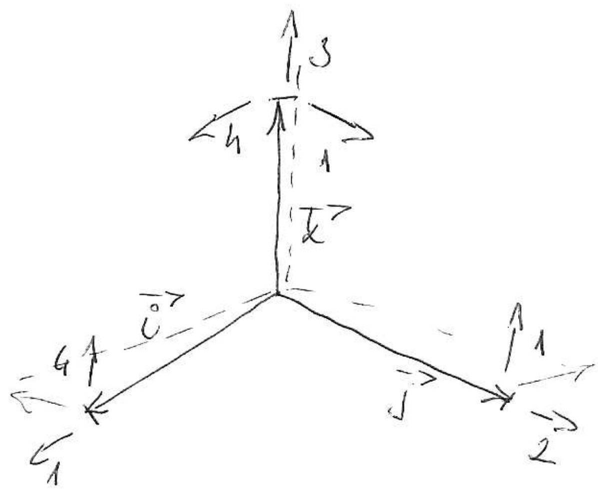
Pl:

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix} 10^{-4}$$

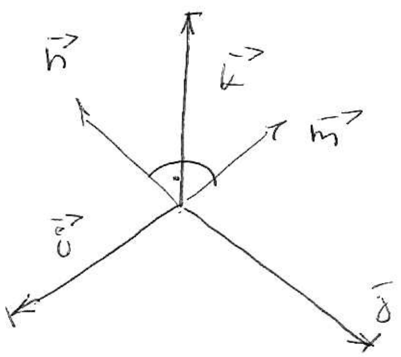
$$\vec{n} = 0{,}6\,\vec{i} + 0{,}8\,\vec{k}$$

$$\vec{m} = -0{,}8\,\vec{i} + 0{,}6\,\vec{k}$$

Szemléltetés:



$+10^{-4}$



$\varepsilon_n, \gamma_{mn}$: ?

$$\varepsilon_n = \vec{n}\,\underline{\underline{A}}\,\vec{n}$$

$$= \begin{bmatrix} 0{,}6 & 0 & 0{,}8 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix} 10^{-4} \begin{bmatrix} 0{,}6 \\ 0 \\ 0{,}8 \end{bmatrix} \right) =$$

$$= \begin{bmatrix} 0{,}6 & 0 & 0{,}8 \end{bmatrix} \begin{bmatrix} 3{,}8 \\ 0{,}8 \\ 4{,}8 \end{bmatrix} 10^{-4} = (0{,}6 \cdot 3{,}8 + 0{,}8 \cdot 4{,}8)\, 10^{-4} =$$

$$= \underline{6{,}12 \cdot 10^{-4}}$$

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$$\frac{1}{2}\gamma_{mn} = \vec{m}\cdot(\underline{\underline{A}}\ \vec{n}) = \begin{bmatrix} -0{,}8 & 0 & 0{,}6 \end{bmatrix} \begin{bmatrix} 3{,}8 \\ 0{,}8 \\ 4{,}8 \end{bmatrix} 10^{-4} = \dots$$

$$= (-0{,}8\cdot 3{,}8 + 0{,}6\cdot 4{,}8)\cdot 10^{-4} = \dots = \underline{-1{,}456\cdot 10^{-4}}$$

<u>Séma:</u>

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix} 10^{-4} \begin{bmatrix} 0{,}6 \\ 0 \\ 0{,}8 \end{bmatrix}$$

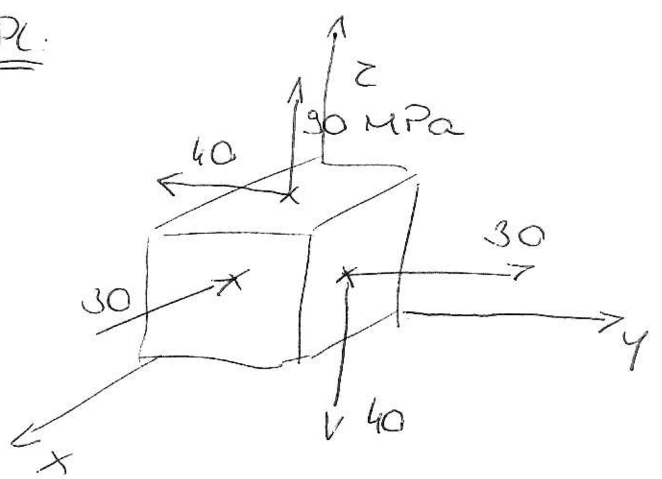
$$\varepsilon_n = (0{,}6^2 + 4\cdot 0{,}6\cdot 0{,}8\cdot 2 + 3\cdot 0{,}8^2)\cdot 10^{-4}$$

$n: \begin{bmatrix} 0{,}6 & 0 & 0{,}8 \end{bmatrix}$

$m: \begin{bmatrix} -0{,}8 & 0 & 0{,}6 \end{bmatrix}$

$$\frac{1}{2}\gamma_{mn} = (-0{,}8\cdot 0{,}6 + 4\cdot 0{,}6^2 - 0{,}8^2\cdot 4 + 3\cdot 0{,}6\cdot 0{,}8)\cdot 10^{-4}$$

<u>Pl:</u>



$$\underline{\underline{T}} = \begin{bmatrix} -30 & 0 & 0 \\ 0 & 30 & -40 \\ 0 & -40 & 90 \end{bmatrix} \text{MPa}$$

$$\vec{n} = 0{,}6\cdot\vec{i} + 0{,}8\,\vec{j}$$

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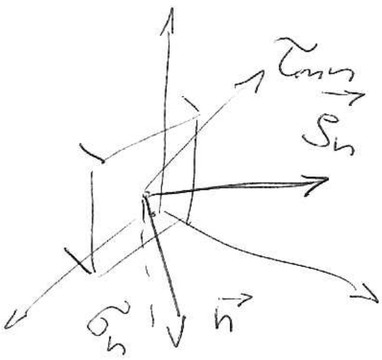


$$\vec{\rho}_n = \underline{\underline{T}} \cdot \vec{n} = \begin{bmatrix} -30 & 0 & 0 \\ 0 & 30 & -40 \\ 0 & -40 & 30 \end{bmatrix} \begin{bmatrix} 0{,}6 \\ 0{,}8 \\ 0 \end{bmatrix} =$$

$$= -18\vec{i} + 24\vec{j} - 32\vec{k}$$

$$\sigma_n = \vec{n}\,\rho\,\vec{n} = (-0{,}6 \cdot 0{,}8 + 0{,}8 \cdot 24) = \underline{8{,}4 \text{ MPa}}$$

$$\tau_{mn} = \vec{m}\,\rho\,\vec{n} = \vec{k}\,\rho\,\vec{n} = \underline{-32 \text{ MPa}}$$

$$\vec{m} = \vec{k}$$

Séma:

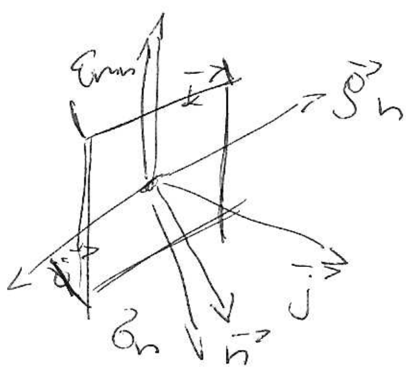
$$\left.\begin{aligned} \sigma_n &= \vec{n}\,\underline{\underline{T}}\,\vec{n} \\ \tau_{mn} &= \vec{m}\,\underline{\underline{T}}\,\vec{n} \end{aligned}\right\} \begin{bmatrix} -30 & 0 & 0 \\ 0 & 30 & -40 \\ 0 & -40 & 30 \end{bmatrix} \begin{bmatrix} 0{,}6 \\ 0{,}8 \\ 0 \end{bmatrix}$$

$$n: \begin{bmatrix} 0{,}6 & 0{,}8 & 0 \end{bmatrix}$$

$$m: \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$\sigma_n = -30 \cdot 0{,}6^2 + 30 \cdot 0{,}8^2 = \underline{8{,}4}$$

$$\tau_{mn} = -40 \cdot 0{,}8 = \underline{-32}$$



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Rudak szilárdságtana

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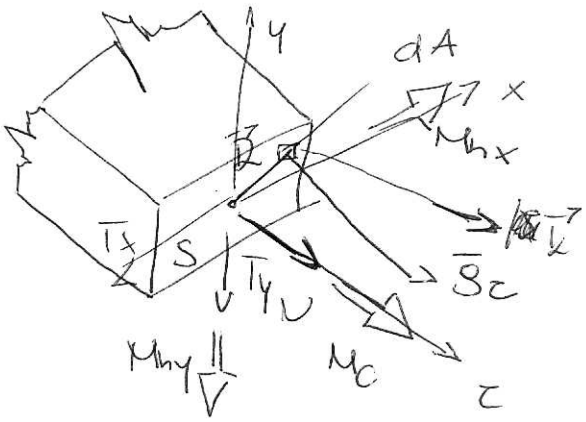
Alapkísérletek:

- megfigyelés $\longrightarrow \underline{\underline{A}}$
- terhelés, egyensúly $\longrightarrow \underline{\underline{T}}$
- alapkísérlet : $\underline{\underline{T}} = \underline{\underline{T}}(\underline{\underline{A}})$
- általánosítás

Feszültségi eredők



$$\vec{\rho}_z = \tau_{xz}\vec{i} + \tau_{yz}\vec{j} + \sigma_z\vec{k}$$

S-be redukálva:

$$\vec{F} = \int \vec{\rho}_z \, dA = \ldots = T_x\vec{i} - T_y\vec{j} + N\vec{k}$$

$$\vec{M}_S = \int \vec{R} \times \vec{\rho}_z \, dA = \ldots = M_{hx}\vec{i} - M_{hy}\vec{j} + M_z\vec{k}$$

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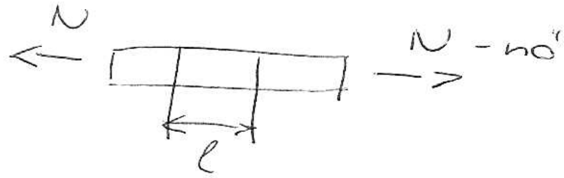
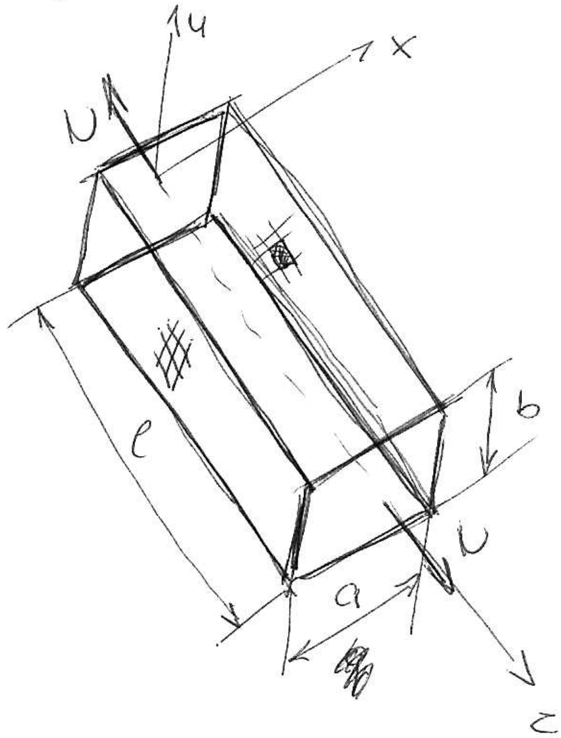
Húzás – nyomás

Alapkísérlet:

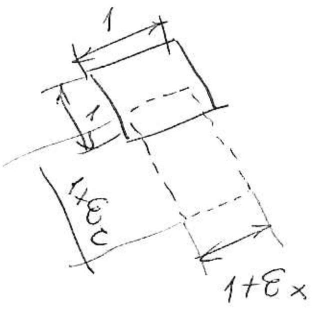




tapasztalat:

- $l' > l$, $a' < a$, $b' < b$

- 



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Alakváltozás:

$\varepsilon \neq 0, \ \gamma = 0$

$\varepsilon =$ áll.

hosszirányú fajlagos nyúlás

$$\varepsilon_h = \varepsilon_z = \frac{l' - l}{l} = \frac{\Delta l}{l}$$

$$\left.\begin{array}{l} \varepsilon_x = \dfrac{\Delta a}{a} \\ \varepsilon_y = \dfrac{\Delta b}{b} \end{array}\right\} \varepsilon_x = \varepsilon_y - \varepsilon_k$$

keresztirányú fajlagos nyúlás

$$\varepsilon_k = \varepsilon_x = \varepsilon_y = -\nu \varepsilon_h$$

$$\varepsilon_x = \varepsilon_y = -\nu \varepsilon_z$$

ν Poisson tényező

$(\sim 0{,}3)$

$$\underline{\underline{A}} = \begin{bmatrix} \varepsilon_x & & \\ & \varepsilon_y & \\ & & \varepsilon_z \end{bmatrix}$$

Feszültség

$\underline{\underline{A}} =$ áll. $\rightarrow \underline{\underline{T}} =$ áll.

palást: terheletlen

$\vec{\rho}_x = \vec{\rho}_y = 0$

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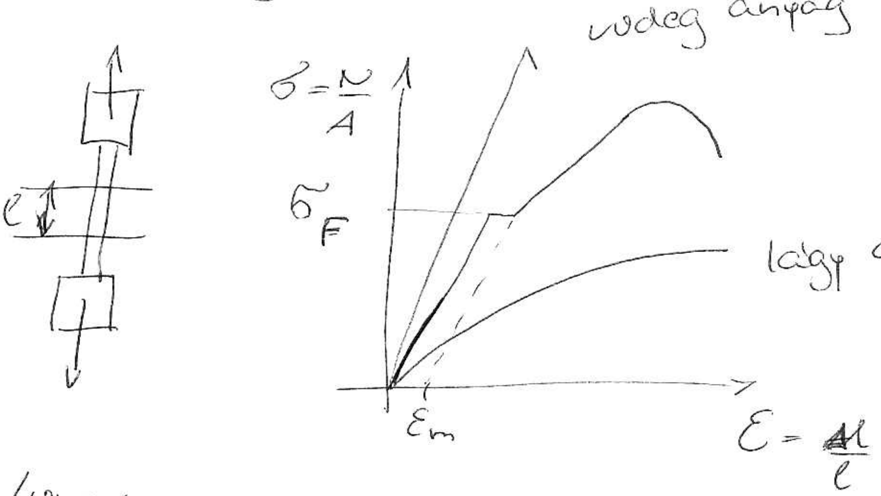
$$\underline{\underline{T}} = \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_z \end{bmatrix} \\ \quad \vec{\rho}_x \quad \vec{\rho}_y \quad \vec{\rho}_z \end{array}$$

$$\vec{F}_s = \int \vec{\rho}_z \, dA = \underbrace{\vec{\rho}_z A = N \vec{k}}$$

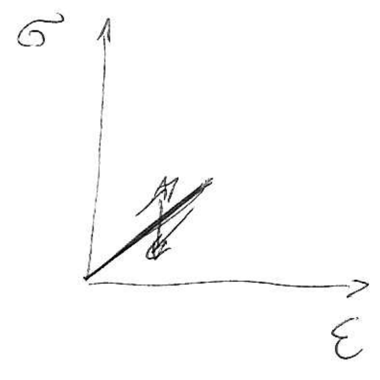
$$\vec{\rho}_z = \frac{N}{A} \vec{k} = \sigma_z k$$

$$\boxed{\sigma_z = \frac{N}{A}}$$

Szakító diagram:



Lineárisan rugalmas test



(Young-modulus)

rugalmassági modulus ($\sim 2 \cdot 10^5$ MPa)

$$\boxed{\sigma_z = E \varepsilon_z}$$ Hooke-törvény

(húzásra vonatkozó)

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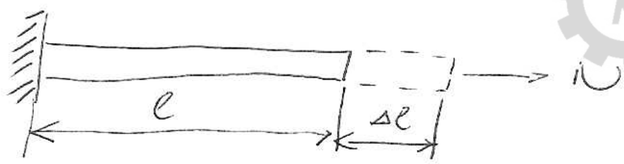
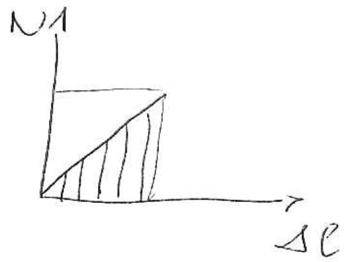
Jellemző alakváltozás



$$\boxed{\Delta l = l\,\varepsilon_z = l\,\frac{\sigma_z}{E} = \frac{N l}{A E}}$$

- Alakváltozási energia

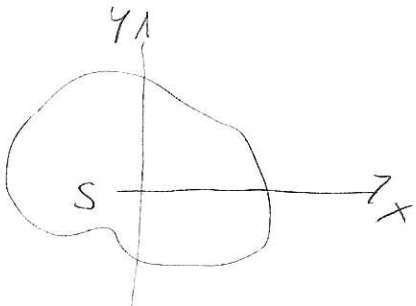


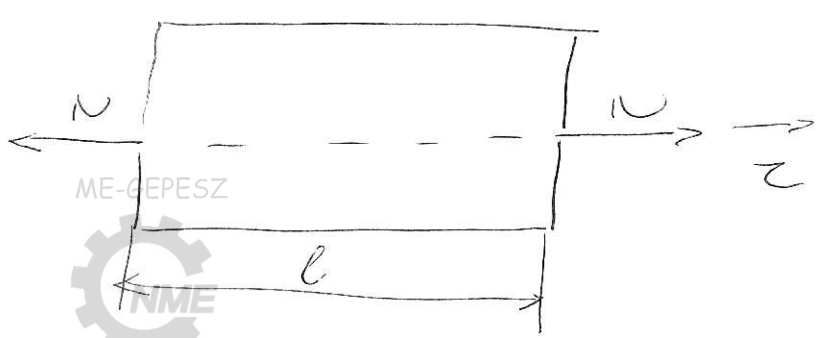
$$W_k = \frac{1}{2} N \Delta l = U$$

$$\boxed{U = \frac{1}{2}\,\frac{N^2 l}{A E}}$$

$$\Delta l = \frac{\partial U}{\partial N}$$

Általánosítás





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Alap: $n_t \geq n$

$$\frac{\sigma_{jell}}{|\sigma_z|} \geq \frac{\sigma_{jell}}{\sigma_{meg}}$$

$$|\sigma_z| \leq \sigma_{meg}$$

- Ellenőrzés

$$\sigma_z = \frac{N}{A}$$

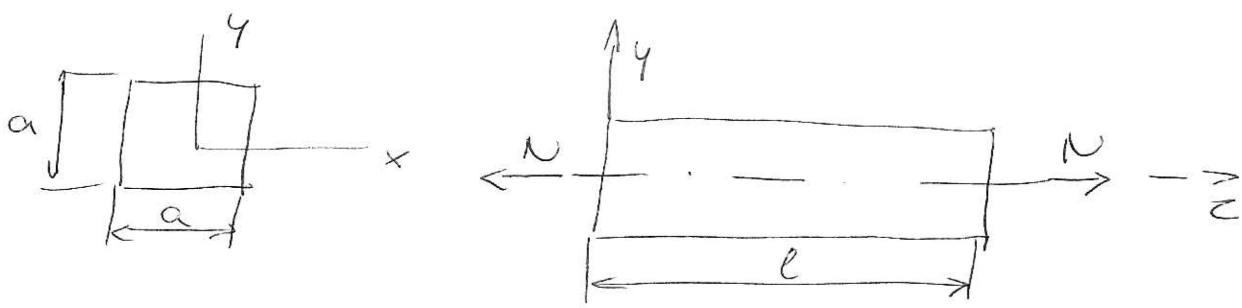
$$\sigma_{meg} = \frac{\sigma_{jell}}{n}$$

$$|\sigma_z| \leq \sigma_{meg} \quad ??$$

- Méretezés

$$\frac{|N|}{A} \leq \sigma_{meg} \rightarrow A \geq \frac{|N|}{\sigma_{meg}} = A_{szüks}$$

<u>Pl.</u>



$l = 200\,mm$

$a = 20\,mm$

$E = 2 \cdot 10^5\,MPa$

$\nu = 0{,}3$

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a.) $\Delta l = 20\ \mu m \qquad \underline{\underline{A}} = ?$

$$\left.\begin{aligned} \varepsilon_z &= \frac{\Delta l}{l} = \frac{20 \cdot 10^{-3}\,mm}{200\,mm} = 10^{-4} \\ \varepsilon_x &= \varepsilon_y = -\nu \varepsilon_z = -0{,}3 \cdot 10^{-4} \end{aligned}\right\}$$

$$\underline{\underline{A}} = \begin{bmatrix} -0{,}3 & & \\ & -0{,}3 & \\ & & 1 \end{bmatrix} 10^{-4}$$

b.) $\underline{\underline{T}} = ?$

$$\sigma_z = E\varepsilon_z = 2 \cdot 10^5 \cdot 10^{-4} = 20\ MPa = \underline{20\ N/mm^2}$$

c.) Δa $= -4{,}5\ \mu m \rightarrow N = ?$

$$\varepsilon_x = \frac{\Delta a}{a} = \frac{-4{,}5 \cdot 10^{-3}\,\mu m}{20\,mm} = -\frac{4{,}5}{2} \cdot 10^{-4}$$

$$\varepsilon_x = -\nu \varepsilon_z \rightarrow \varepsilon_z = -\frac{\varepsilon_x}{\nu} = \frac{4{,}5}{2 \cdot 0{,}3} \cdot 10^{-4} = 7{,}5 \cdot 10^{-4}$$

$$\sigma_z = E\varepsilon_z = 2 \cdot 10^5 \cdot 7{,}5 \cdot 10^{-4} = 150\ MPa$$

$$N = A \cdot \sigma_z = 20 \cdot 20 \cdot 150 = \underline{60 \cdot 10^3\,N} = \underline{60\,kN}$$

d.) $\sigma_{jell} = 120\ MPa \quad , n = 2$

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megfelel-e b/c esetben

NME

a)

$$\sigma_{meg} = \frac{120}{2} = 60 MPa$$

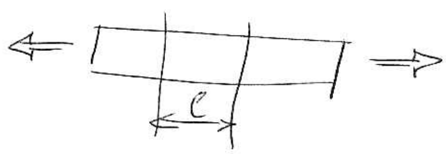
b) $\sigma_z = 20 < 60 \rightarrow$ megfelel

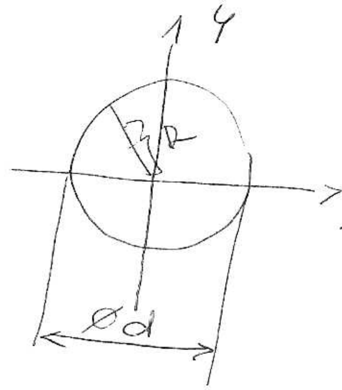
c) $\sigma_z = 150 > 60 \rightarrow$ nem felel meg

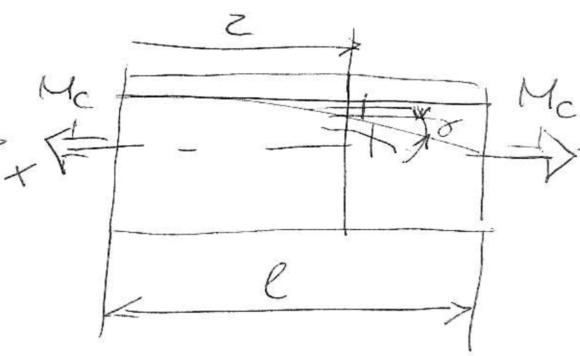
Csavarás

Csak kör és körgyűrű keresztmetszet

Alapkísérlet







tapasztalat:

- $l' = l$; $d' = d$
- Kkm - sík marad
  - elfordul:

$\Phi = \vartheta z$ - elfordulás

ϑ: fajlagos szögelfordulás $\left(\frac{rad}{m}\right)$

- alkotóból csavarvonal

emelkedési szög: γ

Általánosítás: (R, z) tetszőleges

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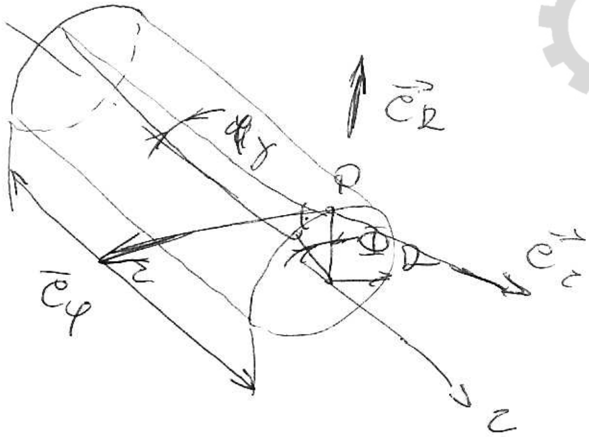


$$\Phi = \vartheta z$$

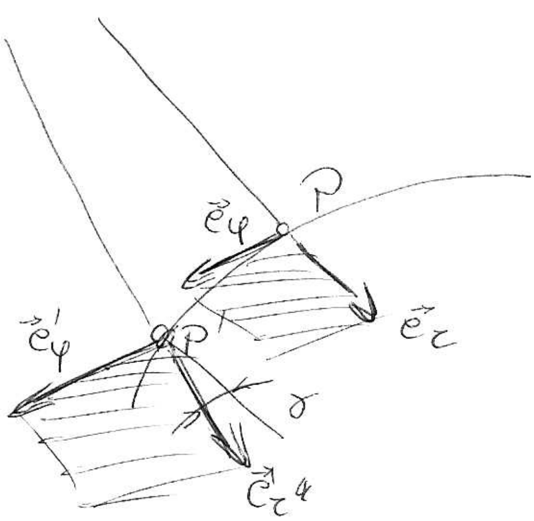
$$\overline{PP'} = R\Phi = z\,\underbrace{\mathrm{tg}\,\gamma}_{\sim\gamma} = z\gamma$$

$$R\vartheta z = \gamma z$$

$$\boxed{\gamma = R\vartheta}$$

(R, φ, z) Henger koord. r.

$$\underline{\underline{A}} = \begin{bmatrix} \varepsilon_R & \frac{1}{2}\gamma_{R\varphi} & \frac{1}{2}\gamma_{Rz} \\ \frac{1}{2}\gamma_{\varphi R} & \varepsilon_\varphi & \frac{1}{2}\gamma_{\varphi z} \\ \frac{1}{2}\gamma_{zR} & \frac{1}{2}\gamma_{z\varphi} & \varepsilon_z \end{bmatrix}$$



$$\gamma_{\varphi z} = \gamma_{z\varphi}$$

most

$$\underset{(R,\varphi,z)}{\underline{\underline{A}}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}\gamma_{\varphi z} \\ 0 & \frac{1}{2}\gamma_{z\varphi} & 0 \end{bmatrix}$$

Elemi négyszög mozgása

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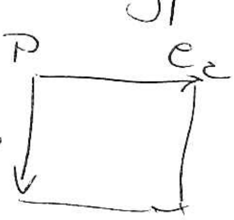
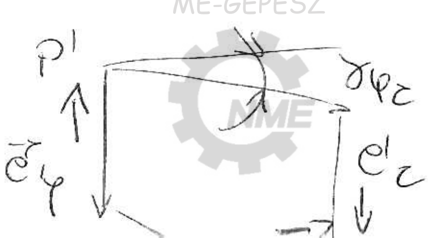
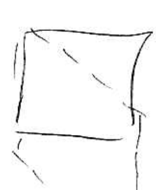
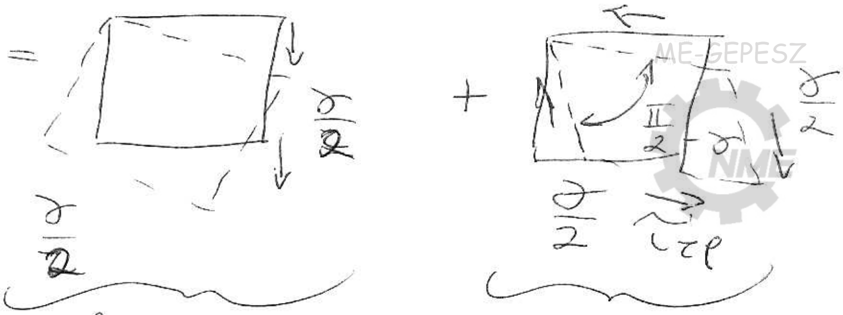




$P = P'$



=

$=$ 

forgás + torzulás

$\tau_{\varphi z}$

$$\underline{\underline{T}}_{(R,\varphi,z)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \tau_{\varphi z} \\ 0 & \tau_{z\varphi} & 0 \end{bmatrix}$$

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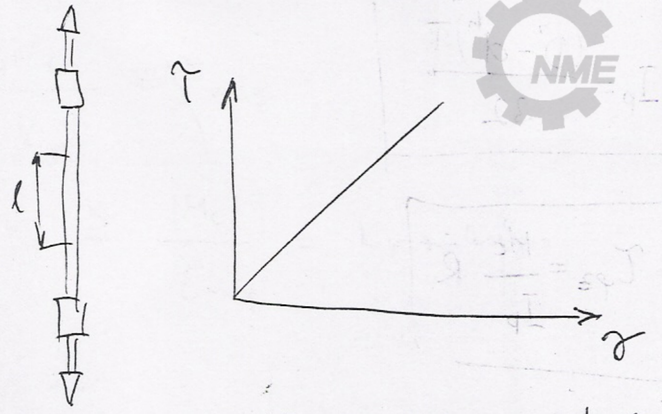
Szilárdságtan

ME-GEPESZ



lin. rugalmas test   Csavarási Hooke-törvény

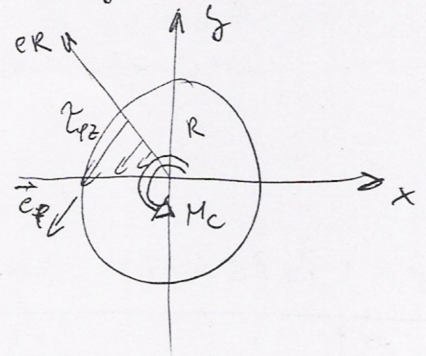
$$\tau = G\gamma$$

$G =$ csúsztató/nyíró rug. modulus

$G \cong 80\,000$ MPa

$$\tau_{\varphi z} = \tau_{z\varphi}$$

$$\tau_{\varphi z} = G\gamma_{\varphi z} = G R \vartheta$$



Egyensúly

$$M_c = \int R\,\tau_{\varphi z}\,dA = G\vartheta \underbrace{\int_A R^2\,dA}_{I_p}$$

$I_p =$ Poláris Másodrendű nyomaték

$$I_p = \int R^2\,dA$$

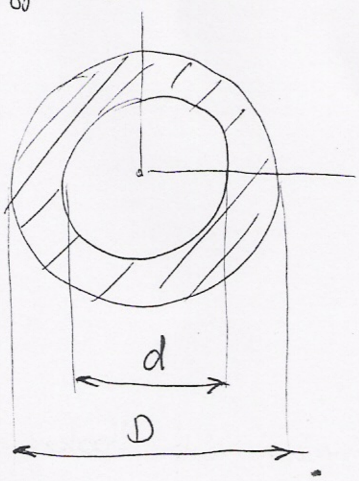
$$M_c = G\vartheta I_p$$

Körre:

$$I_p = \underbrace{\int R^2\,dA}_{2\pi R\,dR} = 2\pi\int_0^{d/2} R^3\,dR = 2\pi \frac{R^4}{4}\Big|_0^{d/2}$$

$$\boxed{I_p = \frac{d^4\pi}{32}}$$

Körgyűrűre:

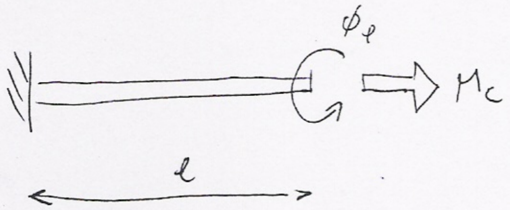


$$\boxed{I_p = \frac{(D^4 - d^4)\pi}{32}}$$

$$\boxed{\tau_p = \tau_{\varphi z} = \frac{M_c}{I_p} R}$$

$$\tau_{max} = \frac{|M_c|}{I_p} R_{max} = \frac{|M_c|}{K_p} \qquad K_p = \frac{I_p}{R_{max}}$$ - poláris keresztmetszeti tényező

- jellemző alakváltozás:



$$\boxed{\Phi_\ell = \vartheta \ell = \frac{M_c \cdot \ell}{I_p G}}$$

alakváltozási energia:

$$U = W_K = \frac{1}{2} \frac{M_c^2 \ell}{I_p \cdot G}$$

Ellenőrzés

τ_{jell} , n

$$\tau_{meg} = \frac{\tau_{jell}}{n}$$

$$\tau_{max} = \frac{|M_c|}{K_p}$$

$\tau_{max} \leq \tau_{meg}$ - megfelel

$\tau_{max} \geq \tau_{meg}$ - nem felel meg

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Méretezés:

$$\frac{|M_c|}{K_p} \le \tau_{meg}$$

$$K_p \ge \frac{|M_c|}{\tau_{meg}} = K_{p\,szükséges}$$

pl. kör

$$K_p = \frac{I_p}{d/2} = \frac{d^4\pi}{32}\,\frac{2}{d} = \frac{d^3\pi}{16}$$

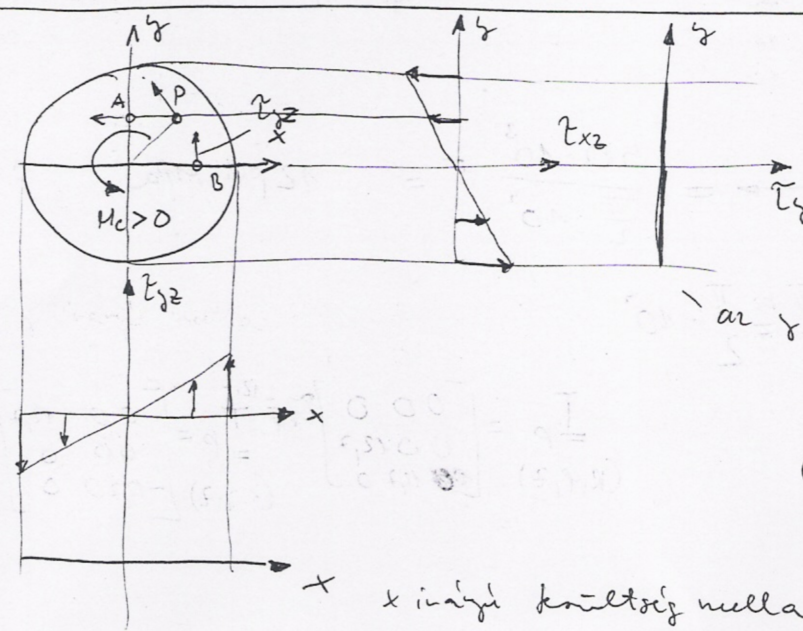
$$K_{p\,szükséges} = \frac{d^3_{szükséges}\,\pi}{16} \longrightarrow d_{szükséges}$$

pl. körgyűrű

$$K_p = \frac{I_p}{D/2} = \frac{(D^4-d^4)\pi}{16D}$$

$\frac{D}{d}$ - előírt $\rightarrow D = \sim$

Feszültségek (R, φ, z) és (x, y, z) KR-ben:



$$\underset{(x,y,z)}{\underline{\underline{T}}_A} = \begin{bmatrix} 0 & 0 & -\tau_A \\ 0 & 0 & 0 \\ -\tau_A & 0 & 0 \end{bmatrix}$$

az y irányú nulla

$$\underset{(x,y,z)}{\underline{\underline{T}}_B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \tau_B \\ 0 & \tau_B & 0 \end{bmatrix}$$

x irányú feszültség nulla

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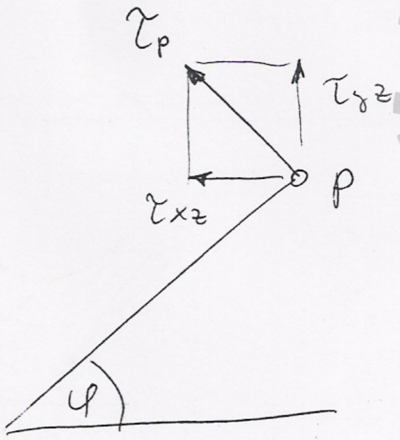
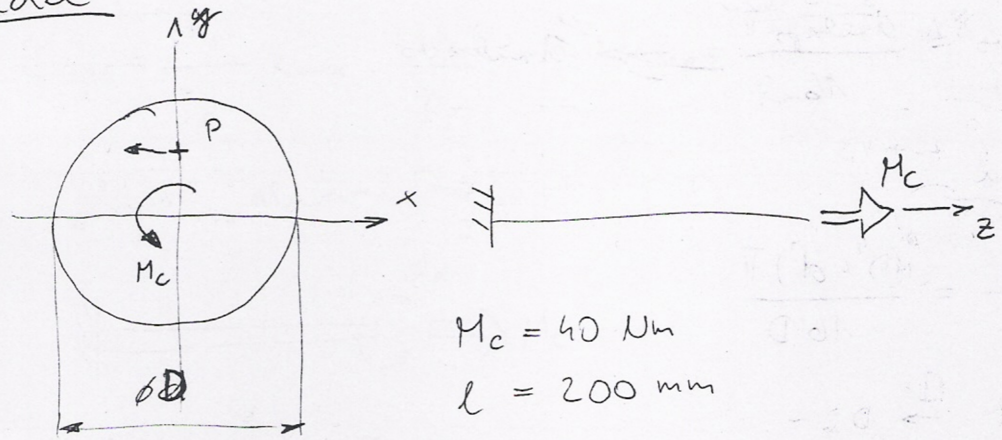


$$\left.\begin{array}{l}\tau_{xz} = -\tau_p \sin\varphi \\ \tau_{yz} = \tau_p \cos\varphi\end{array}\right\}$$

Példa



$M_c = 40$ Nm

$l = 200$ mm

$D = 20$ mm

$R_p = 5$ mm

$G = 80000$ MPa

a) $\underline{\underline{T}}_p \; (R,\varphi,z)$, $\underline{\underline{T}}_p \; (x,y,z)$

$$\tau_p = \frac{M_c}{I_p} R_p \longrightarrow = \frac{40 \cdot 10^3}{\frac{\pi}{2} \cdot 10^4} \, 5 = 12{,}7 \text{ MPa}$$

$$I_p = \frac{D^4 \pi}{32} \Rightarrow \frac{20^4 \pi}{32} = \frac{\pi}{2} \cdot 10^4$$

$$\underset{(R,\varphi,z)}{\underline{\underline{T}}_p} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 12{,}7 \\ 0 & 12{,}7 & 0 \end{bmatrix} \quad \tau_{R\varphi} = 12{,}7 \quad \underset{(x,y,z)}{\underline{\underline{T}}_p} = \begin{bmatrix} 0 & 0 & -12{,}7 \\ 0 & 0 & 0 \\ -12{,}7 & 0 & 0 \end{bmatrix} \quad \tau_{yz} = 12{,}7$$

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$\underset{=}{A_p}_{(x,y,z)} = ?$

$$\gamma_{xz} = \frac{\tau_{xz}}{G} = \frac{-12,7}{8\cdot 10^4} = -1,6\cdot 10^{-4}$$

$$\underline{\underline{A}}_p = \begin{bmatrix} 0 & 0 & -0,8 \\ 0 & 0 & 0 \\ -0,8 & 0 & 0 \end{bmatrix}\cdot 10^{-4}$$

c)
$$\varphi_l = \frac{M_c l}{I_p G} = \frac{40\cdot 10^3\cdot 200}{\frac{\pi}{2}\cdot 10^4\cdot 80000} = \dots = 0,064 \text{ rad}$$
$$= 3,6 \text{ fok}$$

e) $d = ?$ ha $M_c = 60$; $\tau_{meg} = 40\,\text{MPa}$

$$\tau_{max} = \frac{M_c}{K} \leq \tau_{meg}$$

$$K_p \geq \frac{M_c}{\tau_{meg}} = K_{p\,\text{szükséges}}$$

$$K_{p\,\text{szükséges}} = \frac{60\cdot 10^3}{40} = 1,5\cdot 10^3 = \frac{d_{\text{szüks}}^2\,\pi}{16}$$

$$d_{sz} = 10\sqrt[3]{\frac{1,5\cdot 1,6}{\pi}} = 19,69\text{ mm}$$

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