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Ex.1

$$\sqrt{x+2}+\sqrt{x+7}=5$$

$$\begin{matrix} x+2\geq 0 \\ x+7\geq 0 \end{matrix} \Rightarrow \begin{matrix} x\geq -2 \\ x\geq -7 \end{matrix} \Rightarrow x\in[-2;\infty)$$

Fie: $\begin{matrix} \sqrt{x+2}=u \\ \sqrt{x+7}=v \end{matrix} \Rightarrow \begin{matrix} u+v=5 \\ u^2-v^2=-5 \end{matrix} \Rightarrow \begin{matrix} u+v=5 \\ (u+v)(u-v)=-5 \end{matrix} \Rightarrow \begin{matrix} u+v=5 \\ u-v=-1 \end{matrix} \Rightarrow$

$\Rightarrow \quad 2u=4\Rightarrow u=2$

$\sqrt{x+2}=2\Rightarrow x+2=4\Rightarrow x=2$

Verificare:

$$\sqrt{2+2}+\sqrt{2+7}=5$$
$$\sqrt{4}+\sqrt{9}=5$$
$$2+3=5$$
$$5=5(A)$$

$S=\{2\}$

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Ex.2

$$\sqrt{4x-5}=\sqrt{x+1}$$

$$\begin{cases}4x-5\geq 0\\ x+1\geq 0\end{cases}\Rightarrow\begin{cases}4x\geq 5\\ x\geq -1\end{cases}\Rightarrow\begin{cases}x\geq \frac{5}{4}\\ x\geq -1\end{cases}\Rightarrow x\geq \frac{5}{4}$$

$$\sqrt{4x-5}=\sqrt{x+1}$$
$$4x-5=x+1$$
$$3x=6$$
$$x=2$$

Verificare: $\sqrt{4x-5}=\sqrt{x+1}$

$$\sqrt{4\cdot 2-5}=\sqrt{2+1}$$
$$\sqrt{3}=\sqrt{3}$$

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Ex.3

$$\sqrt{6x+7}=\sqrt{x+2}$$

$$\begin{cases} 6x+7\geq 0 \\ x+2\geq 0 \end{cases} \Rightarrow \begin{cases} 6x\geq -7 \\ x\geq -2 \end{cases} \Rightarrow \begin{cases} x\geq -\frac{7}{6} \\ x\geq -2 \end{cases} \Rightarrow x\geq -\frac{7}{6} \Rightarrow x\in [-\frac{7}{6};+\infty[$$

$$\sqrt{6x+7}=\sqrt{x+2}$$

$$6x+7=x+2$$

$$5x=-5$$

$$x=-1\in [-\frac{7}{6};+\infty[$$

Verificare:

$$\sqrt{6x+7}=\sqrt{x+2}$$

$$\sqrt{6\cdot(-1)+7}=\sqrt{-1+2}$$

$$\sqrt{1}=\sqrt{1}$$

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Ex.4

$$\sqrt{x+2}+\sqrt{x-3}=5$$

$$\begin{cases} x+2\geq 0 \\ x-3\geq 0 \end{cases} \Rightarrow \begin{cases} x\geq -2 \\ x\geq 3 \end{cases} \Rightarrow x\geq 3 \Rightarrow x\in[3;+\infty[$$

$$\sqrt{x+2}+\sqrt{x-3}=5$$

$$x+2+2\sqrt{(x+2)(x-3)}+x-3=25$$

$$2\sqrt{(x+2)(x-3)}=26-2x$$

$$\sqrt{(x+2)(x-3)}=13-x$$

$$x^2+2x-3x-6=169-26x+x^2$$

$$25x=175$$

$$x=7\in[3;+\infty[$$

Verificare:

$$\sqrt{7+2}+\sqrt{7-3}=5$$

$$\sqrt{9}+\sqrt{4}=5$$

$$3+2=5$$

$$5=5$$

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Ex.5

$$\sqrt{3x+3}+\sqrt{2x+6}=2$$

$$\begin{cases}3x+3\geq 0\\2x+6\geq 0\end{cases}\Rightarrow\begin{cases}3x\geq -3\\2x\geq -6\end{cases}\Rightarrow\begin{cases}x\geq -1\\x\geq -3\end{cases}\Rightarrow x\geq -1\Rightarrow x\in[-1;+\infty[$$

$$\sqrt{3x+3}+\sqrt{2x+6}=2$$

$$3x+3+2\sqrt{(3x+3)(2x+6)}+2x+6=4$$

$$2\sqrt{(3x+3)(2x+6)}=-5-5x$$

$$-5-5x\geq 0$$

$$5x\leq -5$$

$$x\leq -1$$

$$x\in\{-1\}$$

$$4(6x^2+18x+6x+18)=25+50x+25x^2$$

$$24x^2+72x+24x+72=25+50x+25x^2$$

$$x^2-46x-47=0$$

$$x_1=-1\in\{-1\}$$

$$x_2=47\notin\{-1\}$$

Verificare:

$$\sqrt{3\cdot(-1)+3}+\sqrt{2\cdot(-1)+6}=2$$

$$\sqrt{0}+\sqrt{4}=2$$

$$0+2=2$$

$$2=2$$

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Ex.6

$$\sqrt{x+6}-\sqrt{x+2}=\sqrt{2x+8}$$

$$\begin{cases} x+6\geq 0 \\ x+2\geq 0 \\ 2x+8\geq 0 \end{cases} \Rightarrow \begin{cases} x\geq -6 \\ x\geq -2 \\ 2x\geq -8 \end{cases} \Rightarrow \begin{cases} x\geq -6 \\ x\geq -2 \\ x\geq -4 \end{cases} \Rightarrow x\in[-2;+\infty[$$

$$\sqrt{x+6}-\sqrt{x+2}=\sqrt{2x+8}$$

$$x+6-2\sqrt{(x+6)(x+2)}+x+2=2x+8$$

$$-2\sqrt{(x+6)(x+2)}=0$$

$$(x+6)(x+2)=0$$

$$x_1=-6\notin[-2;+\infty[$$

$$x_2=-2\in[-2;+\infty[$$

Verificare:

$$\sqrt{-2+6}+\sqrt{-2+2}=\sqrt{2\cdot(-2)+8}$$

$$\sqrt{4}+\sqrt{0}=\sqrt{4}$$

$$2+0=2$$

$$2=2$$

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Ex.7

$$\sqrt{2x-2}-\sqrt{x+2}=\sqrt{x-6}$$

$$\begin{cases} 2x+2\geq 0 \\ x+2\geq 0 \\ x-6\geq 0 \end{cases} \Rightarrow \begin{cases} 2x\geq -2 \\ x\geq -2 \\ x\geq 6 \end{cases} \Rightarrow \begin{cases} x\geq -1 \\ x\geq -2 \\ x\geq 6 \end{cases} \Rightarrow x\in[6;+\infty[$$

$$\sqrt{2x+2}-\sqrt{x+2}=\sqrt{x-6}$$

$$2x+2-2\sqrt{(2x+2)(x+2)}+x+2=x-6$$

$$-2\sqrt{(2x+2)(x+2)}=-2x-10$$

$$\sqrt{(2x+2)(x+2)}=x+5$$

$$x+5\geq 0$$

$$x\geq -5$$

$$2x^2+4x+2x+4=x^2+10x+25$$

$$x^2-4x-21=0$$

$$x_1=7\in[6;+\infty[$$

$$x_2=-3\notin[6;+\infty[$$

Verificare:

$$\sqrt{2\cdot 7+2}-\sqrt{7+2}=\sqrt{7-6}$$

$$\sqrt{16}-\sqrt{9}=\sqrt{1}$$

$$4-3=1$$

$$1=1$$

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Ex.8

$$\sqrt[3]{x-1}+\sqrt[3]{2x-1}=1$$

$$x-1+3\cdot\sqrt[3]{x-1}\cdot\sqrt[3]{2x-1}(\sqrt[3]{x-1}+\sqrt[3]{2x-1})+2x-1=1^3$$

$$3\cdot\sqrt[3]{x-1}\cdot\sqrt[3]{2x-1}\cdot 1=3-3x$$

$$\sqrt[3]{(x-1)(2x-1)}=1-x$$

$$2x^2-2x-x+1=1-3x+3x^2-x^3$$

$$x^3-x^2=0$$

$$x^2(x-1)=0$$

$$x_1=0$$

$$x_2=1$$

Verificare:

$$\sqrt[3]{0-1}+\sqrt[3]{2\cdot 0-1}=1$$

$$-1-1=1$$

$$-2=1(F)$$

$$\sqrt[3]{1-1}+\sqrt[3]{2\cdot 1-1}=1$$

$$0+1=1(A)$$

$$S=\{1\}$$

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Ex.9 $\sqrt[3]{15+2x}+\sqrt[3]{19-3x}=4$

Fie: $\sqrt[3]{15+2x}=u$, $\sqrt[3]{19-3x}=v$ $\Rightarrow$ $u+v=4$ $\Rightarrow$ $u=4-v$

$$\frac{u^3-15}{2}=\frac{v^3-19}{-3} \Rightarrow \frac{(4-v)^3-15}{2}=\frac{v^3-19}{-3}$$

$$\frac{64-48v^2+12v-v^3-15}{2}=\frac{v^3-19}{-3} \Rightarrow -192+144v^2-36v+3v^3+45=2v^3-38$$

$$v^3+144v^2-36v-109=0 \Rightarrow (v-1)(v^2-35v+109)=0 \Rightarrow v_1=1$$

$$v_{2,3}=\frac{35\pm\sqrt{1225-436}}{2}=\frac{35\pm\sqrt{789}}{2}$$

$v_2=31{,}545$

$v_3=3{,}455$

$\sqrt[3]{19-3x}=1$	$\sqrt[3]{19-3x}=31{,}545$	$\sqrt[3]{19-3x}=3{,}455$
$19-3x=1$	$19-3x=31390{,}02$	$19-3x=41{,}2424$
$-3x=-18$	$-3x=31371{,}02$	$-3x=22{,}2424$
$x=6$	$x=-10457{,}007$	$x=-7{,}4141$

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Ex.10 $\sqrt[3]{2x^2+x-9}+\sqrt[3]{100-x-2x^2}=7$

Fie: $2x^2+x=t$

$\sqrt[3]{t-9}=u$

$\sqrt[3]{100-t}=v$

$$\begin{cases} u+v=7 \\ 91=u^3+v^3 \end{cases} \Rightarrow \begin{cases} u+v=7 \\ 91=(u+v)(u^2-uv+v^2) \end{cases} \Rightarrow \begin{cases} u+v=7 \\ 91=(u+v)[(u+v)^2-3uv] \end{cases} \Rightarrow \begin{cases} u+v=7 \\ 91=7[49-3uv] \end{cases}$$

$$\Rightarrow \begin{cases} u+v=7 \\ 13=49-3uv \end{cases} \Rightarrow \begin{cases} u+v=7 \\ 3uv=36 \end{cases} \Rightarrow \begin{cases} u+v=7 \\ uv=12 \end{cases} \Rightarrow u^2-7u+12=0 \Rightarrow \begin{cases} u_1=3 \\ u_2=4 \end{cases}$$

$$\Rightarrow \begin{cases} \sqrt[3]{t-9}=3 \\ \sqrt[3]{t-9}=4 \end{cases} \Rightarrow \begin{cases} t-9=27 \\ t-9=64 \end{cases} \Rightarrow \begin{cases} t=36 \\ t=73 \end{cases} \Rightarrow \begin{cases} 2x^2+x-36=0 \\ 2x^2+x-73=0 \end{cases} \Rightarrow \begin{cases} 2x^2+x-36=0 \\ 2x^2+x-73=0 \end{cases}$$

$$\Rightarrow \begin{cases} x_1=4; x_2=-\dfrac{9}{2} \\ x_3=5{,}797; x_4=-6{,}297 \end{cases}$$

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Ex.11

$$\sqrt[3]{2-x}=1-\sqrt{x-1}$$

$$x\in[0;+\infty]$$

$$2-x=1-3\sqrt{x-1}+3(x-1)-(x-1)\sqrt{x-1}$$

$$2-x=1-3\sqrt{x-1}+3x-3-x\sqrt{x-1}+\sqrt{x-1}$$

$$4-4x=-x\sqrt{x-1}-2\sqrt{x-1}$$

$$-4\sqrt{(x-1)^2}+x\sqrt{x-1}+2\sqrt{x-1}=0$$

$$\sqrt{x-1}(-4\sqrt{x-1}+x+2)=0$$

$$x_1=1 \qquad \text{sau}$$

$$-4\sqrt{x-1}+x+2=0$$

$$x+2=4\sqrt{x-1}$$

$$x^2+4x+4=16x-16$$

$$x^2-12x+20=0$$

$$\Delta=144-80=64$$

$$x_{2,3}=\frac{12\pm 8}{2}$$

$$x_2=10;\ x_3=2$$

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Ex.12 $\sqrt[3]{1+\sqrt{x}}+\sqrt[3]{8-\sqrt{x}}=3$

$\sqrt[3]{1+\sqrt{x}}+\sqrt[3]{8-\sqrt{x}}=3$

$1+\sqrt{x}+3\cdot\sqrt[3]{1+\sqrt{x}}\cdot\sqrt[3]{8-\sqrt{x}}(\sqrt[3]{1+\sqrt{x}}+\sqrt[3]{8-\sqrt{x}})+8-\sqrt{x}=27$

$3\cdot\sqrt[3]{(1+\sqrt{x})(8-\sqrt{x})}\cdot 3=18$

$\sqrt[3]{(1+\sqrt{x})(8-\sqrt{x})}=2$

$(1+\sqrt{x})(8-\sqrt{x})=8$

$8-\sqrt{x}+8\sqrt{x}-\sqrt{x^2}=8$

$7\sqrt{x}-\sqrt{x^2}=0$

$\sqrt{x}(7-\sqrt{x})=0$

$x_1=0$

$x_2=49$

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Ex.13 Să se determine mulţimea: $A=\{x\in R\,|\,\sqrt{x+3}+\sqrt{9-x^2}=\sqrt[3]{x^2+18}-3\}$

$$\begin{cases} x+3\geq 0 \\ 9-x^2\geq 0 \\ \sqrt[3]{x^2+18}-3\geq 0 \end{cases} \Rightarrow \begin{cases} x+3\geq 0 \\ 9-x^2\geq 0 \\ \sqrt[3]{x^2+18}\geq 3 \end{cases} \Rightarrow \begin{cases} x+3\geq 0 \\ 9-x^2\geq 0 \\ x^2+18\geq 27 \end{cases} \Rightarrow \begin{cases} x+3\geq 0 \\ 9-x^2\geq 0 \Rightarrow D=\{\pm 3\} \\ x^2-9\geq 0 \end{cases}$$

x	$-\infty$		-3			3			$+\infty$
$x+3$	-	-	0	+	+	+	+	+	+
$9-x^2$	-	-	0	+	+	0	-	-	-
x^2-9	+	+	0	-	-	0	+	+	+

Soluţii posibile: -3 sau +3

$\sqrt{-3+3}+\sqrt{9-(-3)^2}=\sqrt[3]{(-3)^2+18}-3$

$\sqrt{0}+\sqrt{9-9}=\sqrt[3]{9+18}-3$

$0+0=\sqrt[3]{27}-3$

$0=3-3$

$0=0$

$\sqrt{3+3}+\sqrt{9-3^2}=\sqrt[3]{3^2+18}-3$

$\sqrt{6}+\sqrt{9-9}=\sqrt[3]{9+18}-3$

$\sqrt{6}+0=\sqrt[3]{27}-3$

$\sqrt{6}=3-3$

$\sqrt{6}=0$

$A=\{-3\}$

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Ex.14 $\sqrt{x^2-3x+2} > x+1$

$x^2-3x+2 \geq 0 \Rightarrow x \in (-\infty;1] \cup [2;+\infty)$

x	$-\infty$		-1		1		2		$+\infty$
$x+1$	-	-	0	+	+	+	+	+	+
x^2-3x+2	+	+	+	+	0	-	0	+	+

Cazul: $x \in (-\infty;-1)$

$\sqrt{x^2-3x+2} > x+1$

$+ > -$

Soluţia este: $x \in (-\infty;-1)$

Cazul: $x \in [-1;1] \cup [2;+\infty)$

$\sqrt{x^2-3x+2} > x+1$

$x^2-3x+2 > x^2+2x+1$

$-5x > -1$

$x < \frac{1}{5}$

Soluţia este: $x \in [-1;\frac{1}{5})$

Soluţia generală este: $x \in (-\infty;-1) \cup [-1,\frac{1}{5}) = (-\infty;\frac{1}{5})$

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Ex.15

$$\sqrt[2013]{x+\sqrt{x^2-1}}+\sqrt[2013]{x-\sqrt{x^2-1}}=2$$

$$\begin{cases}\sqrt[2013]{x+\sqrt{x^2-1}}=a\\ \sqrt[2013]{x-\sqrt{x^2-1}}=b\\ a+b=2\end{cases}\Rightarrow\begin{cases}x+\sqrt{x^2-1}=a^{2013}\\ x-\sqrt{x^2-1}=b^{2013}\\ a+b=2\end{cases}\Rightarrow\begin{cases}2x=a^{2013}+b^{2013}\\ 2\sqrt{x^2-1}=a^{2013}-b^{2013}\\ a+b=2\end{cases}\Rightarrow\begin{cases}x=\dfrac{a^{2013}+b^{2013}}{2}\\ \sqrt{x^2-1}=\dfrac{a^{2013}-b^{2013}}{2}\\ a+b=2\end{cases}$$

$$\begin{cases}x=\dfrac{a^{2013}+b^{2013}}{2}\\ x^2-1=\left(\dfrac{a^{2013}-b^{2013}}{2}\right)^2\\ a+b=2\end{cases}\Rightarrow\begin{cases}x=\dfrac{a^{2013}+b^{2013}}{2}\\ x^2=1+\left(\dfrac{a^{2013}-b^{2013}}{2}\right)^2\\ a+b=2\end{cases}\Rightarrow\begin{cases}\left(\dfrac{a^{2013}+b^{2013}}{2}\right)^2=1+\left(\dfrac{a^{2013}-b^{2013}}{2}\right)^2\\ a+b=2\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}\dfrac{a^{4026}+2a^{2013}b^{2013}+b^{4026}}{4}=1+\dfrac{a^{4026}-2a^{2013}b^{2013}+b^{4026}}{4}\\ a+b=2\end{cases}\Rightarrow\begin{cases}\dfrac{2a^{2013}b^{2013}}{4}=1+\dfrac{-2a^{2013}b^{2013}}{4}\\ a+b=2\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}a^{2013}b^{2013}=1\\ a+b=2\end{cases}\Rightarrow\begin{cases}ab=1\\ a+b=2\end{cases}\Rightarrow a=b=1\Rightarrow x=1$$