



Ex:1

$$\frac{1}{4} x^{2^{\frac{1}{\log_2 x}}} = 2^{\frac{1}{4} \log_2^2 x}$$

$$x \in (0; +\infty)$$

$$\log_2 \left(\frac{1}{4} x^{2^{\frac{1}{\log_2 x}}} \right) = \log_2 \left(2^{\frac{1}{4} \log_2^2 x} \right)$$

$$\log_2 \frac{1}{4} + \log_2 x^{2^{\frac{1}{\log_2 x}}} = \left(\frac{1}{4} \log_2^2 x \right) (\log_2 2)$$

$$\log_2 2^{-2} + \frac{1}{2} \log_2 x \cdot \log_2 x = \frac{1}{4} \log_2^2 x$$

$$-2 + \frac{1}{2} \log_2^2 x = \frac{1}{4} \log_2^2 x$$

$$-8 + 2 \log_2^2 x = \log_2^2 x$$

$$\log_2^2 x = 8$$

$$\log_2 x = 2\sqrt{2}$$

$$x = 2^{2\sqrt{2}}$$

$$\log_2 x = -2\sqrt{2}$$

$$x = 2^{-2\sqrt{2}}$$

$$x = \frac{1}{2^{2\sqrt{2}}}$$



Ex:2 $\log_{x+1}(x^2 - 3x + 1) = 1$

$$\begin{aligned}x^2 - 3x + 1 &> 0 \\x + 1 &> 0 \\x + 1 &\neq 1\end{aligned}$$

 \Leftrightarrow

$$\begin{aligned}x &\in \left(-\infty; \frac{3 - \sqrt{5}}{2}\right) \cup \left(\frac{3 + \sqrt{5}}{2}; +\infty\right) \\x &> -1 \\x &\neq 0\end{aligned}$$

 \Leftrightarrow

$$x \in (-1; 0) \cup \left(0; \frac{3 - \sqrt{5}}{2}\right) \cup \left(\frac{3 + \sqrt{5}}{2}; +\infty\right)$$

$$\begin{aligned}x^2 - 3x + 1 &= (x + 1)^1 \\x^2 - 4x &= 0 \\x(x - 4) &= 0 \\x_1 = 0 &\text{ nu este soluție} \\x_2 &= 4\end{aligned}$$



Ex:3

$$\log_3(x^2 - 4x + 3) = \log_3(3x + 21)$$

$$\begin{aligned}x^2 - 4x + 3 &> 0 \\ 3x + 21 &> 0\end{aligned}$$

 \Leftrightarrow

$$\begin{aligned}x &\in (-\infty; 1) \cup (3; +\infty) \\ x &\in (-7; +\infty)\end{aligned}$$

 \Leftrightarrow

$$x \in (-7; 1) \cup (3; +\infty)$$

$$x^2 - 4x + 3 = 3x + 21$$

$$x^2 - 7x - 18 = 0$$

$$\Delta = 49 + 72 = 121$$

$$x_{1,2} = \frac{7 \pm 11}{2}$$

$$x_1 = 9$$

$$x_2 = -2$$