



Semnul funcției de gradul I:

$$f : R \rightarrow R$$

$$f(x) = ax + b$$

$$a \neq 0$$

x	$-\infty$	$-\frac{b}{a}$	$+\infty$
$ax + b$	semn contrar lui a	0	semnul lui a



$$f : \mathbb{R} \rightarrow \mathbb{R}$$

Semnul funcției de gradul al II-lea:

$$f(x) = ax^2 + bx + c$$

$$a \neq 0$$

$$\Delta > 0$$

x	$-\infty$	x_1	x_2	$+\infty$
$ax^2 + bx + c$	semnul lui a	0	0	semnul lui a

$$\Delta = 0$$

x	$-\infty$	$x_1 = x_2$	$+\infty$
$ax^2 + bx + c$	semnul lui a	0	semnul lui a

$$\Delta < 0$$

x	$-\infty$	$+\infty$
$ax^2 + bx + c$	semnul lui a	



Ex.1 Să se rezolve inecuația: $x^2 - 5x + 2 \geq 0$

$$x^2 - 5x + 2 \geq 0$$

$$\Delta = 5^2 - 4 \cdot 1 \cdot 2 = 25 - 8 = 17$$

$$x_{1,2} = \frac{5 \pm \sqrt{17}}{2}$$

x	$-\infty$			$\frac{5 - \sqrt{17}}{2}$			$\frac{5 + \sqrt{17}}{2}$			$+\infty$
$x^2 - 5x + 2$	+	+	+	0	-	-	0	+	+	+

$$x \in \left(-\infty; \frac{5 - \sqrt{17}}{2}\right] \cup \left[\frac{5 + \sqrt{17}}{2}; +\infty\right)$$



Ex.2 Să se rezolve inecuația: $\frac{2x^2 - x}{x-1} \leq 2$

$$\frac{2x^2 - x}{x-1} \leq 2$$

$$2x^2 - 3x + 2 = 0$$

$$x - 1 = 0$$

$$\frac{2x^2 - x}{x-1} - 2 \leq 0$$

$$\Delta = b^2 - 4ac = 9 - 16 = -7$$

$$x = 1$$

$$\frac{2x^2 - x - 2x + 2}{x-1} \leq 0$$

$$\frac{2x^2 - 3x + 2}{x-1} \leq 0$$

x	$-\infty$		1		$+\infty$
$2x^2 - 3x + 2$	+	+	+	+	+
$x - 1$	-	-	0	+	+
$\frac{2x^2 - 3x + 2}{x-1}$	-	-		+	+

$$x \in (-\infty; 1)$$



Ex.3 Să se rezolve inecuația: $\frac{x^2 - 8x + 7}{x^2 - 12x + 20} < 0$

$$\frac{x^2 - 8x + 7}{x^2 - 12x + 20} < 0$$

$$x^2 - 8x + 7 = 0$$

$$x_{1,2} = \frac{8 \pm \sqrt{64 - 28}}{2} = \frac{8 \pm 6}{2}$$

$$x_1 = 7$$

$$x_2 = 1$$

$$x^2 - 12x + 20 = 0$$

$$x_{1,2} = \frac{12 \pm \sqrt{144 - 80}}{2} = \frac{12 \pm 8}{2}$$

$$x_1 = 10$$

$$x_2 = 2$$

x	$-\infty$		1		2		7		10		$+\infty$
$x^2 - 8x + 7$	+	+	0	-	-	-	0	+	+	+	+
$x^2 - 12x + 20$	+	+	+	+	0	-	-	-	0	+	+
$\frac{x^2 - 8x + 7}{x^2 - 12x + 20}$	+	+	0	-		+	0	-		+	+

$$x \in (1;2) \cup (7;10)$$



Ex.4 Să se rezolve inecuația: $\frac{2x^2 - 6x - 36}{-x^2 - x + 2} \leq 0$

$$\frac{2x^2 - 6x - 36}{-x^2 - x + 2} \leq 0$$

$$\frac{x^2 - 3x - 18}{-x^2 - x + 2} \leq 0$$

$$x^2 - 3x - 18 = 0$$

$$x_{1,2} = \frac{3 \pm \sqrt{9 + 72}}{2} = \frac{3 \pm 9}{2}$$

$$x_1 = 6$$

$$x_2 = -3$$

$$-x^2 - x + 2 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{1 + 8}}{-2} = \frac{1 \pm 3}{-2}$$

$$x_1 = -2$$

$$x_2 = 1$$

x	$-\infty$		-3		-2		1		6		$+\infty$
$x^2 - 3x - 18$	+	+	0	-	-	-	-	-	0	+	+
$-x^2 - x + 2$	-	-	-	-	0	+	0	-	-	-	-
$\frac{x^2 - 3x - 18}{-x^2 - x + 2}$	-	-	0	+		-		+	0	-	-

$$x \in (-\infty; -3] \cup (-2; -1) \cup [6; +\infty)$$



Ex.5 Să se rezolve inecuația: $\frac{x^2 - 16}{x^2 - 2x - 24} \leq 0$

$$\frac{x^2 - 16}{x^2 - 2x - 24} \leq 0$$

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$x_{1,2} = \pm 4$$

$$x^2 - 2x - 24 = 0$$

$$\Delta = 4 + 96 = 100$$

$$x_{1,2} = \frac{2 \pm 10}{2}$$

$$x_1 = 6$$

$$x_2 = -4$$

x	-∞		-4	4		6	+∞			
$x^2 - 16$	+	+	0	-	-	0	+	+	+	+
$x^2 - 2x - 24$	+	+	0	-	-	-	-	0	+	+
$\frac{x^2 - 16}{x^2 - 2x - 24}$	+	+		+	+	0	-		+	+

$$x \in [4;6)$$



Ex.6 Să se rezolve inecuația: $\frac{2x^2 + x - 15}{-x^2 + 15x - 50} \geq 0$

$$\frac{2x^2 + x - 15}{-x^2 + 15x - 50} \geq 0$$

$$2x^2 + x - 15 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 + 120}}{4} = \frac{-1 \pm 11}{4}$$

$$x_1 = \frac{10}{4} = \frac{5}{2}$$

$$x_2 = \frac{-12}{4} = -3$$

$$-x^2 + 15x - 50 = 0$$

$$\Delta = 225 - 200 = 25$$

$$x_{1,2} = \frac{-15 \pm 5}{-2}$$

$$x_1 = 5$$

$$x_2 = 10$$

x	$-\infty$		-3		$\frac{5}{2}$		5		10		$+\infty$
$2x^2 + x - 15$	+	+	0	-	0	+	+	+	+	+	+
$-x^2 + 15x - 50$	-	-	-	-	-	-	0	+	0	-	-
$\frac{2x^2 + x - 15}{-x^2 + 15x - 50}$	-	-	0	+	0	-		+		-	-

$$x \in [-3; \frac{5}{2}] \cup (5; 10)$$



Ex.7 Să se rezolve inecuația: $\frac{x^2 - 8x + 15}{x^2 - 3x - 4} < 0$

$$\frac{x^2 - 8x + 15}{x^2 - 3x - 4} < 0$$

$$x^2 - 8x + 15 = 0$$

$$\Delta = 64 - 60 = 4$$

$$x_{1,2} = \frac{8 \pm 2}{2}$$

$$x_1 = 5$$

$$x_2 = 3$$

$$x^2 - 3x - 4 = 0$$

$$\Delta = 9 + 16 = 25$$

$$x_{1,2} = \frac{3 \pm 5}{2}$$

$$x_1 = 4$$

$$x_2 = -1$$

x	$-\infty$		-1		3		4		5		$+\infty$
$x^2 - 8x + 15$	+	+	+	+	0	-	-	-	0	+	+
$x^2 - 3x - 4$	+	+	0	-	-	-	0	+	+	+	+
$\frac{x^2 - 8x + 15}{x^2 - 3x - 4}$	+	+		-	0	+		-	0	+	+

$$x \in (-1;3) \cup (4;5)$$



Ex.8 Să se rezolve inecuația: $\frac{x^2 - 7x + 15}{x^2 + 7x - 18} \leq 0$

$$\frac{x^2 - 7x + 15}{x^2 + 7x - 18} \leq 0$$

$$x^2 - 7x + 15 = 0$$

$$\Delta = 49 - 60 = -11 < 0$$

$$x^2 + 7x - 18 = 0$$

$$\Delta = 49 + 72 = 121$$

$$x_{1,2} = \frac{-7 \pm 11}{2}$$

$$x_1 = \frac{4}{2} = 2$$

$$x_2 = \frac{-18}{2} = -9$$

x	$-\infty$		-9				2		$+\infty$
$x^2 - 7x + 15$	+	+	+	+	+	+	+	+	+
$x^2 + 7x - 18$	+	+	0	-	-	-	0	+	+
$\frac{x^2 - 7x + 15}{x^2 + 7x - 18}$	+	+		-	-	-		+	+

$$x \in (-9; 2)$$



Ex.9 Să se rezolve inecuația: $\frac{x^2 - x - 6}{x^2 - x - 2} \geq 0$

$$\frac{x^2 - x - 6}{x^2 - x - 2} \geq 0$$

$$x^2 - x - 6 = 0$$

$$\Delta = 1 + 24 = 25$$

$$x_{1,2} = \frac{1 \pm 5}{2}$$

$$x_1 = 3$$

$$x_2 = -2$$

$$x^2 - x - 2 = 0$$

$$\Delta = 1 + 8 = 9$$

$$x_{1,2} = \frac{1 \pm 3}{2}$$

$$x_1 = 2$$

$$x_2 = -1$$

x	$-\infty$		-2		-1		2		3		$+\infty$
$x^2 - x - 6$	+	+	0	-	-	-	-	-	0	+	+
$x^2 - x - 2$	+	+	+	+	0	-	0	+	+	+	+
$\frac{x^2 - x - 6}{x^2 - x - 2}$	+	+	0	-		+		-	0	+	+

$$x \in (-\infty; -2] \cup (-1; 2) \cup [3; +\infty)$$



Ex.10 Să se rezolve inecuația: $\frac{-x^2 + 3x + 18}{x - 4} > 0$

$$\frac{-x^2 + 3x + 18}{x - 4} > 0$$

$$-x^2 + 3x + 18 = 0$$

$$\Delta = 9 + 72 = 81$$

$$x_{1,2} = \frac{-3 \pm \sqrt{81}}{-2} = \frac{-3 \pm 9}{-2}$$

$$x_1 = \frac{6}{-2} = -3$$

$$x_2 = \frac{-12}{-2} = 6$$

$$x - 4 = 0$$

$$x = 4$$

x	$-\infty$		-3		4		6		$+\infty$	
$-x^2 + 3x + 18$	-	-	0	+	+	+	0	-	-	-
$x - 4$	-	-	-	-	0	+	+	+	+	+
$\frac{-x^2 + 3x + 18}{x - 4}$	+	+	0	-		+	0	-	-	-

$$x \in (-\infty; -3) \cup (4; 6)$$



Ex11 Să se rezolve inecuația: $\frac{-x^4 + 4x^2 - 4}{x^3 - 4x} \leq 0$

$$\frac{-x^4 + 4x^2 - 4}{x^3 - 4x} \leq 0$$

$$\frac{x^4 - 4x^2 + 4}{x^3 - 4x} \geq 0$$

$$\frac{(x^2 - 2)^2}{x(x^2 - 4)} \geq 0$$

$$(x^2 - 2)^2 = 0$$

$$x^2 - 2 = 0$$

$$x^2 = 2$$

$$x_1 = x_2 = \sqrt{2}$$

$$x_3 = x_4 = -\sqrt{2}$$

$$x = 0$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x_{1,2} = \pm 2$$

x	-∞		-2		-√2		0		√2		2		+∞	
$(x^2 - 2)^2$	+	+	+	+	0	+	+	+	0	+	+	+	+	+
x	-	-	-	-	-	-	0	+	+	+	+	+	+	+
$x^2 - 4$	+	+	0	-	-	-	-	-	-	-	0	+	+	+
$\frac{(x^2 - 2)^2}{x(x^2 - 4)}$	-	-		+	0	+		-	0	-		+	+	+

$$x \in (-2; 0) \cup \{\sqrt{2}\} \cup (2; +\infty)$$