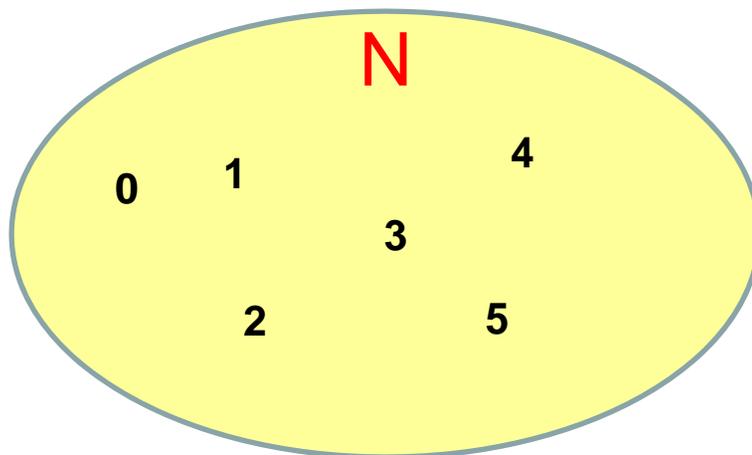




## Mulțimea numerelor naturale

$$N = \{0;1;2;3;4;\dots\}$$

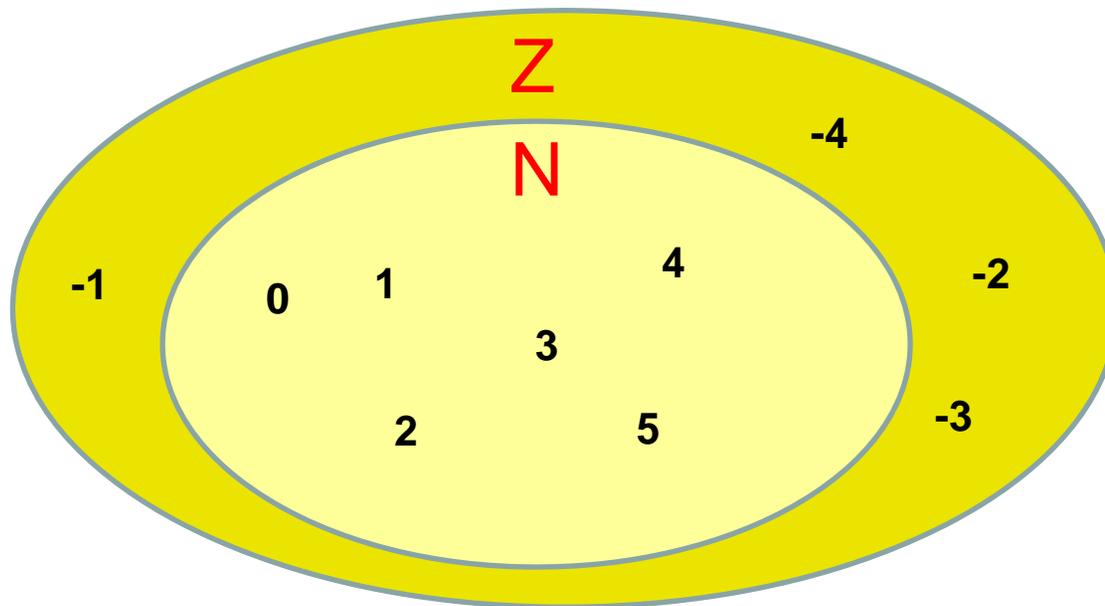
$$N^* = \{1;2;3;4;\dots\}$$





## Mulțimea numerelor întregi

$$\mathbb{Z} = \{ \dots; -3; -2; -1; 0; 1; 2; 3; \dots \}$$





Rezolvați ecuația în numerelor întregi:  $x^2 - 3x(y^2 + y - 1) + 4y^2 + 4y - 6 = 0$

$$x^2 - 3x(y^2 + y - 1) + 4y^2 + 4y - 6 = 0$$

$$x^2 - 3xy^2 - 3xy + 3x + 4y^2 + 4y - 6 = 0$$

$$(4 - 3x)y^2 + (4 - 3x)y + x^2 + 3x - 6 = 0$$

$$(4 - 3x)y^2 + (4 - 3x)y = -x^2 - 3x + 6$$

$$y^2 + y = \frac{x^2 + 3x - 6}{3x - 4} \in \mathbb{Z}$$

$$\frac{x^2 + 3x - 6}{3x - 4} = \frac{x^2 - 2 + 3x - 4}{3x - 4} = \frac{x^2 - 2}{3x - 4} + 1 = \frac{1}{3} \left( \frac{3x^2 - 6}{3x - 4} + 1 \right) = \frac{1}{3} \left( \frac{3x^2 - 6}{3x - 4} \right) + 1 = \frac{1}{3} \left( \frac{3x^2 - 4x + 4x - 6}{3x - 4} \right) + 1 =$$

$$= \frac{1}{3} \left( x + \frac{1}{3} \cdot \frac{12x - 18}{3x - 4} \right) + 1 = \frac{1}{3} \left( x + \frac{1}{3} \cdot \frac{12x - 16 - 2}{3x - 4} \right) + 1 = \frac{1}{3} \left[ x + \frac{1}{3} \cdot \left( 4 - \frac{2}{3x - 4} \right) \right] + 1 \in \mathbb{Z} \Rightarrow$$

$$\Rightarrow (3x - 4) \in \{\pm 1; \pm 2\}$$

$$\downarrow$$

$$x \in \{1; 2\}$$

$$y^2 + y = 2 \Rightarrow y^2 + y - 2 = 0$$

$$\downarrow$$

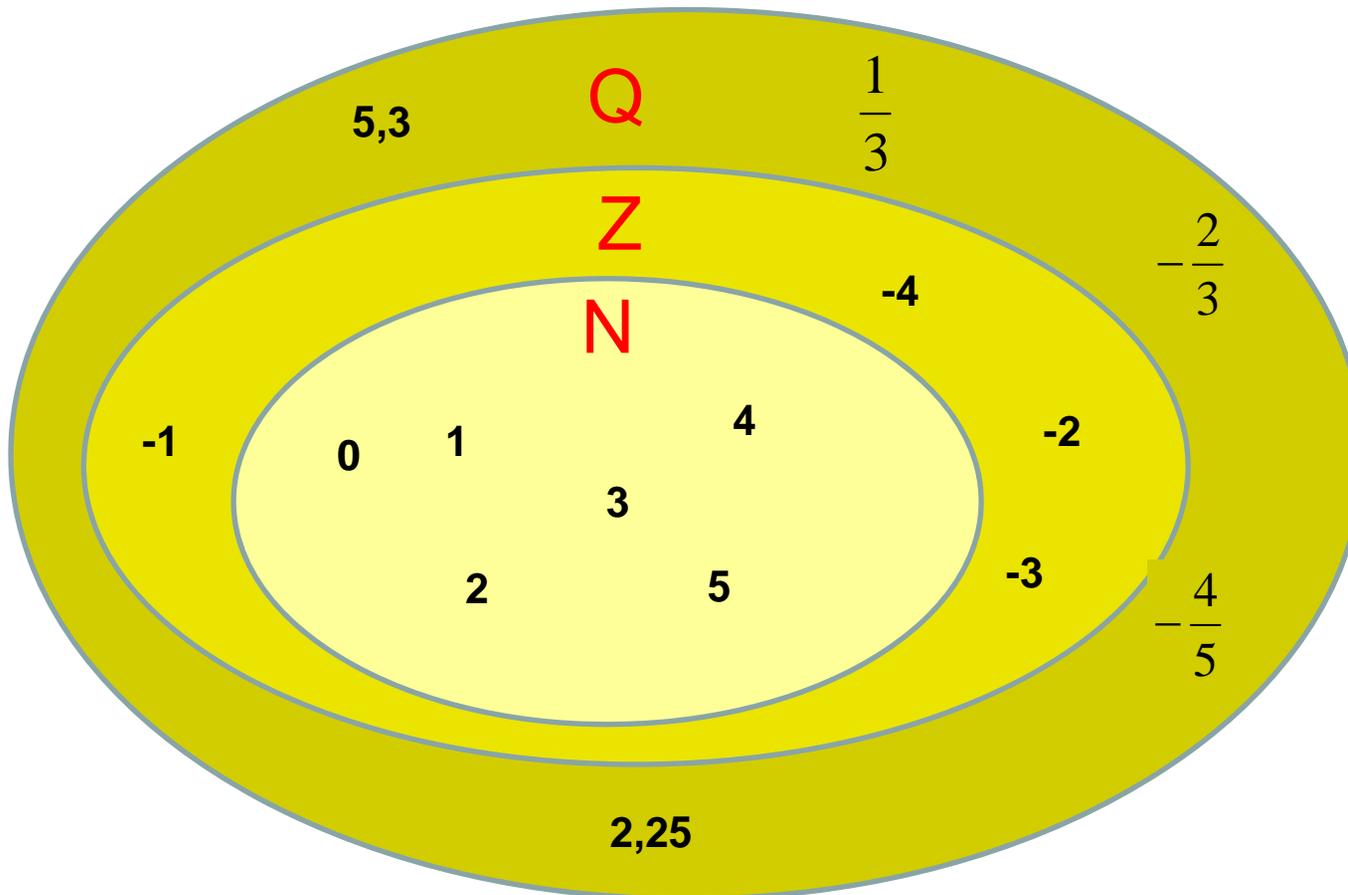
$$y \in \{-2; 1\}$$

$$S = \{(1; -2); (1; 1); (2; -2); (2; 1); \}$$



Mulțimea numerelor raționale

$$Q = \left\{ \frac{m}{n} / m; n \in Z; n \neq 0 \right\}$$





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1a)  $3 = \frac{3}{1} = 3,0000$

1e)  $-\frac{1}{4} = -0,25$

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1b)  $\frac{15}{8} = 1,875$

1f)  $-\frac{2}{3} = -0,(6) \approx -0,6666$

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1c)  $\frac{1}{4} = 0,25$

1g)  $-\frac{29}{11} = -2,(63) \approx -2,6363$

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1d)  $0 = \frac{0}{1} = 0,0000$

1h)  $-\frac{123}{13} = -9,(461538) \approx -9,461538461538$

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$$2a) \quad 1,33(4) = \frac{1334 - 133}{900} = \frac{1201}{900} \approx 1,3344$$

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$$2b) \quad 0,(7) = \frac{7}{9} \approx 0,7777$$

---

$$2c) \quad -0,(14) = -\frac{14}{99} \approx -0,1414$$

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$$2d) \quad 2,073(83) = \frac{207383 - 2073}{99000} = \frac{205310}{99000} = \frac{20531}{9900} \approx 2,0738383$$

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$$2e) \quad -0,01(023) = -\frac{1023 - 1}{99900} = -\frac{1022}{99900} \approx -0,01023023$$

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$$2f) \quad -2,001(7) = -\frac{20017 - 2001}{9000} = -\frac{18016}{9000} \approx -2,00177$$



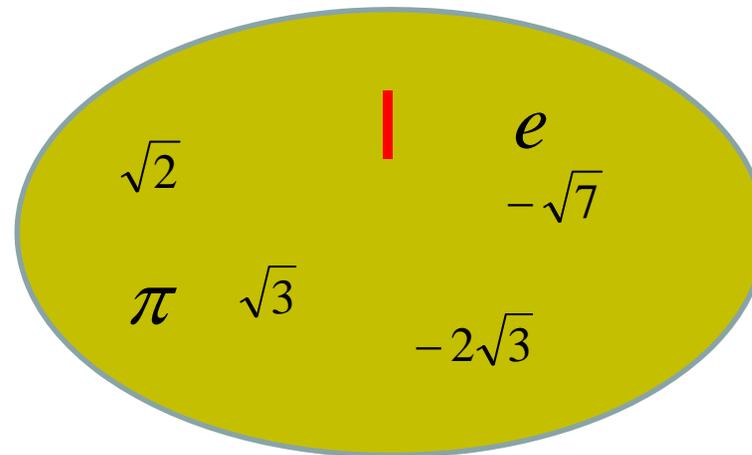
Mulțimea numerelor iraționale: ***I***

Ex:

$$\sqrt{2} = 1,4142135\dots$$

$$\pi = 3,1415926\dots$$

$$e = 2,7182818\dots$$





Mulțimea numerelor reale:  $R = Q \cup I$

Partea întreaga și partea fracțională a unui număr real:

$$x = [x] + \{x\} \quad \forall x \in R$$

Ex 1:

$$\begin{aligned} [\sqrt{1}] + [\sqrt{2}] + \dots + [\sqrt{30}] &= [\sqrt{1}] + [\sqrt{2}] + \sqrt{3} + [\sqrt{4}] + [\sqrt{5}] + \dots + [\sqrt{8}] + \\ &+ [\sqrt{9}] + [\sqrt{10}] + \dots + [\sqrt{15}] + [\sqrt{16}] + [\sqrt{17}] + \dots + [\sqrt{24}] + [\sqrt{25}] + \dots + [\sqrt{30}] = \\ &= 1 \cdot 3 + 5 \cdot 2 + 7 \cdot 3 + 9 \cdot 4 + 6 \cdot 5 = 3 + 10 + 21 + 36 + 30 = 100 \end{aligned}$$



Ex 2:

$$\left[ \frac{1}{50} \right] + \left[ \frac{2}{50} \right] + \dots + \left[ \frac{99}{50} \right] = \left[ \frac{1}{50} \right] + \left[ \frac{2}{50} \right] + \dots + \left[ \frac{49}{50} \right] + \left[ \frac{50}{50} \right] + \dots + \left[ \frac{99}{50} \right] =$$

$$= 49 \cdot 0 + 50 \cdot 1 = 50$$

Ex 3:

$$\{x\} = x - [x]$$

$$\left\{ \frac{1}{50} \right\} + \left\{ \frac{2}{50} \right\} + \dots + \left\{ \frac{99}{50} \right\} = \frac{1}{50} - \left[ \frac{1}{50} \right] + \frac{2}{50} - \left[ \frac{2}{50} \right] + \dots + \frac{99}{50} - \left[ \frac{99}{50} \right] =$$

$$= \frac{1 + 2 + \dots + 99}{50} - 50 = \frac{99 \cdot 100}{2 \cdot 50} - 50 = 99 - 50 = 49$$



Ex 4:

$$[(\sqrt{3} - \sqrt{2})^2] = [(3 - 2\sqrt{6} + 2)] = [5 - 2\sqrt{6}] = 5 + [-2\sqrt{6}] = 5 - 5 = 0$$

$$-2\sqrt{6} = -2 \cdot 2,4 = -4,8 \Rightarrow [-2\sqrt{6}] = -5$$

Ex 5:

$$\{x\} = x - [x]$$

$$\{(\sqrt{3} - \sqrt{2})^2\} = (\sqrt{3} - \sqrt{2})^2 - [(\sqrt{3} - \sqrt{2})^2] = 3 - 2\sqrt{6} + 2 - 0 = 5 - 2\sqrt{6}$$



Ex 6:

$$\begin{aligned}
 [(\sqrt{n+2} + \sqrt{n+5})^2] &= [n+2 + 2\sqrt{(n+2)(n+5)} + n+5] = \\
 &= [2n+7 + \sqrt{4(n^2+7n+10)}] = 2n+7 + [\sqrt{4n^2+28n+40}] = \\
 &= 2n+7 + 2n+6 = 4n+13
 \end{aligned}$$

$$4n^2 + 24n + 36 \leq 4n^2 + 28n + 40 < 4n^2 + 28n + 49$$

$$(2n+6)^2 \leq 4n^2 + 28n + 40 < (2n+7)^2$$

$$2n+6 \leq \sqrt{4n^2 + 28n + 40} < 2n+7 \Rightarrow [\sqrt{4n^2 + 28n + 40}] = 2n+6$$



Ex 7:

$$[\sqrt{(n+1)(n+2)}] = [\sqrt{n^2 + 3n + 2}] = n + 1$$

$$n^2 + 2n + 1 \leq n^2 + 3n + 2 < n^2 + 4n + 4$$

$$(n+1)^2 \leq n^2 + 3n + 2 < (n+2)^2$$

$$n+1 \leq \sqrt{n^2 + 3n + 2} < n+2 \Rightarrow [\sqrt{n^2 + 3n + 2}] = n + 1$$



Ex 8

Determinați ordinul mulțimii:  $A = \{x / x = \frac{n^2 + 1}{2n^2 + n + 1}; n \in \{1; 2, \dots; 100\}\}$

Dem.

$\exists k; n \in \{1; 2, \dots; 100\}$

$k \neq n$

astfel încât

$$\frac{n^2 + 1}{2n^2 + n + 1} = \frac{k^2 + 1}{2k^2 + k + 1}$$

$$(n^2 + 1)(k^2 + k + 1) = (k^2 + 1)(2n^2 + n + 1)$$

$$2k^2n^2 + kn^2 + n^2 + 2k^2 + k + 1 = 2k^2n^2 + k^2n + k^2 + 2n^2 + n + 1$$

$$kn^2 + n^2 + 2k^2 + k - k^2n - k^2 - 2n^2 - n = 0$$

$$(kn^2 - k^2n) - (n^2 - k^2) - (n - k) = 0$$

$$kn(n - k) - (n - k)(n + k) - (n - k) = 0$$

$$kn - (n + k) - 1 = 0$$

$$kn - n - k - 1 = 0$$

$$kn - n - k + 1 - 2 = 0$$

$$(n - 1)(k - 1) = 2$$

$$1 \cdot 2 = 2$$

$$2 \cdot 1 = 2$$

$$n=3; k=2$$

sau

$$n=2; k=3$$

Mulțimea are 99 de elemente!