Inversa functiilor trigonometrice

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$$\sin(\arcsin x) = x, \forall x \in [-1;1]$$

$$\sin(\arccos x) = \sqrt{1-x^2}, \forall x \in [-1;1]$$

$$\sin(arctgx) = \frac{x}{\sqrt{1+x^2}}, \forall x \in R$$

$$\sin(arcctgx) = \frac{1}{\sqrt{1+x^2}}, \forall x \in R$$

$$\cos(\arcsin x) = \sqrt{1-x^2}, \forall x \in [-1;1]$$

$$\cos(\arccos x) = x, \forall x \in [-1;1]$$

$$\cos(arctgx) = \frac{1}{\sqrt{1+x^2}}, \forall x \in R$$

$$\cos(arcctgx) = \frac{x}{\sqrt{1+x^2}}, \forall x \in R$$

$$tg(\arcsin x) = \frac{x}{\sqrt{1-x^2}}, \forall x \in (-1;1)$$

$$tg(\arccos x) = \frac{\sqrt{1-x^2}}{x}, \forall x \in [-1;1]-\{0\}$$

$$tg(arctgx) = x, \forall x \in R$$

$$tg(arcctgx) = \frac{1}{x}, \forall x \in R-\{0\}$$

$$ctg(\arcsin x) = \frac{\sqrt{1-x^2}}{x}, \forall x \in [-1;1]-\{0\}$$

$$ctg(\arccos x) = \frac{x}{\sqrt{1-x^2}}, \forall x \in (-1;1)$$

$$ctg(arctgx) = \frac{1}{x}, \forall x \in R-\{0\}$$

$$ctg(arcctgx) = x, \forall x \in R$$

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Ex. Calculaţi: $\sin(\arcsin\frac{3}{5}+\arccos\frac{4}{5})$

$$\sin(\arcsin\frac{3}{5}+\arccos\frac{4}{5})=\sin(\arccos\frac{3}{5})\cos(\arccos\frac{4}{5})+\sin(\arccos\frac{4}{5})\cos(\arcsin\frac{3}{5})=$$

$$=\frac{3}{5}\cdot\frac{4}{5}+\sqrt{1-\frac{16}{25}}\cdot\sqrt{1-\frac{9}{25}}=\frac{12}{25}+\frac{3}{5}\cdot\frac{4}{5}=\frac{24}{25}$$

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Ex. Calculaţi: $\cos\left(\arcsin\frac{5}{13}-\arccos\frac{12}{13}\right)$

$$\cos\left(\arcsin\frac{5}{13}-\arccos\frac{12}{13}\right)=\cos\left(\arcsin\frac{5}{13}\right)\cos\left(\arccos\frac{12}{13}\right)+\sin\left(\arcsin\frac{5}{13}\right)\sin\left(\arccos\frac{12}{13}\right)=$$

$$=\sqrt{1-\frac{25}{169}}\cdot\frac{12}{13}+\frac{5}{13}\cdot\sqrt{1-\frac{144}{169}}=\frac{144}{169}+\frac{25}{169}=1$$

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Ex. Calculaţi: $tg(arctg3-arctg2)$

$$tg(arctg3-arctg2)=\frac{tg(arctg3)-tg(arctg2)}{1+tg(arctg3)\cdot tg(arctg2)}=\frac{3-2}{1+3\cdot 2}=\frac{1}{7}$$

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Ex. Arătaţi egalitatea: $arctg\frac{1}{3}+arctg\frac{1}{5}+arctg\frac{1}{7}+arctg\frac{1}{8}=\frac{\pi}{4}$

$$tg[arctg\frac{1}{3}+arctg\frac{1}{5}]=\frac{\frac{1}{3}+\frac{1}{5}}{1-\frac{1}{3}\cdot\frac{1}{5}}=\frac{\frac{5+3}{15}}{\frac{15-1}{15}}=\frac{8}{14}=\frac{4}{7}$$

$$tg[arctg\frac{1}{7}+arctg\frac{1}{8}]=\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\cdot\frac{1}{8}}=\frac{\frac{8+7}{56}}{\frac{56-1}{56}}=\frac{15}{55}=\frac{3}{11}$$

$$tg[arctg\frac{1}{3}+arctg\frac{1}{5}+arctg\frac{1}{7}+arctg\frac{1}{8}]=tg\frac{\pi}{4}$$

$$\frac{tg[arctg\frac{1}{3}+arctg\frac{1}{5}]+tg[arctg\frac{1}{7}+arctg\frac{1}{8}]}{1-tg[arctg\frac{1}{3}+arctg\frac{1}{5}]\cdot tg[arctg\frac{1}{7}+arctg\frac{1}{8}]}=1$$

$$\frac{\frac{4}{7}+\frac{3}{11}}{1-\frac{4}{7}\cdot\frac{3}{11}}=1$$

$$\frac{\frac{44+21}{77}}{\frac{77-12}{77}}=1$$

$$\frac{65}{65}=1$$

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Ex.

a) Demonstraţi identitatea: $arcctgx - arcctg(x+1) = arcctg(1+x+x^2)$

b) Calculaţi : $S_n = \sum_{k=1}^{n} arcctg(k^2+k+1)$

$$arcctgx - arcctg(x+1) = arcctg(1+x+x^2)$$

$$ctg[arcctgx - arcctg(x+1)] = ctg[arcctg(1+x+x^2)]$$

$$\frac{x(x+1)+1}{(x+1)-x} = 1+x+x^2$$

$$\frac{x^2+x+1}{x+1-x} = 1+x+x^2$$

$$x^2+x+1 = 1+x+x^2$$

$$S_n = \sum_{k=1}^{n} arcctg(k^2+k+1) = \sum_{k=1}^{n} [arcctg(k) - arcctg(k+1)] =$$

$$= (arcctg1 - arcctg2) + (arcctg2 - arcctg3) + \ldots +$$

$$+ [arcctg(n) - arcctg(n+1)] = \frac{\pi}{4} - arcctg(n+1)$$

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Ex.

a) Demonstraţi identitatea: $arcctg(2x-1)-arcctg(2x+1)=arcctg(2x^2)$

b) Calculaţi : $S_n=\sum_{k=1}^{n} arcctg(2k^2)$

$$arcctg(2x-1)-arcctg(2x+1)=arcctg(2x^2)$$

$$ctg[arcctg(2x-1)-arcctg(2x+1)]=ctg[arcctg(2x^2)]$$

$$\frac{(2x-1)(2x+1)+1}{(2x+1)-(2x-1)}=2x^2$$

$$\frac{4x^2-1+1}{2x+1-2x+1}=2x^2$$

$$\frac{4x^2}{2}=2x^2$$

$$S_n=\sum_{k=1}^{n} arcctg(2k^2)=\sum_{k=1}^{n}[arcctg(2k-1)-arcctg(2k+1)]=$$

$$=(arcctg1-arcctg3)+(arcctg3-arcctg5)+\ldots+$$

$$+[arcctg(2n-1)-arcctg(2n+1)]=\frac{\pi}{4}-arcctg(2n+1)$$

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Ex.

a) Demonstraţi identitatea: $arctg\frac{x}{x+1} - arctg\frac{x-1}{x} = arctg\frac{1}{2x^2}$

b) Calculaţi : $S_n = \sum_{k=1}^{n} arctg\frac{1}{2k^2}$

$$arctg\frac{x}{x+1} - arctg\frac{x-1}{x} = arctg\frac{1}{2x^2}$$

$$tg(arctg\frac{x}{x+1} - arctg\frac{x-1}{x}) = tg(arctg\frac{1}{2x^2})$$

$$\frac{\frac{x}{x+1} - \frac{x-1}{x}}{1+\frac{x}{x+1}\cdot\frac{x-1}{x}} = \frac{1}{2x^2}$$

$$\frac{\frac{x^2 - x^2 + 1}{x(x+1)}}{\frac{x^2 + x + x^2 - x}{x(x+1)}} = \frac{1}{2x^2}$$

$$\frac{1}{2x^2} = \frac{1}{2x^2}$$

$$S_n = \sum_{k=1}^{n} arctg\frac{1}{2k^2} = \sum_{k=1}^{n}(arctg\frac{k}{k+1} - arctg\frac{k-1}{k}) =$$

$$= (arctg\frac{1}{2} - arctg0) + (arctg\frac{2}{3} - arctg\frac{1}{2}) + ... +$$

$$+ (arctg\frac{n}{n+1} - arctg\frac{n-1}{n}) = arctg\frac{n}{n+1} - arctg0 =$$

$$= arctg\frac{n}{n+1}$$