

Formule pentru transformarea sumelor în produse

molnar1956@yahoo.com



Ex.

Transformați în produse: $\cos 2x + \cos 4x + \cos 6x$

$$\begin{aligned}\cos 2x + \cos 4x + \cos 6x &= 2 \cos \frac{2x+6x}{2} \cos \frac{2x-6x}{2} + \cos 4x = 2 \cos 4x \cos 2x + \cos 4x = \\ &= 2 \cos 4x \cdot \left(\cos 2x + \frac{1}{2}\right) = 2 \cos 4x \cdot \left(\cos 2x + \cos \frac{\pi}{3}\right) = 2 \cos 4x \cdot 2 \cos \frac{2x + \frac{\pi}{3}}{2} \cos \frac{2x - \frac{\pi}{3}}{2} = \\ &= 4 \cos 4x \cos\left(x + \frac{\pi}{6}\right) \cos\left(x - \frac{\pi}{6}\right)\end{aligned}$$

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Ex.

Transformați în produse: $1 + \cos x + \cos 2x$

$$\begin{aligned} 1 + \cos x + \cos 2x &= (1 + \cos 2x) + \cos x = 2 \cos^2 x + \cos x = 2 \cos x \left[\cos x + \frac{1}{2} \right] = \\ &= 2 \cos x \left[\cos x + \cos \frac{\pi}{3} \right] = 2 \cos x \cdot 2 \cdot \cos \frac{x + \frac{\pi}{3}}{2} \cos \frac{x - \frac{\pi}{3}}{2} = 4 \cos x \cos \left(\frac{x}{2} + \frac{\pi}{6} \right) \cos \left(\frac{x}{2} - \frac{\pi}{6} \right) \end{aligned}$$

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Ex.

Transformați în produse: $\sin x + \sin 3x + \sin 5x$

$$\begin{aligned}\sin x + \sin 3x + \sin 5x &= (\sin x + \sin 5x) + \sin 3x = 2 \sin \frac{x+5x}{2} \cos \frac{x-5x}{2} + \sin 3x = \\ &= 2 \sin 3x \cos 2x + \sin 3x = 2 \sin 3x \cdot \left[\cos 2x + \frac{1}{2} \right] = 2 \sin 3x \cdot \left[\cos 2x + \cos \frac{\pi}{3} \right] = \\ &= 2 \sin 3x \cdot 2 \cos \frac{2x + \frac{\pi}{3}}{2} \cos \frac{2x - \frac{\pi}{3}}{2} = 4 \sin 3x \cdot \cos \left(x + \frac{\pi}{6} \right) \cos \left(x - \frac{\pi}{6} \right)\end{aligned}$$

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Ex.

Transformați în produse: $1 + \sin x + \cos x$

$$\begin{aligned}1 + \sin x + \cos x &= 2 \sin \frac{x}{2} \cos \frac{x}{2} + 2 \cos^2 \frac{x}{2} = 2 \cos \frac{x}{2} \cdot (\sin \frac{x}{2} + \cos \frac{x}{2}) = 2 \cos \frac{x}{2} \cdot [\cos(\frac{\pi}{2} - \frac{x}{2}) + \cos \frac{x}{2}] = \\&= 2 \cos \frac{x}{2} \cdot 2 \cos \frac{\frac{\pi}{2} - \frac{x}{2} + \frac{x}{2}}{2} \cos \frac{\frac{\pi}{2} - \frac{x}{2} - \frac{x}{2}}{2} = 4 \cos \frac{x}{2} \cos \frac{\pi}{4} \cos(\frac{\pi}{4} - \frac{x}{2}) = 4 \cdot \frac{\sqrt{2}}{2} \cos \frac{x}{2} \cos(\frac{\pi}{4} - \frac{x}{2}) = \\&= 2\sqrt{2} \cos \frac{x}{2} \cos(\frac{\pi}{4} - \frac{x}{2})\end{aligned}$$

$$\cos \alpha = \pm \sqrt{\frac{1 + \cos 2\alpha}{2}}$$

$$\sin \alpha = \pm \sqrt{\frac{1 - \cos 2\alpha}{2}}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

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Ex.

Transformați în produse: $1 - \sin x - \cos x$

$$\begin{aligned}1 - \sin x - \cos x &= -2 \sin \frac{x}{2} \cos \frac{x}{2} + 2 \sin^2 \frac{x}{2} = 2 \sin \frac{x}{2} \cdot (\sin \frac{x}{2} - \cos \frac{x}{2}) = 2 \sin \frac{x}{2} \cdot [\cos(\frac{\pi}{2} - \frac{x}{2}) - \cos \frac{x}{2}] = \\ &= 2 \sin \frac{x}{2} \cdot [-2 \sin \frac{\frac{\pi}{2} - \frac{x}{2} + \frac{x}{2}}{2} \sin \frac{\frac{\pi}{2} - \frac{x}{2} - \frac{x}{2}}{2}] = -4 \sin \frac{x}{2} \sin \frac{\pi}{4} \sin(\frac{\pi}{4} - \frac{x}{2}) = -4 \cdot \frac{\sqrt{2}}{2} \sin \frac{x}{2} \sin(\frac{\pi}{4} - \frac{x}{2}) = \\ &= -2\sqrt{2} \sin \frac{x}{2} \sin(\frac{\pi}{4} - \frac{x}{2})\end{aligned}$$

$$\cos \alpha = \pm \sqrt{\frac{1 + \cos 2\alpha}{2}}$$

$$\sin \alpha = \pm \sqrt{\frac{1 - \cos 2\alpha}{2}}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

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Ex.

Transformați în produse: $\sin^2 2x - \sin^2 3x$

$$\begin{aligned}\sin^2 2x - \sin^2 3x &= \frac{1 - \cos 4x}{2} - \frac{1 - \cos 6x}{2} = \frac{\cos 6x - \cos 4x}{2} = \\ &= -\frac{1}{2} \cdot 2 \cdot \sin \frac{6x + 4x}{2} \cdot \sin \frac{6x - 4x}{2} = -\sin 5x \cdot \sin x\end{aligned}$$

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Ex.

Fie numerele reale x, y și z . Arătați că:

$$\sin x + \sin y - \sin(x + y) = 4 \sin \frac{x}{2} \sin \frac{y}{2} \sin \frac{x + y}{2}$$

$$\begin{aligned} \sin x + \sin y - \sin(x + y) &= 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2} - 2 \sin \frac{x + y}{2} \cos \frac{x + y}{2} = \\ &= 2 \sin \frac{x + y}{2} \left[\cos \frac{x - y}{2} - \cos \frac{x + y}{2} \right] = -2 \sin \frac{x + y}{2} \cdot 2 \cdot \sin \frac{x - y}{2} \sin \frac{x + y}{2} = \\ &= -4 \sin \frac{x + y}{2} \sin \frac{x}{2} \sin \frac{-y}{2} = 4 \sin \frac{x}{2} \sin \frac{y}{2} \sin \frac{x + y}{2} \end{aligned}$$