Ex. Demonstraţi identitatea: $\dfrac{\sin 2x}{\cos x + \cos^2 x} = 2tg\,\dfrac{x}{2}$

$$\frac{\sin 2x}{\cos x + \cos^2 x} = 2tg\,\frac{x}{2}$$

$$\frac{2\sin x \cdot \cos x}{\cos x(1+\cos x)} = 2tg\,\frac{x}{2}$$

$$\frac{\sin x}{1+\cos x} = tg\,\frac{x}{2}$$

Ex.　Demonstraţi identitatea: $\dfrac{tgx}{tg\,2x - tgx} = \cos 2x$

$$\frac{tgx}{\dfrac{2tgx}{1-tg^2x}-tgx} = \cos 2x \Leftrightarrow \frac{tgx}{tgx\left(\dfrac{2}{1-tg^2x}-1\right)} = \cos 2x \Leftrightarrow \frac{1}{\dfrac{2}{1-tg^2x}-1} = \cos 2x \Leftrightarrow \frac{1}{\dfrac{2-1+tg^2x}{1-tg^2x}} = \cos 2x \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{\dfrac{1+tg^2x}{1-tg^2x}} = \cos 2x \Leftrightarrow \frac{1-tg^2x}{1+tg^2x} = \cos 2x$$

Ex.　　Demonstraţi identitatea: $\quad ctg\,2x + tgx = \dfrac{1}{\sin 2x}$

$$ctg\,2x + tgx = \frac{1}{\sin 2x}$$

$$\frac{1}{tg\,2x} + tgx = \frac{1}{\dfrac{2tgx}{1+tg^2x}}$$

$$\frac{1}{\dfrac{2tgx}{1-tg^2x}} + tgx = \frac{1+tg^2x}{2tgx}$$

$$\frac{1-tg^2x}{2tgx} + tgx = \frac{1+tg^2x}{2tgx}$$

$$1-tg^2x + 2tg^2x = 1 + tg^2x$$

Ex. Demonstraţi identitatea: $\dfrac{\sin 2x}{1+\cos 2x} \cdot \dfrac{\cos x}{1+\cos x} = tg\,\dfrac{x}{2}$

$$\frac{\sin 2x}{1+\cos 2x} \cdot \frac{\cos x}{1+\cos x} = tg\,\frac{x}{2}$$

$$\frac{2\sin x\cos x}{1+\cos 2x} \cdot \frac{\cos x}{1+\cos x} = \frac{\sin x}{1+\cos x}$$

$$\frac{2\cos^2 x}{1+\cos 2x} = 1$$

$$2\cos^2 x = 1+\cos 2x$$

$$2\cos^2 x - 1 = \cos 2x$$

Ex. Calculaţi $\dfrac{2+3\cos x}{4-5\sin x}$ dacă $tg\dfrac{x}{2}=-\dfrac{2}{3}$

$$\sin x = \frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{2\cdot(-\frac{2}{3})}{1+(-\frac{2}{3})^2} = \frac{-\frac{4}{3}}{1+\frac{4}{9}} = \frac{-\frac{4}{3}}{\frac{13}{9}} = -\frac{12}{13}$$

$$\cos x = \frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{1-(-\frac{2}{3})^2}{1+(-\frac{2}{3})^2} = \frac{1-\frac{4}{9}}{1+\frac{4}{9}} = \frac{\frac{5}{9}}{\frac{13}{9}} = \frac{5}{13}$$

$$\frac{2+3\cos x}{4-5\sin x} = \frac{2+3\cdot\frac{5}{13}}{4-5\cdot(-\frac{12}{13})} = \frac{2+\frac{15}{13}}{4+\frac{60}{13}} = \frac{\frac{41}{13}}{\frac{112}{13}} = \frac{41}{112}$$

Ex.　　　Calculaţi　　$p\sin 2\alpha + q\cos q$　　dacă　$tg\dfrac{x}{2}=\dfrac{p}{q}$

$$\sin 2\alpha = \frac{2tg\dfrac{\alpha}{2}}{1+tg^2\dfrac{\alpha}{2}} = \frac{2\cdot\dfrac{p}{q}}{1+\dfrac{p^2}{q^2}} = \frac{\dfrac{2p}{q}}{\dfrac{q^2+p^2}{q^2}} = \frac{2pq}{p^2+q^2}$$

$$\cos 2\alpha = \frac{1-tg\dfrac{\alpha}{2}}{1+tg^2\dfrac{\alpha}{2}} = \frac{1-\dfrac{p^2}{q^2}}{1+\dfrac{p^2}{q^2}} = \frac{\dfrac{q^2-p^2}{q^2}}{\dfrac{q^2+p^2}{q^2}} = \frac{q^2-p^2}{p^2+q^2}$$

$$p\sin 2\alpha + q\cos q = p\cdot\frac{2pq}{p^2+q^2} + q\cdot\frac{q^2-p^2}{p^2+q^2} = \frac{2p^2q+q^3-p^2q}{p^2+q^2} =$$

$$= \frac{q(2p^2+q^2-p^2)}{p^2+q^2} = \frac{q(p^2+q^2)}{p^2+q^2} = q$$

Formule pentru tangenta sumei,
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Ex. Calculaţi $E = tg(4a - b)$ dacă $tga = \dfrac{1}{5}; \quad tgb = \dfrac{1}{239}$

$$E = tg(4a-b) = \frac{tg\,4a - tgb}{1 + tg\,4a \cdot tgb} = \frac{\dfrac{2tg\,2a}{1 - tg^2\,2a} - tgb}{1 + \dfrac{2tg\,2a}{1 - tg^2\,2a} \cdot tgb} = \frac{\dfrac{\dfrac{5}{6}}{1 - \dfrac{25}{144}} - \dfrac{1}{239}}{1 + \dfrac{\dfrac{5}{6}}{1 - \dfrac{25}{144}} \cdot \dfrac{1}{239}} = \frac{\dfrac{\dfrac{5}{6}}{\dfrac{119}{144}} - \dfrac{1}{239}}{1 + \dfrac{\dfrac{5}{6}}{\dfrac{119}{144}} \cdot \dfrac{1}{239}} =$$

$$= \frac{\dfrac{720}{714} - \dfrac{1}{239}}{1 + \dfrac{720}{714} \cdot \dfrac{1}{239}} = \frac{\dfrac{28561}{119239}}{\dfrac{28561}{119239}} = 1$$

$$tg\,2a = \frac{2tga}{1 - tg^2 a} = \frac{\dfrac{2}{5}}{1 - \dfrac{1}{25}} = \frac{\dfrac{2}{5}}{\dfrac{24}{25}} = \frac{5}{12}$$