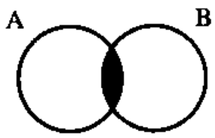
SETS

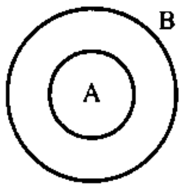
8.1 Sets

1. ∩ 'intersection'
   A ∩ B is shaded.

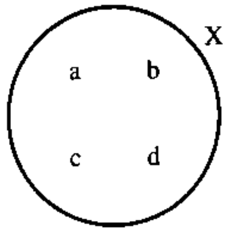


2. ∪ 'union'
   A ∪ B is shaded.

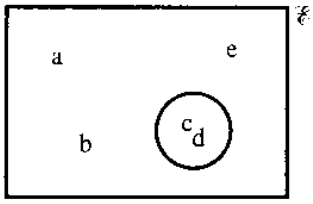
3. ⊂ 'is a subset of'
   A ⊂ B
   [B ⊄ A means 'B is *not* a subset of A']



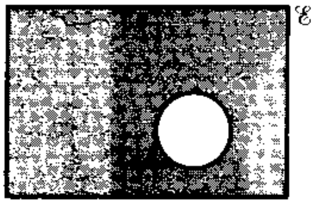
4. ∈ 'is a member of'
   'belongs to'
   b ∈ X
   [e ∉ X means 'e is not a member of set X']



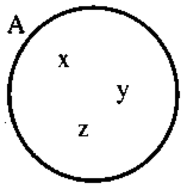
5. $\mathscr{E}$ 'universal set'
   $\mathscr{E} = \{a, b, c, d, e\}$



6. A′ 'complement of'
   'not in A'
   A′ is shaded
   $(A \cup A' = \mathscr{E})$



7. $n(A)$ 'the number of elements in set A'
   $n(A) = 3$



8. $A = \{x : x \text{ is an integer}, 2 \leqslant x \leqslant 9\}$

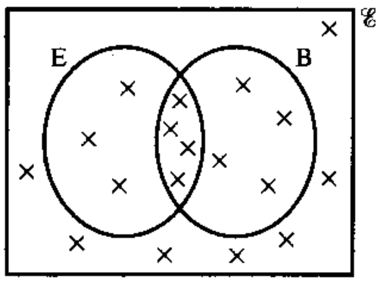
   A is the [set of] elements x [such that] x is an integer and $2 \leqslant x \leqslant 9$.

   The set A is {2, 3, 4, 5, 6, 7, 8, 9}.

9. $\varnothing$ or { } 'empty set'
   (Note: $\varnothing \subset A$ for any set A)

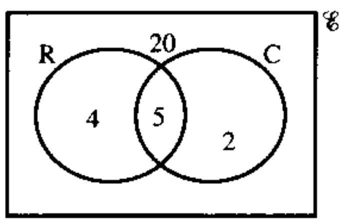
Exercise 1

1. In the Venn diagram,
   $\mathscr{E}$ = {people in an hotel}
   B = {people who like bacon}
   E = {people who like eggs}



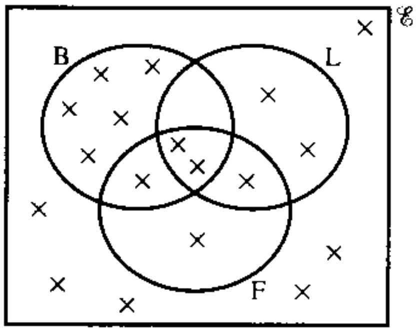
   (a) How many people like bacon?
   (b) How many people like eggs but not bacon?
   (c) How many people like bacon and eggs?
   (d) How many people are in the hotel?
   (e) How many people like neither bacon nor eggs?

2. In the Venn diagram,
   $\mathscr{E}$ = {boys in the fourth form}
   R = {members of the rugby team}
   C = {members of the cricket team}



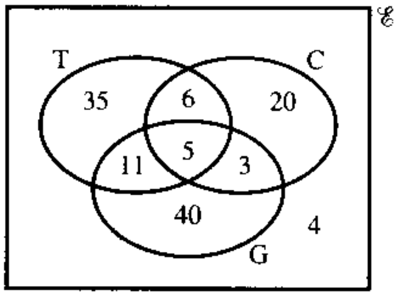
   (a) How many are in the rugby team?
   (b) How many are in both teams?
   (c) How many are in the rugby team but not in the cricket team?
   (d) How many are in neither team?
   (e) How many are there in the fourth form?

3. In the Venn diagram,
ℰ = {cars in a street}
B = {blue cars}
L = {cars with left-hand drive}
F = {cars with four doors}



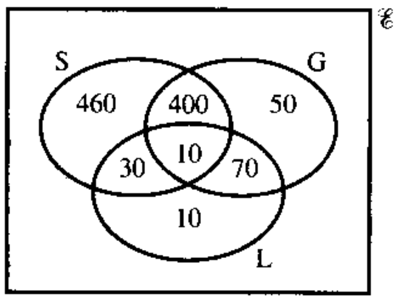
(a) How many cars are blue?
(b) How many blue cars have four doors?
(c) How many cars with left-hand drive have four doors?
(d) How many blue cars have left-hand drive?
(e) How many cars are in the street?
(f) How many blue cars with left-hand drive do not have four doors?

4. In the Venn diagram,
ℰ = {houses in the street}
C = {houses with central heating}
T = {houses with a colour T.V.}
G = {houses with a garden}



(a) How many houses have gardens?
(b) How many houses have a colour T.V. and central heating?
(c) How many houses have a colour T.V. and central heating and a garden?
(d) How many houses have a garden but not a T.V. or central heating?
(e) How many houses have a T.V. and a garden but not central heating?
(f) How many houses are there in the street?

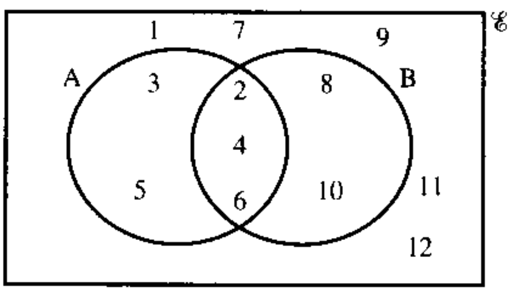
5. In the Venn diagram,
ℰ = {children in a mixed school}
G = {girls in the school}
S = {children who can swim}
L = {children who are left-handed}



(a) How many left-handed children are there?
(b) How many girls cannot swim?
(c) How many boys can swim?
(d) How many girls are left-handed?
(e) How many boys are left-handed?
(f) How many left-handed girls can swim?
(g) How many boys are there in the school?

Example

ℰ = {1, 2, 3, ..., 12}, A = {2, 3, 4, 5, 6} and B = {2, 4, 6, 8, 10}.



(a) $A \cup B = \{2, 3, 4, 5, 6, 8, 10\}$
(b) $A \cap B = \{2, 4, 6\}$
(c) $A' = \{1, 7, 8, 9, 10, 11, 12\}$
(d) $n(A \cup B) = 7$
(e) $B' \cap A = \{3, 5\}$

Exercise 2

In this exercise, be careful to use set notation only when the answer *is* a set.

1. If $M = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $N = \{5, 7, 9, 11, 13\}$,

find:

(a) $M \cap N$ (b) $M \cup N$ (c) $n(N)$ (d) $n(M \cup N)$

State whether true or false:

(e) $5 \in M$ (f) $7 \in (M \cup N)$

(g) $N \subset M$ (h) $\{5, 6, 7\} \subset M$

2. If $A = \{2, 3, 5, 7\}$, $B = \{1, 2, 3, \ldots, 9\}$,

find:

(a) $A \cap B$ (b) $A \cup B$ (c) $n(A \cap B)$ (d) $\{1, 4\} \cap A$

State whether true or false:

(e) $A \in B$ (f) $A \subset B$ (g) $9 \subset B$ (h) $3 \in (A \cap B)$

3. If $X = \{1, 2, 3, \ldots, 10\}$, $Y = \{2, 4, 6, \ldots, 20\}$ and
$Z = \{x : x \text{ is an integer}, 15 \leqslant x \leqslant 25\}$,

find:

(a) $X \cap Y$ (b) $Y \cap Z$ (c) $X \cap Z$

(d) $n(X \cup Y)$ (e) $n(Z)$ (f) $n(X \cup Z)$

State whether true or false:

(g) $5 \in Y$ (h) $20 \in X$

(i) $n(X \cap Y) = 5$ (j) $\{15, 20, 25\} \subset Z$.

4. If $D = \{1, 3, 5\}$, $E = \{3, 4, 5\}$, $F = \{1, 5, 10\}$,

find:

(a) $D \cup E$ (b) $D \cap F$ (c) $n(E \cap F)$

(d) $(D \cup E) \cap F$ (e) $(D \cap E) \cup F$ (f) $n(D \cup F)$

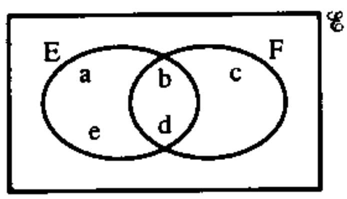
State whether true or false:

(g) $D \subset (E \cup F)$ (h) $3 \in (E \cap F)$ (i) $4 \notin (D \cap E)$

5. Find:

(a) $n(E)$ (b) $n(F)$ (c) $E \cap F$

(d) $E \cup F$ (e) $n(E \cup F)$ (f) $n(E \cap F)$

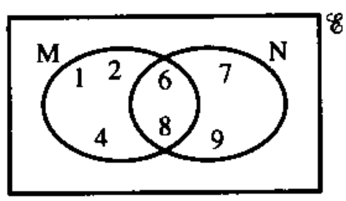


6. Find:

(a) $n(M \cap N)$ (b) $n(N)$ (c) $M \cup N$

(d) $M' \cap N$ (e) $N' \cap M$ (f) $(M \cap N)'$

(g) $M \cup N'$ (h) $N \cup M'$ (i) $M' \cup N'$



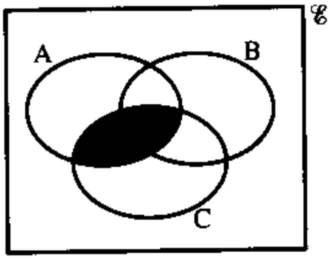
Example

On a Venn diagram, shade the regions:

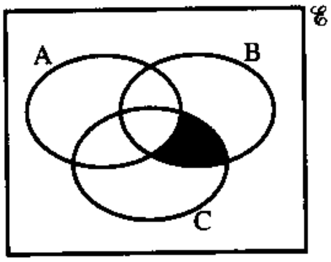
(a) $A \cap C$ (b) $(B \cap C) \cap A'$

where A, B, C are interesting sets.

(a) $A \cap C$



(b) $(B \cap C) \cap A'$
[find $(B \cap C)$ first]



Exercise 3

1. Draw six diagrams similar to Figure 1 and shade the following sets:
   (a) $A \cap B$ (b) $A \cup B$ (c) A'
   (d) $A' \cap B$ (e) $B' \cap A$ (f) $(B \cup A)'$

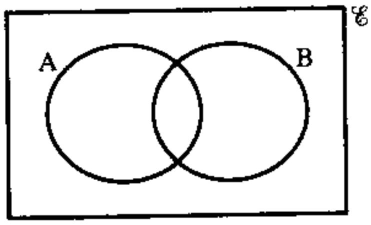


Figure 1

2. Draw four diagrams similar to Figure 2 and shade the following sets:
   (a) $A \cap B$ (b) $A \cup B$ (c) $B' \cap A$ (d) $(B \cup A)'$

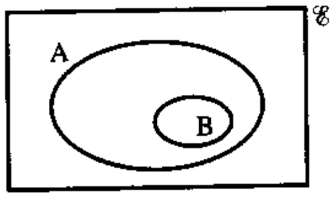


Figure 2

3. Draw four diagrams similar to Figure 3 and shade the following sets:
   (a) $A \cup B$ (b) $A \cap B$ (c) $A \cap B'$ (d) $(B \cup A)'$

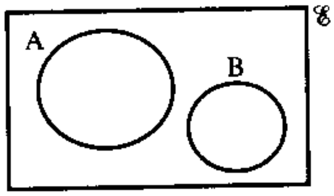


Figure 3

4. Draw eleven diagrams similar to Figure 4 and shade the following sets:
   (a) $A \cap B$ (b) $A \cup C$ (c) $A \cap (B \cap C)$
   (d) $(A \cup B) \cap C$ (e) $B \cap (A \cup C)$ (f) $A \cap B'$
   (g) $A \cap (B \cup C)'$ (h) $(B \cup C) \cap A$ (i) $C' \cap (A \cap B)$
   (j) $(A \cup C) \cup B'$ (k) $(A \cup C) \cap (B \cap C)$

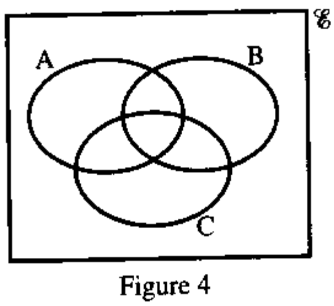


Figure 4

5. Draw nine diagrams similar to Figure 5 and shade the following sets:

(a) $(A \cup B) \cap C$ (b) $(A \cap B) \cup C$ (c) $(A \cup B) \cup C$
(d) $A \cap (B \cup C)$ (e) $A' \cap C$ (f) $C' \cap (A \cup B)$
(g) $(A \cap B) \cap C$ (h) $(A \cap C) \cup (B \cap C)$ (i) $(A \cup B \cup C)'$

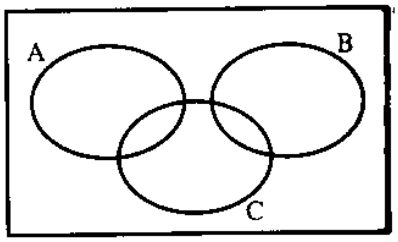
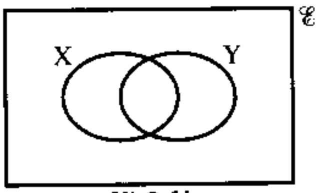


Figure 5

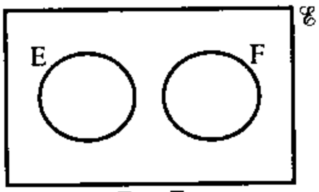
6. Copy each diagram and shade the region indicated.

(a)



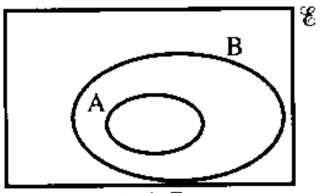
$X' \cap Y$

(b)



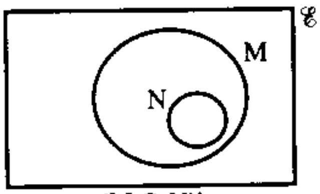
$E \cup F$

(c)



$A \cap B$

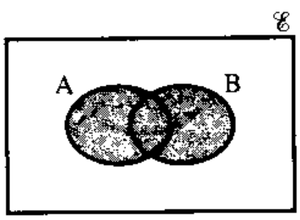
(d)



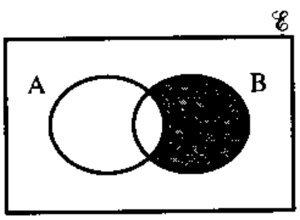
$(M \cap N)'$

7. Describe the region shaded.

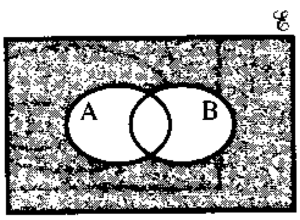
(a)



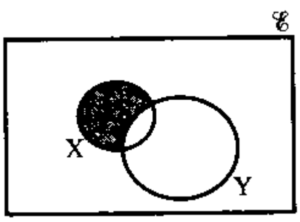
(b)



(c)



(d)



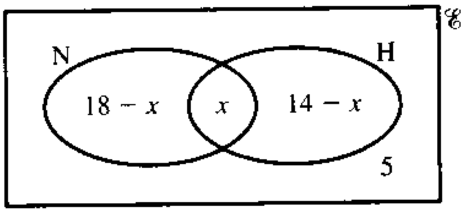
8.2 Logical problems

Example 1

In a form of 30 girls, 18 play netball and 14 play hockey, whilst 5 play neither.
Find the number who play both netball and hockey.

Let $\mathscr{E}$ = {girls in the form}
N = {girls who play netball}
H = girls who play hockey}

and x = the number of girls who play both netball and hockey



The number of girls in each portion of the universal set is shown in the Venn diagram.

Since $n(\mathscr{E}) = 30$

$$18 - x + x + 14 - x + 5 = 30$$
$$37 - x = 30$$
$$x = 7$$

∴ Seven girls play both netball and hockey.

Example 2

If A = {sheep}
B = {sheep dogs}
C = {'intelligent' animals}
D = {animals which make good pets}

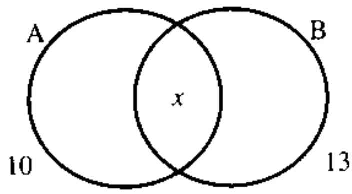
(a) Express the following sentences in set language:
(i) No sheep are 'intelligent' animals.
(ii) All sheep dogs make good pets.
(iii) Some sheep make good pets.

(b) Interpret the following statements:
(i) $B \subset C$
(ii) $B \cup C = D$

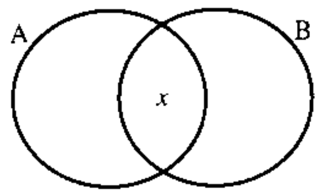
(a) (i) $A \cap C = \varnothing$
(ii) $B \subset D$
(iii) $A \cap D \neq \varnothing$

(b) (i) All sheep dogs are intelligent animals.
(ii) Animals which make good pets are either sheep dogs or 'intelligent' animals (or both).

Exercise 4



1. In the Venn diagram $n(A) = 10$, $n(B) = 13$, $n(A \cap B) = x$ and $n(A \cup B) = 18$.
   (a) Write in terms of x the number of elements in A but not in B.
   (b) Write in terms of x the number of elements in B but not in A.
   (c) Add together the number of elements in the three parts of the diagram to obtain the equation $10 - x + x + 13 - x = 18$.
   (d) Hence find the number of elements in both A and B.



2. In the Venn diagram $n(A) = 21$, $n(B) = 17$, $n(A \cap B) = x$ and $n(A \cup B) = 29$.
   (a) Write down in terms of x the number of elements in each part of the diagram.
   (b) Form an equation and hence find x.
3. The sets M and N intersect such that $n(M) = 31$, $n(N) = 18$ and $n(M \cup N) = 35$. How many elements are in both M and N?
4. The sets P and Q intersect such that $n(P) = 11$, $n(Q) = 29$ and $n(P \cup Q) = 37$. How many elements are in both P and Q?

5. The sets A and B intersect such that $n(A \cap B) = 7$, $n(A) = 20$ and $n(B) = 23$. Find $n(A \cup B)$.

6. Twenty boys in a form all play either football or basketball (or both). If thirteen play football and ten play basketball, how many play both sports?

7. Of the 53 staff at a school, 36 drink tea, 18 drink coffee and 10 drink neither tea nor coffee. How many drink both tea and coffee?

8. Of the 32 pupils in a class, 18 play golf, 16 play the piano and 7 play both. How many play neither?

9. Of the pupils in a class, 15 can spell 'parallel', 14 can spell 'Pythagoras', 5 can spell both words and 4 can spell neither. How many pupils are there in the class?

10. In a school, students must take at least one of these subjects: Maths, Physics or Chemistry. In a group of 50 students, 7 take all three subjects, 9 take Physics and Chemistry only, 8 take Maths and Physics only and 5 take Maths and Chemistry only. Of these 50 students, x take Maths only, x take Physics only and $x + 3$ take Chemistry only. Draw a Venn diagram, find x, and hence find the number taking Maths.

11. All of 60 different vitamin pills contain at least one of the vitamins A, B and C. Twelve have A only, 7 have B only, and 11 have C only. If 6 have all three vitamins and there are x having A and B only, B and C only and A and C only, how many pills contain vitamin A?

12. The IGCSE results of the 30 members of a Rugby squad were as follows:
All 30 players passed at least two subjects, 18 players passed at least three subjects, and 3 players passed four subjects or more. Calculate:
(a) how many passed exactly two subjects,
(b) what fraction of the squad passed exactly three subjects.

13. In a group of 59 people, some are wearing hats, gloves or scarves (or a combination of these), 4 are wearing all three, 7 are wearing just a hat and gloves, 3 are wearing just gloves and a scarf and 9 are wearing just a hat and scarf. The number wearing only a hat or only gloves is x, and the number wearing only a scarf or none of the three items is $(x - 2)$. Find x and hence the number of people wearing a hat.

14. In a street of 150 houses, three different newspapers are delivered: T, G and M. Of these, 40 receive T, 35 receive G, and 60 receive M; 7 receive T and G, 10 receive G and M and 4 receive T and M; 34 receive no paper at all. How many receive all three?
Note: If '7 receive T and G', this information does not mean 7 receive T and G *only*.

15. If S = {Scottish men}, G = {good footballers}, express the following statements in words:
(a) $G \subset S$
(b) $G \cap S = \varnothing$
(c) $G \cap S \neq \varnothing$
(Ignore the truth or otherwise of the statements.)

16. Given that $\mathscr{E}$ = {pupils in a school}, B = {boys}, H = {hockey players}, F = {football players}, express the following in words:
(a) $F \subset B$ (b) $H \subset B'$ (c) $F \cap H \neq \varnothing$ (d) $B \cap H = \varnothing$
Express in set notation:
(e) No boys play football.
(f) All pupils play either football or hockey.

17. If $\mathscr{E}$ = {living creatures}, S = {spiders}, F = {animals that fly}, T = {animals which taste nice}, express in set notation:
(a) No spiders taste nice.
(b) All animals that fly taste nice.
(c) Some spiders can fly.
Express in words:
(d) $S \cup F \cup T = \mathscr{E}$ (e) $T \subset S$

18. $\mathscr{E}$ = {tigers}, T = {tigers who believe in fairies}, X = {tigers who believe in Eskimos}, H = {tigers in hospital}.
Express in words:
(a) $T \subset X$ (b) $T \cup X = H$ (c) $H \cap X = \varnothing$
Express in set notation:
(d) All tigers in hospital believe in fairies.
(e) Some tigers believe in both fairies and Eskimos.

19. $\mathscr{E}$ = {school teachers}, P = {teachers called Peter}, B = {good bridge players}, W = {women teachers}. Express in words:
(a) $P \cap B = \varnothing$ (b) $P \cup B \cup W = \mathscr{E}$ (c) $P \cap W \neq \varnothing$
Express in set notation:
(d) Women teachers cannot play bridge well.
(e) All good bridge players are women called Peter.