Sets

1. (*162.*) How many elements do the set $A \times A$, the set $A \times B$ and the set $B \times B$ contain if

   $A = \{ 5;\ 6;\ 11 \}$ and $B = \{ 2;\ 6;\ 18;\ 24 \}$?

2. (*163.*) How many elements are there in the union and in the intersection of the following two sets?

   $A =$ {two-digit numbers divisible by 7},
   $B =$ {two-digit numbers divisible by 3}.

3. (*164.*) How many elements are there in a set which has at least 1000 more subsets than its elements?

4. (*165.*) A set has 10 elements. What is the ratio between the number of subsets which have 2 elements and the number of subsets which have 3 elements?

5. (*166.*) A set has 15 elements. What is the ratio between the number of subsets which have 6 elements and the number of subsets which have 9 elements?

6. (*167.*) A and B are sets. Under what conditions will it be true that $A \setminus B = B \setminus A$?

7. (*168.*) How many subsets are there in the set $A = \{ a; b; c; d; e \}$, of which c is an element?

8. (*169.*) How many subsets are there in the set $A = \{ a; b; c; d; e \}$, of which at least one of b and c is an element?

9. *(170.)* $A = \{10; 20; 30\}$ is a set. Find sets B, C and D for which the following connections hold: $A \cup B = \{10; 20; 30; 40; 50\}$; $A \cap C = \{20\}$; $A \setminus D =$

10. *(171.)* A set has 1; 2; 3 or 4 more subsets than its elements. How many elements can it have in each case?

11. *(172.)* Determine the sets $A \setminus B$ and $B \setminus A$ if

$A = \{a; b; c; d\}$; $A \cup B = \{a; b; c; d\}$; $A \cap B = \{a; c\}$!

12. *(173.)* Is it true that for every set A and B

$((A \setminus B) \cup (B \setminus A)) \subset A \cup B$?

13. *(174.)* Let H be the set of positive integers smaller than 1000. How many elements are there in the union of all the subsets of H?

14. *(175.)* What can the set B be if $A = \{a; b; c; d\}$; $A \cup B = A$ and $A \cap B = B$?

15. *(176.)* What is the connection between the sets A and B in the following cases?

a) $A \setminus B = \emptyset$ and $A \cup B = A$;

b) $A \setminus B = \emptyset$ and $A \cap B = A$;

c) $A \setminus B = \emptyset$ and $B \setminus A = \emptyset$.

16. *(177.)* Determine the elements of the set $(A \times A) \cap (B \times B)$ if $A = \{1; 2\}$, $B = \{1; 2; 3; 4\}$!

17. *(178.)* How many subsets are there in the set of one-digit positive integers for which

a) both 4 and 5 are elements of the subsets;
b) 4 is an element of the subsets, but 5 is not;
c) 5 is an element of the subsets, but 4 is not;
d) neither 4 nor 5 is an element of the subsets.

18. *(179.)* Give a finite and an infinite set, such that their intersections with the set of integers is the set $M = \{-2; -1; 0; 1; 2\}$

19. *(180.)* Sketch the graph of the elements of the set $A = \{a \in \mathbf{Z} \mid |a| \leqq 10\}$ on a coordinate line. Give the set of real numbers for which twice as many elements of A are larger than smaller. ($\mathbf{Z}$ denotes the set of integers.)

20. *(181.)* Prove that if a set has 53 elements, then the number of subsets which have 16 elements is equal to the number of subsets which have 37 elements.

21. *(182.)* Let H the set of inscribed quadrilaterals, D the set of deltoids, and T the set of trapezoids.
Determine the set $H \cap D \cap T$!

22. *(183.)* Let C be the set of centrally symmetrical, and T the set of axially symmetrical quadrilaterals. Determine the set $C \cap T$!

23. *(184.)* Let 3 closed intervals be given on the coordinate line, such that any two of them have at least one common point. Prove that in this case all three have at least one common point.

24. *(185.)* Three closed intervals divide the coordinate line into parts. What is the maximum number of these parts?

25. *(186.)* Let $A = \{1; 2; 3; 4; 5; 6; 7; 8; 9; 10\}$. Determine 6 subsets of A such that the intersection of any two of these subsets contains exactly one element.

26. *(187.)* Give an example of three sets where the intersection of each pair has infinitely many common elements, but the intersection of the three sets is empty.

27. *(188.)* Let A be the set of those ten-digit numbers where only the numbers 1, 2 and 3 appear among the digits, but each appears at least once. How many elements does the set A have?

28. (*189.*) Prove that if a set has n elements, then the number of subsets which have an even number of elements is equal to the number of subsets which have an odd number of elements. (The number of elements in the empty set is even.)

29. (*190.*) Let $M = \{1; 2; 3; 4; 5; 6; 7; 8\}$. How many subsets may be chosen from the set M so that the intersection of any two of the subsets is not empty?

30. (*191.*) Prove that if $A \cap C = \emptyset$, then $A \setminus (B \setminus C) = (A \setminus B) \setminus C$!

31. (*192.*) Do the following two statements have the same meaning?
a) For every element a of set A there exists an element b of set B such that $a < b$.
b) There exists an element b of set B such that $a < b$ for every element a of set A.

32. (*193.*) The set A consists of those natural numbers which are divisible either by 2 or by 6. The set B consists of those natural numbers which are divisible either by 3 or by 4. Which numbers are the elements of $A \cap B$?

33. (*194.*) Let H be the set of positive divisors of the number 210. Give a subset H_1 of H which contains 6 elements, none of which is a divisor of another element of H_1.

34. (*195.*) Let A, B and C be sets for which $A \cap C = B \cap C$ and $A \cup C = B \cup C$. Prove that $A = B$.

35. (*196.*) Let A, B and C be sets for which $A \cap C = B \cap C$ and $A \setminus C = B \setminus C$. Prove that $A = B$.

36. (*197.*) Let A, B and C be sets for which $A \cup C = B \cup C$ and $A \setminus C = B \setminus C$. Does it follow that $A = B$?

37. (*198.*) Let A, B and C be sets for which $A \cup C = B \cup C$ and $C \setminus A = C \setminus B$. Prove that $A = B$.

38. (*199.*) Let $M = \left\{\frac{2}{1}; \frac{4}{3}; \frac{6}{5}; \ldots; \frac{1002}{1001}\right\}$. How many elements are there in the set M for which $|x-1| < 0{,}1$?

39. (*200.*) Let $A = \{0; 1; 2; 3; 4\}$. Determine the sets X and Y so that the following conditions hold:
a) $X \cap Y = A$;
b) $X \cup Y = \mathbf{N}$, where **N** denotes the set of natural numbers,
c) if $x \in X \setminus A$, then there exists a $y \in Y \setminus A$, for which $x - y = 1$.

40. (*201.*) Let A, B and C be sets for which $A \cup B \cup C = \{1; 2; 3; 4; 5; 6; 7; 8; 9; 10\}$; $A \setminus B = \{2; 4; 6; 8; 10\}$; $A \setminus C = \{1; 3; 5; 7; 9\}$.
Prove that $B \cap C = \emptyset$ and $B \cup C = A$!

41. (*202.*) Let A, B and C be subsets of the set $M = \{1; 2; 3; 4; 5; 6\}$. We know the following: $A \cap B = \{2\}$; $(A \cup B) \cap C = \{5; 6\}$; $A \setminus C = \{2; 3; 4\}$; $C \setminus B = \{1; 5\}$.

Determine the sets A, B and C.

42. (*203.*) Let $M \subset \{1; 2; 3; 4\}$ and $M \subset \{3; 4; 5; 6\}$. Prove that $M \subset \{3; 4\}$!

43. (*204.*) Let M be a set for which $\{1; 2; 3; 4\} \subset M$, and $\{3; 4; 5; 6\} \subset M$. Prove that $\{1; 2; 3; 4; 5; 6\} \subset M$!

44. (*205.*) Determine the sets A and B if
$A \cup B = \{1; 2; 3; 4; 5\}$; $A \cap B = \{3; 5\}$; $A \setminus B = \{1\}$; and $B \setminus A = \{2; 4\}$!

45. (*206.*) Determine the sets A and B if $A \cup B = \{1; 2; 3; 4; 5\}$; $A \cap B \not\subset \{3; 4; 5\}$; $A \setminus B = \{1; 4\}$; and $|A| = |B|$, where $|A|$ denotes the number of elements in A.

46. (*207.*) a and b are real numbers for which $\{a^2+a; a\} = \{b^2+b; b\}$. Prove that $a = b$.

47. (*208.*) a, b, c and d are real numbers for which $\{ab;\ a+b\} = \{cd;\ c+d\}$. Is it true that $\{a;\ b\} = \{c;\ d\}$?

48. (*209.*) Sketch the set of points $P(x;y)$, for which $|x|+|y|<1$ in the rectangular coordinate system.

49. (*210.*) Determine the sets $A \cap B$ and $A \cup B$ if
$A = \{x \in \mathbf{R} \mid x^2+2x-4 \leqq 0\}$ and $B = \{x \in \mathbf{R} \mid x^2+4x-1 \leqq 0\}$!
(**R** denotes the set of real numbers.)

50. (*212.*) Let M be the set of solutions of the inequality $x^2+y^2 \leqq 5$ in the underlying set **R**x**R**. How many elements are there in the set $M \cap (\mathbf{Z} \times \mathbf{Z})$?
(**Z** denotes the set of integers and **R** the set of real numbers.)

51. (*213.*) If H is any non-empty set of natural numbers, then it must have a smallest element. Let us denote this by $m(H)$. Let $A \subset \mathbf{N}$; $B \subset \mathbf{N}$ and $A \cap B \neq \emptyset$. Prove that $m(A)+m(B) \leqq m(A \cup B)+m(A \cap B)$!

52. (*218.*) Sketch the set of points $P(x;y)$ for which the following conditions hold:
$x \in \mathbf{R} \setminus \{0\}$; $y \in \mathbf{R} \setminus \{0\}$; $\dfrac{x}{|x|} + \dfrac{y}{|y|} = 0$
in the rectangular co-ordinate system.

(**R** denotes the set of real numbers.)

53. (*219.*) Determine the set $A \cap B$ if $A = \{a+b\sqrt{2};\ a \in \mathbf{Q};\ b \in \mathbf{Q}\}$ and $B = \{a+b\sqrt{3};\ a \in \mathbf{Q};\ b \in \mathbf{Q}\}$!
(**Q** denotes the set of rational numbers.)

54. (*220.*) Determine the set $A \cap B$ if $A = \{x \in \mathbf{N} \mid 2x \leqq 4x-6\}$ and $B = \{x \in \mathbf{N} \mid 4x-11 \leqq 2x+11\}$!
(**N** denotes the set of natural numbers.)

55. (*221.*) 32 students attend the same class. The languages taught to this class are English and Russian and everybody has to learn at least one language. Nine students are learning both languages. Prove that the number of students learning Russian and the number of students learning English cannot be equal.

56. (*223.*) Two problems were set in a mathematics competition. The first problem was solved by 70% of the competitors and the second was solved by 60% of them. Every competitor solved at least one of the problems and 9 of them solved both problems. How many people took part in the competition?

57. (*224.*) Out of 28 students in a class, 8 will take an entrance examination in mathematics, 6 in physics, and 4 in both subjects. How many students will not take an examination in either of these subjects?