

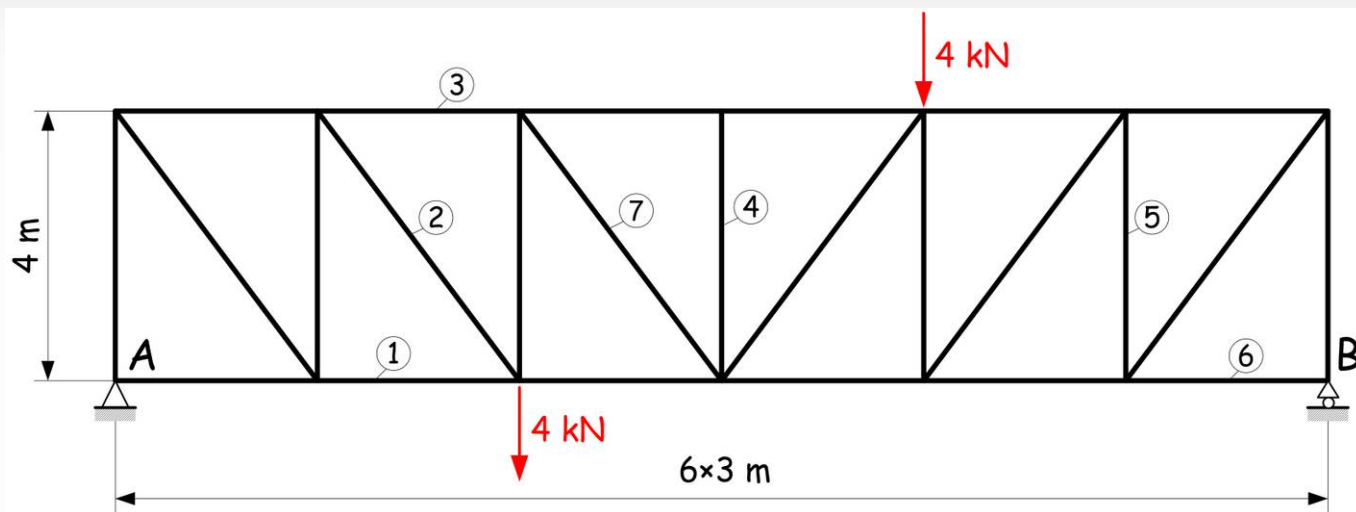
Mechanikai alapismeretek 1. Statika

Építésmérnök O/BSc, Építőművész BA

RÁCSOS TARTÓK RÚDERŐINEK SZÁMÍTÁSA



Rácsos tartó támaszerői



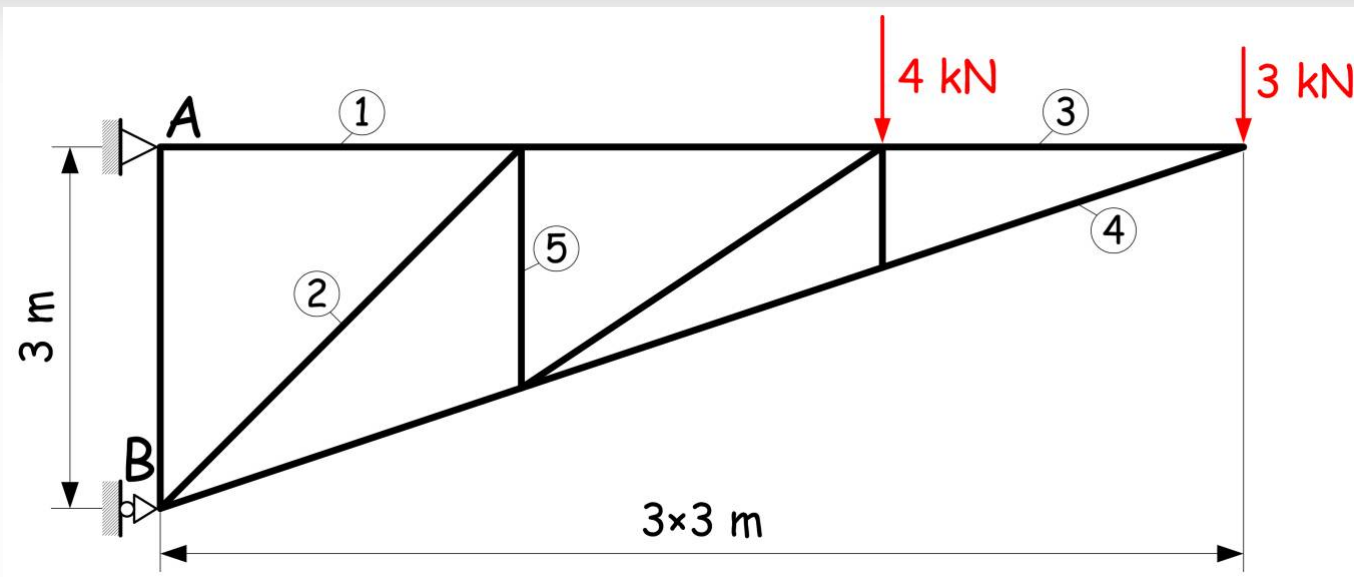
$$\sum M_A = 0 = 4,0 \text{ kN} \cdot 6,0 \text{ m} + 4,0 \text{ kN} \cdot 12,0 \text{ m} - B \cdot 18,0 \text{ m}$$

$$B = \frac{4,0 \text{ kN} \cdot 6,0 \text{ m} + 4,0 \text{ kN} \cdot 12,0 \text{ m}}{18,0 \text{ m}} = \frac{72,0 \text{ kNm}}{18,0 \text{ m}} = 4,0 \text{ kN} (\uparrow)$$

$$\sum F_y = 0 = -A_y + 4,0 \text{ kN} + 4,0 \text{ kN} - B = -A_y + 4,0 \text{ kN}$$

$$A_y = 4,0 \text{ kN} (\uparrow)$$

Rácsos tartó támaszerői



$$\sum M_A = 0 = 4,0 \text{ kN} \cdot 6,0 \text{ m} + 3,0 \text{ kN} \cdot 9,0 \text{ m} - B \cdot 3,0 \text{ m}$$

$$B = \frac{4,0 \text{ kN} \cdot 6,0 \text{ m} + 3,0 \text{ kN} \cdot 9,0 \text{ m}}{3,0 \text{ m}} = \frac{51,0 \text{ kNm}}{3,0 \text{ m}} = 17,0 \text{ kN} (\rightarrow)$$

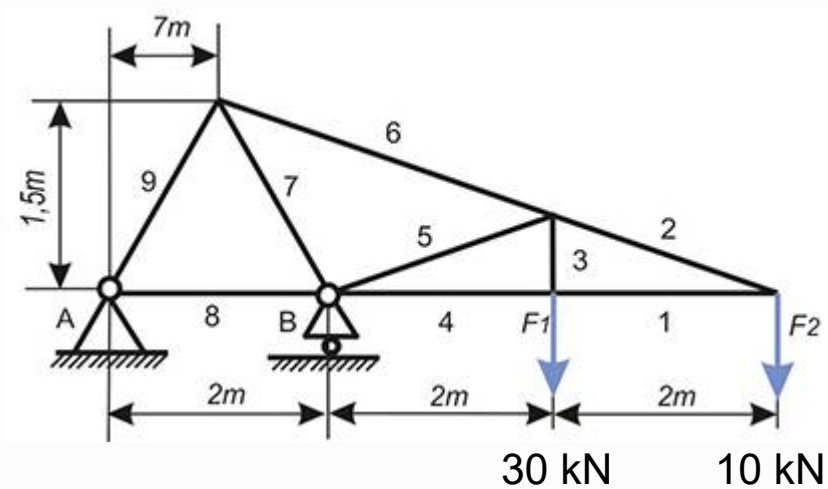
$$\sum F_x = 0 = B - A_x = 17,0 \text{ kN} - A_x \Rightarrow A_x = 17,0 \text{ kN} (\leftarrow)$$

$$\sum F_y = 0 = -A_y + 4,0 \text{ kN} + 3,0 \text{ kN} = -A_y + 7,0 \text{ kN} \Rightarrow A_y = 7,0 \text{ kN} (\uparrow)$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(17,0 \text{ kN})^2 + (7,0 \text{ kN})^2} = 18,38 \text{ kN}$$

$$\text{tg } \alpha = \frac{A_y}{A_x} = \frac{7,0}{17,0} \Rightarrow \alpha = 22,4^\circ$$

Rácsos tartó

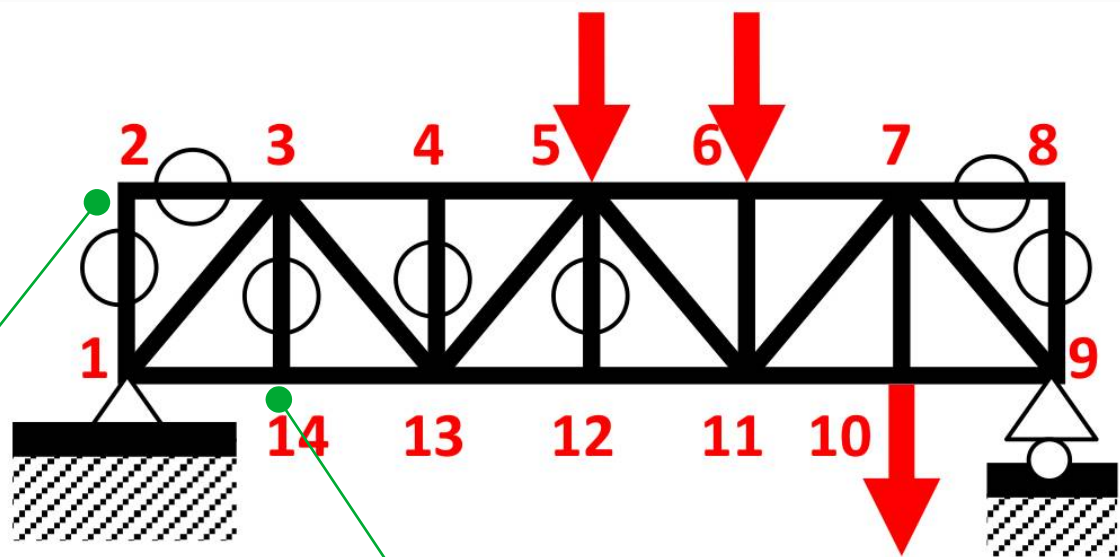


Rácsos tartó rúderőinek számítása

Csomóponti módszer

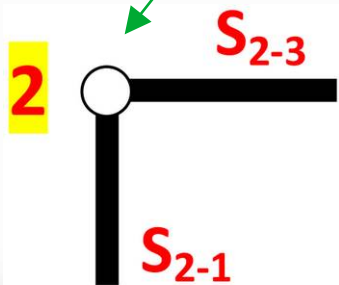
$$\sum F_{ix} = 0$$

$$\sum F_{iy} = 0$$



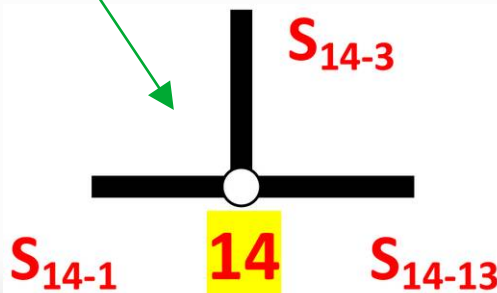
vakrudak megállapítása (szemrevételezéssel)

vakrúd az olyan rúd, amiben nem ébred rúderő ($S = 0$)



$$S_{2-1} = 0 \text{ kN}$$

$$S_{2-3} = 0 \text{ kN}$$



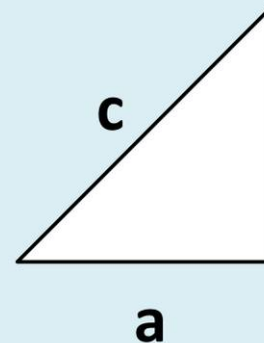
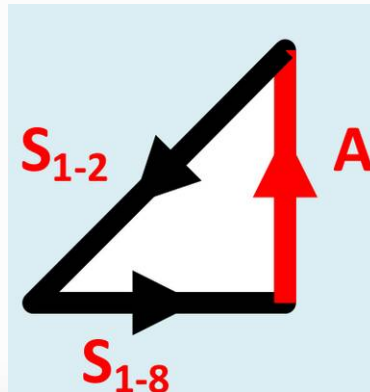
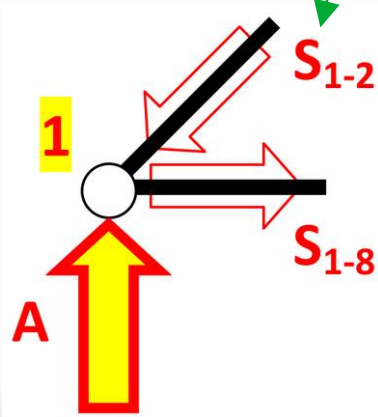
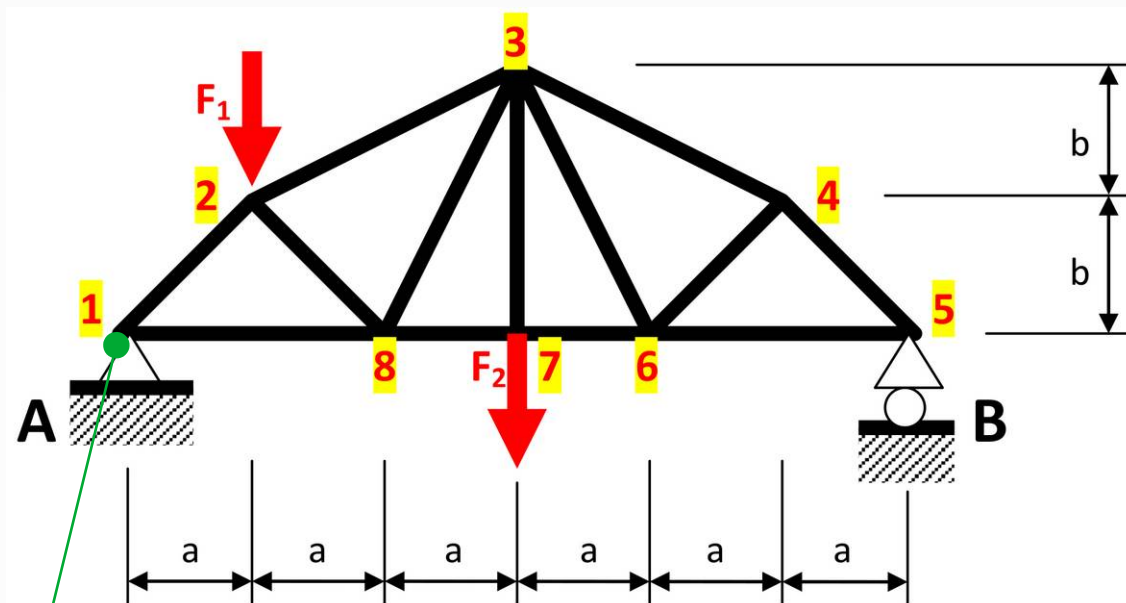
$$S_{14-3} = 0 \text{ kN}$$

Rácsos tartó rúderőinek számítása

Csomóponti módszer

$$\sum F_{ix} = 0$$

$$\sum F_{iy} = 0$$



$$S_{1-2} / A = c / b$$

$$S_{1-2} = A \times c / b$$

NYOMOTT

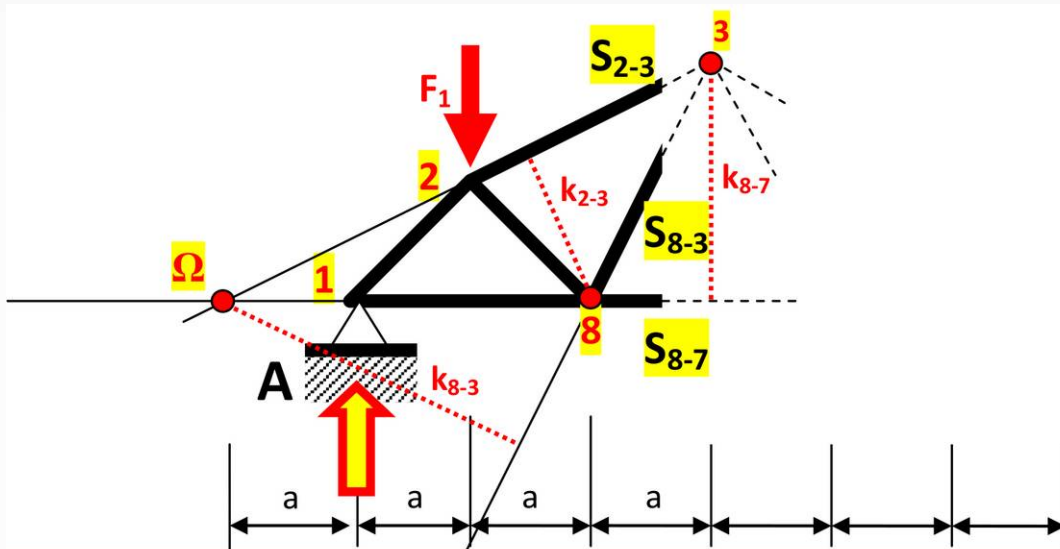
$$S_{1-8} / A = a / b$$

$$S_{1-8} = A \times a / b$$

HÚZOTT

Hármas átmetszés

$$\Sigma \mathbf{M}_i = 0$$



$$\Sigma F_{iy} = 0$$

$$\sum M_i^8 = 0$$

$$+ A \times 2a - F_1 \times a + S_{2-3} \times k_{2-3} = 0$$

$$S_{2-3} = (-A \times 2a + F_1 \times a) / k_{2-3}$$

$$\Sigma M_i^{\Omega} = 0$$

$$-A \times a + F_1 \times 2a + S_{8-3} \times k_{8-3} = 0$$

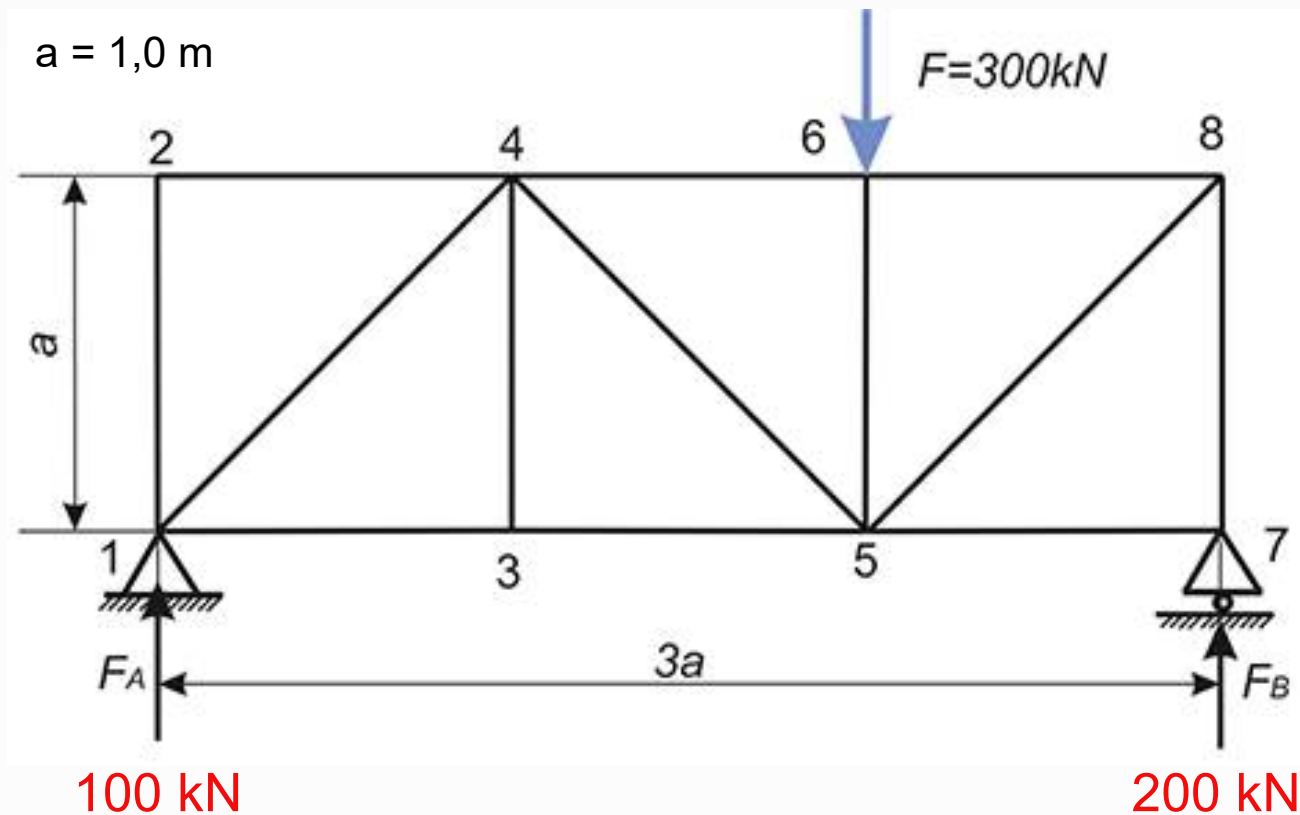
$$S_{8-3} = (+A \times a - F_1 \times 2a) / k_{8-3}$$

$$\sum M_i^3 = 0$$

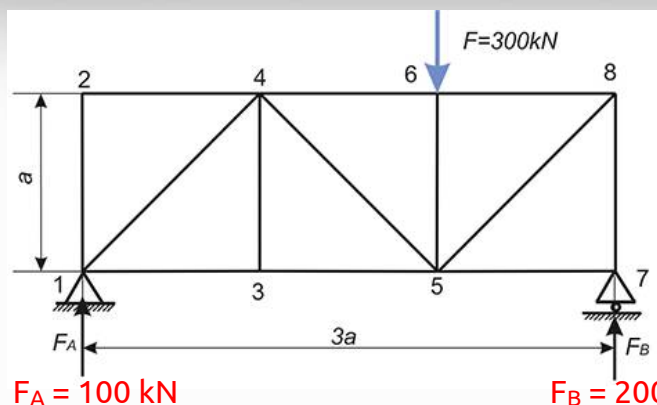
$$+ A \times 3a - F_1 \times 2a + S_{8-7} \times k_{8-7} = 0$$

$$S_{8-7} = (-A \times 3a + F_1 \times 2a) / k_{8-7}$$

Rácsos tartó – csomóponti módszer

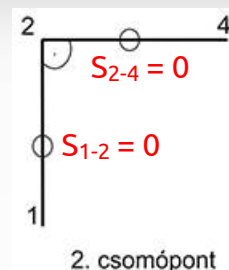


Rácsos tartó – csomóponti módszer

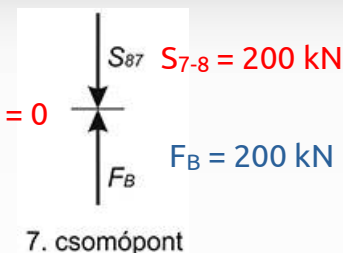


$$F_A = 100 \text{ kN}$$

$$F_B = 200 \text{ kN}$$

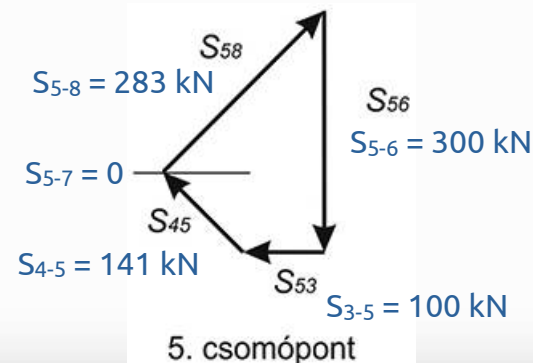
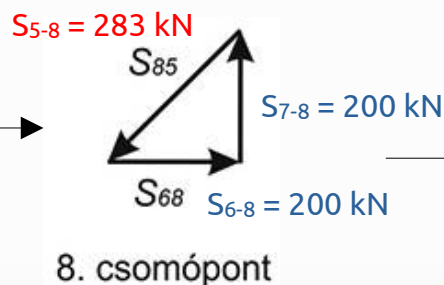
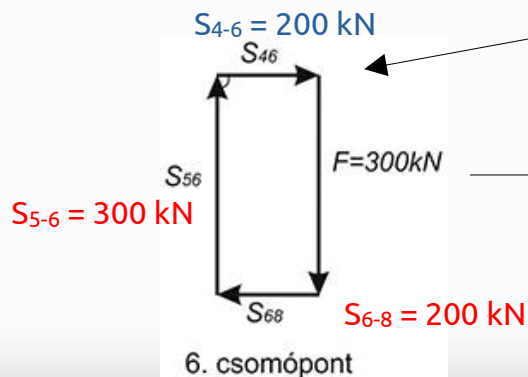
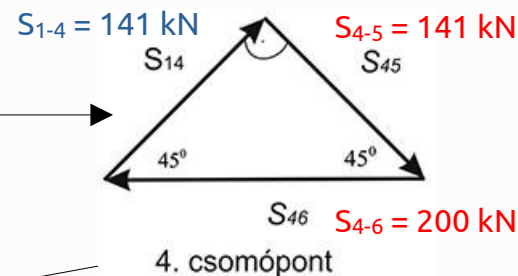
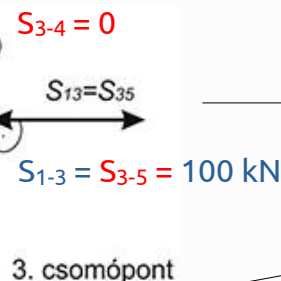
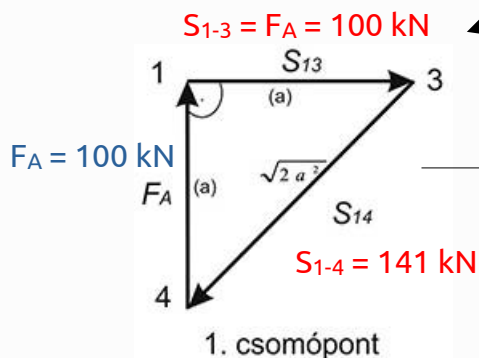


$$S_{5-7} = 0$$

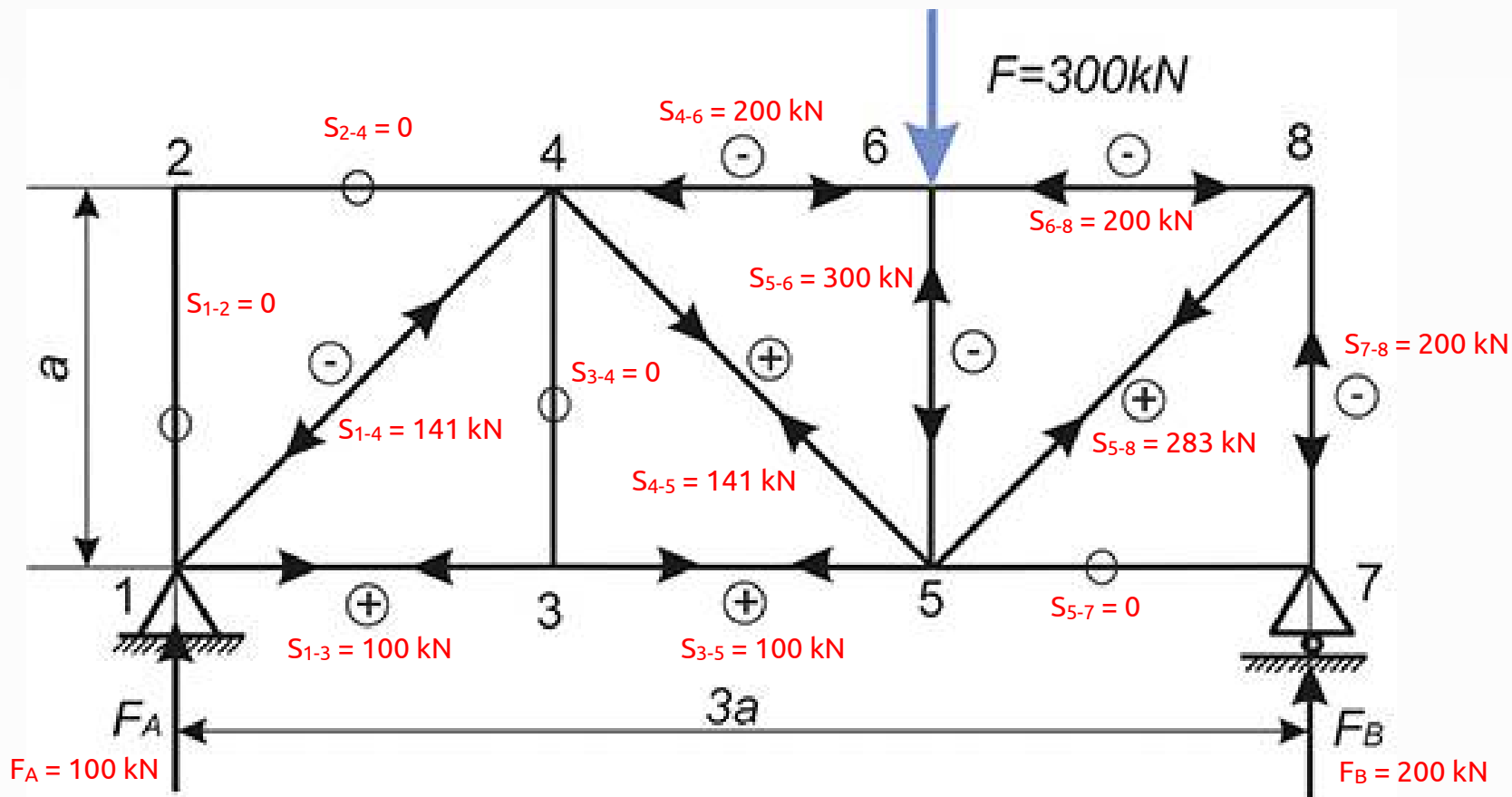


$$\sum F_{ix} = 0$$

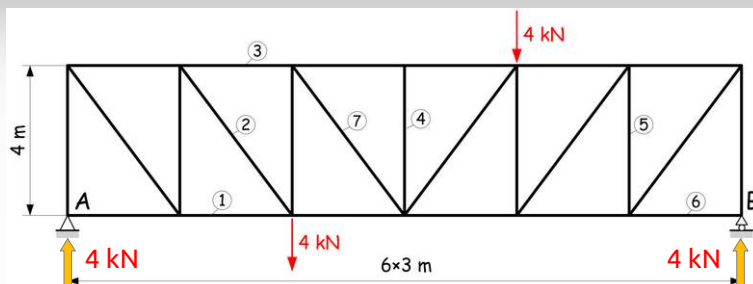
$$\sum F_{iy} = 0$$



Rácsos tartó – csomóponti módszer



Rúderő számítása – hármas átmetszés



$$\sum M_i = 0$$

$$\sum F_{ix} = 0$$

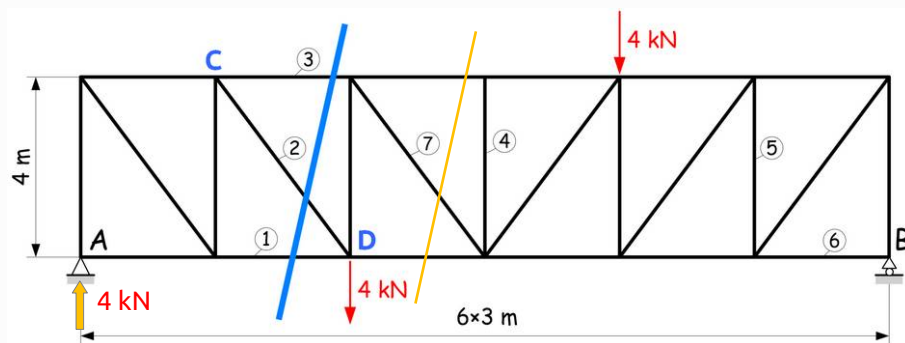
$$\sum F_{iy} = 0$$

$$\sum M_A = 0 = 4,0 \text{ kN} \cdot 6,0 \text{ m} + 4,0 \text{ kN} \cdot 12,0 \text{ m} - B \cdot 18,0 \text{ m}$$

$$B = \frac{4,0 \text{ kN} \cdot 6,0 \text{ m} + 4,0 \text{ kN} \cdot 12,0 \text{ m}}{18,0 \text{ m}} = \frac{72,0 \text{ kNm}}{18,0 \text{ m}} = 4,0 \text{ kN} (\uparrow)$$

$$\sum F_y = 0 = -A_y + 4,0 \text{ kN} + 4,0 \text{ kN} - B = -A_y + 4,0 \text{ kN}$$

$$A_y = 4,0 \text{ kN} (\uparrow)$$



$$\sum F_y = 0 = A_y + F_1 + S_{7y} = -4,0 \text{ kN} + 4,0 \text{ kN} + S_{7y}$$

$$S_{7y} = 0 \text{ [vakrúd]}$$

$$S_7 = 0 \text{ [vakrúd]}$$

$$\sum M_C = 0 = A_y \cdot 3,0 \text{ m} - S_1 \cdot 4,0 \text{ m}$$

$$S_1 = 3,0 \text{ kN} (\rightarrow) \text{ [húzott]}$$

$$\sum M_D = 0 = A_y \cdot 6,0 \text{ m} - S_3 \cdot 4,0 \text{ m}$$

$$S_3 = 6,0 \text{ kN} (\leftarrow) \text{ [nyomott]}$$

$$\sum F_x = 0 = S_1 + S_{2x} + S_3 = 3,0 \text{ kN} + S_{2x} - 6,0 \text{ kN}$$

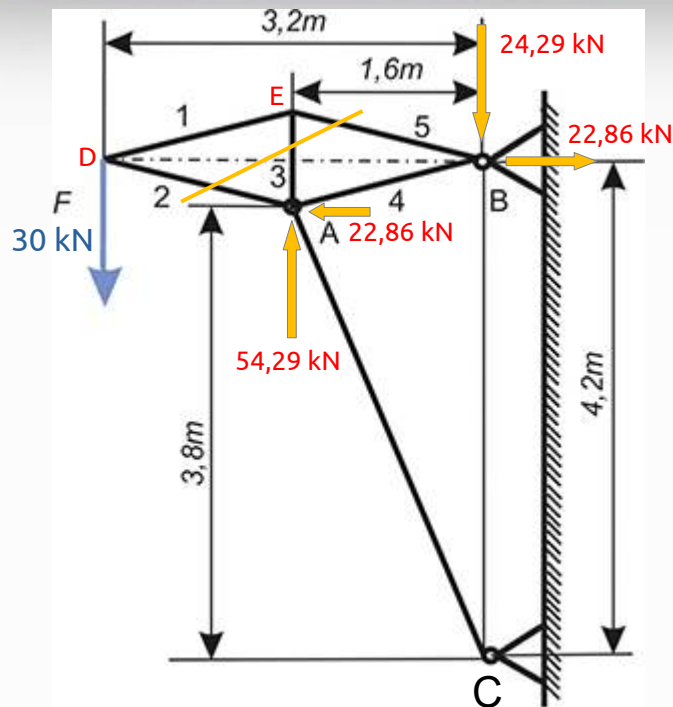
$$S_{2x} = 3,0 \text{ kN} (\rightarrow) \text{ [húzott]}$$

$$\sum F_y = 0 = A_y + S_{2y} = -4,0 \text{ kN} + S_{2y}$$

$$S_{2y} = 4,0 \text{ kN} (\downarrow) \text{ [húzott]}$$

$$S_2 = 5,0 \text{ kN} \text{ [húzott]}$$

Rácsos tartó – hármas átmetszés



$$\Sigma M_C = 0 = -F \cdot 3,2m + B_x \cdot 4,2 = -30kN \cdot 3,2m + B_x \cdot 4,2$$

$$B_x = \frac{96kNm}{4,2m} = 22,86kN (\rightarrow)$$

$$\Sigma F_x = 0 = B_x - C_x$$

$$C_x = -B_x = -22,86kN (\leftarrow)$$

$$C_y = C_x \cdot \frac{3,8m}{1,6m} = -22,86kN \cdot \frac{3,8m}{1,6m} = -54,29 (\uparrow)$$

$$\Sigma F_y = 0 = F + C_y + B_y = 30kN - 54,29kN + B_y$$

$$B_y = 54,29kN - 30kN = 24,29kN (\downarrow)$$

$$A = C$$

E csomópont:

$$\Sigma F_x = 0 = S_{1x} + S_{5x} = S_{1x} + 60kN$$

$$S_{1x} = -60kN (\leftarrow) [\text{húzott}]$$

$$S_{1y} = S_{1x} \cdot \frac{0,4m}{1,6m} = 15kN (\downarrow)$$

$$\Sigma F_y = 0 = S_{1y} + S_{5y} + S_3 = 15kN + 15kN + S_3$$

$$S_3 = -30kN (\uparrow) [\text{nyomott}]$$

D csomópont:

$$\Sigma F_x = 0 = S_{1x} + S_{2x} = 60kN + S_{2x}$$

$$S_{2x} = -60kN (\leftarrow) [\text{nyomott}]$$

$$S_{2y} = S_{2x} \cdot \frac{0,4m}{1,6m} = -15kN (\uparrow)$$

$$\Sigma F_y = 0 = F + S_{1y} + S_{2y} = 30kN - 15kN - 15kN$$

$$\Sigma M_A = 0 = -F \cdot 1,6m + S_{5x} \cdot 0,8m = -30kN \cdot 1,6m + S_{5x} \cdot 0,8m$$

$$S_{5x} = \frac{48kNm}{0,8m} = 60kN (\rightarrow) [\text{húzott}]$$

$$S_{5y} = S_{5x} \cdot \frac{0,4m}{1,6m} = 15kN (\downarrow)$$

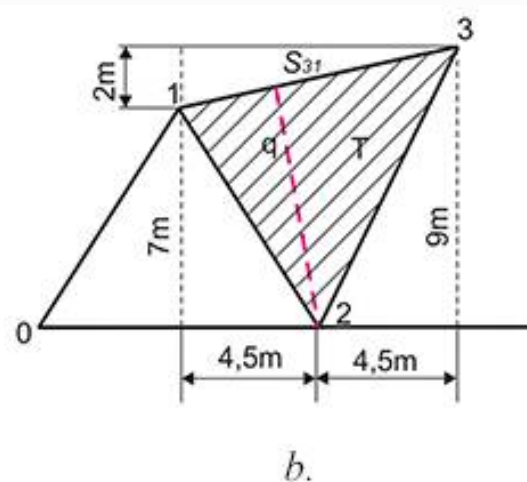
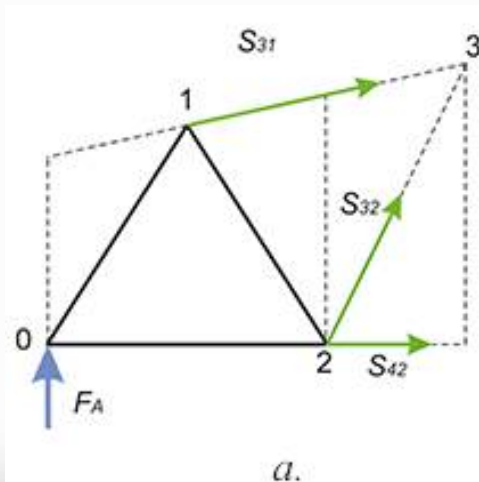
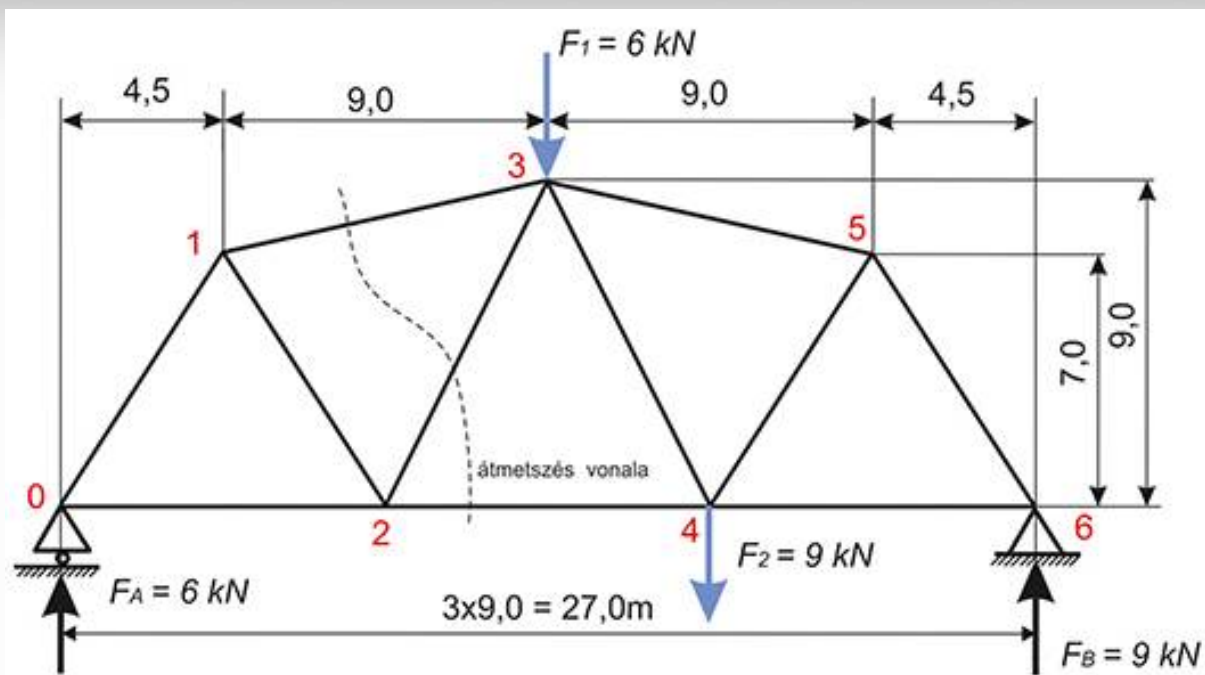
B csomópont:

$$\Sigma F_x = 0 = B_x + S_{5x} + S_{4x} = 22,86kN - 60kN + S_{4x}$$

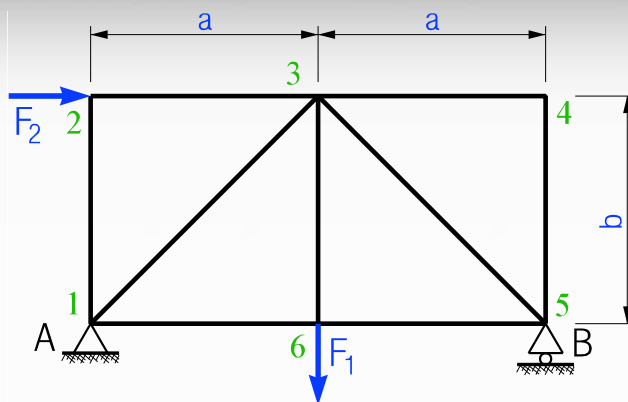
$$S_{4x} = 37,14kN (\rightarrow) [\text{nyomott}]$$

$$S_{4y} = S_{4x} \cdot \frac{0,4m}{1,6m} = 9,29kN (\uparrow)$$

Rácsos tartó



Csomóponti módszer – példa A / 1



$$\begin{aligned} F_1 &= 60 \text{ kN} \\ F_2 &= 20 \text{ kN} \\ a &= b = 3,0 \text{ m} \end{aligned}$$

$$\sum M_A = 0 = F_1 \cdot a + F_2 \cdot b - B \cdot 2a = 60 \text{ kN} \cdot 3,0 \text{ m} + 20 \text{ kN} \cdot 3,0 \text{ m} - B \cdot 6,0 \text{ m} = 180 \text{ kNm} + 60 \text{ kNm} - B \cdot 6,0 \text{ m} = 240 \text{ kNm} - B \cdot 6,0 \text{ m}$$

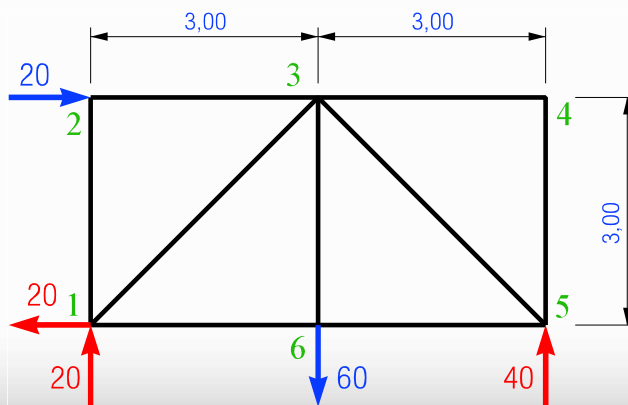
$$B = \frac{-240 \text{ kNm}}{6,0 \text{ m}} = -40,0 \text{ kN} (\uparrow)$$

$$\sum F_x = 0 = F_2 + A_x = 20 \text{ kN} + A_x$$

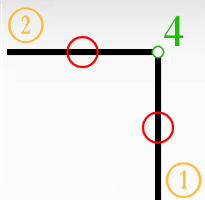
$$A_x = -20,0 \text{ kN} (\leftarrow)$$

$$\sum F_y = 0 = A_y + F_1 + B = A_y + 60 \text{ kN} - 40 \text{ kN}$$

$$A_y = -20,0 \text{ kN} (\uparrow)$$

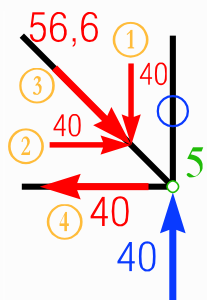


Csomóponti módszer – példa A / 2



$$\sum F_x = 0 \Rightarrow S_{3-4} = \mathbf{0\,kN} \text{ (vakrúd)}$$

$$\sum F_y = 0 \Rightarrow S_{4-5} = \mathbf{0\,kN} \text{ (vakrúd)}$$



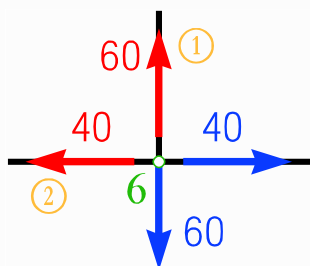
$$\sum F_y = 0 = S_{4-5} + B + S_{3-5,y}$$

$$S_{3-5,y} = \mathbf{-40,0\,kN} \text{ (↓) (nyomott)}$$

$$S_{3-5,x} = \frac{a}{b} \cdot S_{3-5,y} = \mathbf{-40,0\,kN} \text{ (→) (nyomott)}$$

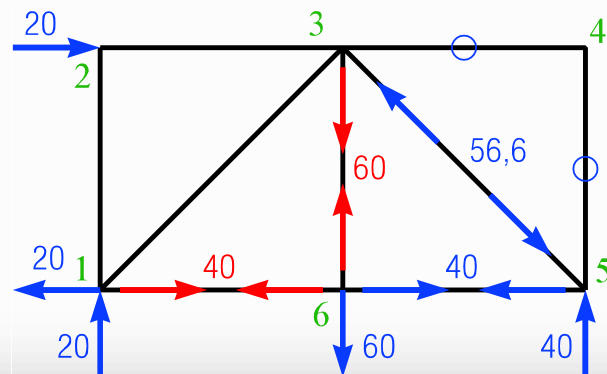
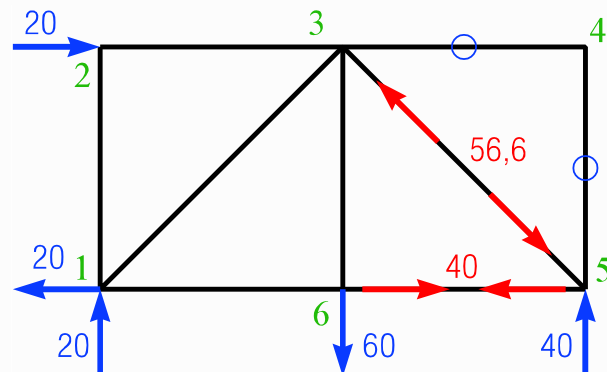
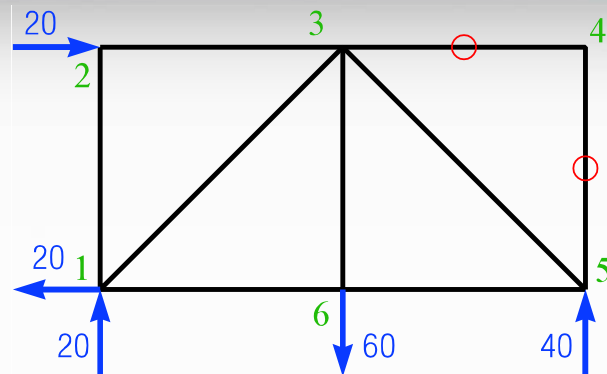
$$S_{3-5} = \sqrt{S_{3-5,x}^2 + S_{3-5,y}^2} = \mathbf{-56,6\,kN} \text{ (nyomott)}$$

$$\sum F_x = 0 = S_{3-5,x} + S_{5-6} \Rightarrow S_{5-6} = \mathbf{40,0\,kN} \text{ (húzott)}$$

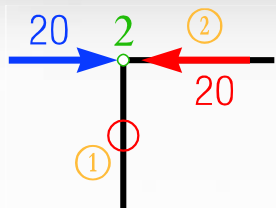


$$\sum F_x = 0 = S_{1-6} + S_{5-6} \Rightarrow S_{1-6} = \mathbf{40,0\,kN} \text{ (←) (húzott)}$$

$$\sum F_y = 0 = S_{3-6} + F_1 \Rightarrow S_{3-6} = \mathbf{60,0\,kN} \text{ (↑) (húzott)}$$

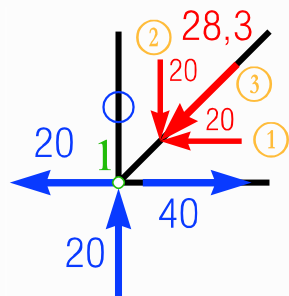


Csomóponti módszer – példa A / 3



$$\sum F_x = 0 = F_2 + S_{2-3} \Rightarrow S_{2-3} = -20,0 \text{ kN} (\leftarrow) (\text{nyomott})$$

$$\sum F_y = 0 \Rightarrow S_{1-2} = 0 \text{ kN} (\text{vakrúd})$$

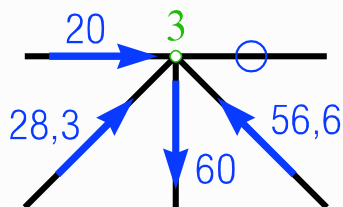


$$\sum F_x = 0 = S_{1-3,x} + S_{1-6} + A_x$$

$$S_{1-3,x} = -20,0 \text{ kN} (\leftarrow) (\text{nyomott})$$

$$S_{1-3,y} = \frac{b}{a} \cdot S_{1-3,x} = -20,0 \text{ kN} (\downarrow) (\text{nyomott})$$

$$S_{1-3} = \sqrt{S_{1-3,x}^2 + S_{1-3,y}^2} = -28,3 \text{ kN} (\text{nyomott})$$



Ellenőrzés:

$$\sum F_x = 0 = S_{2-3} + S_{3-4} + S_{1-3,x} + S_{3-5,x}$$

$$\sum F_y = 0 = S_{1-3,y} + S_{3-6} + S_{3-5,y}$$

