Mechanikai alapismeretek 1. Statika

Építészmérnök O/BSc, Építőművész BA

HÁROMCSUKLÓS TARTÓK IGÉNYBEVÉTELI ÁBRÁI

Pécsi Tudományegyetem · Magyarország első egyeteme® 1367
Műszaki és Informatikai Kar · Építőmérnök Tanszék
Weinreich Zoltán, 2013-20

Háromcsuklós tartó – Torda, sóbánya

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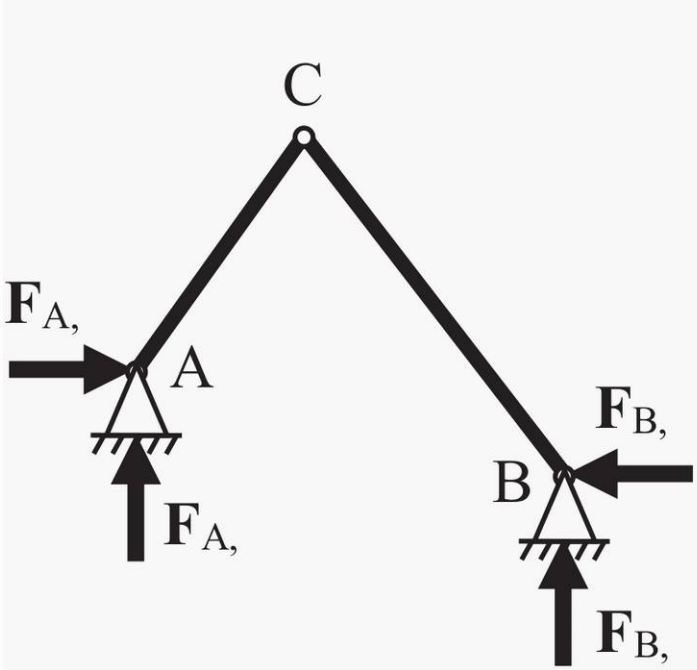
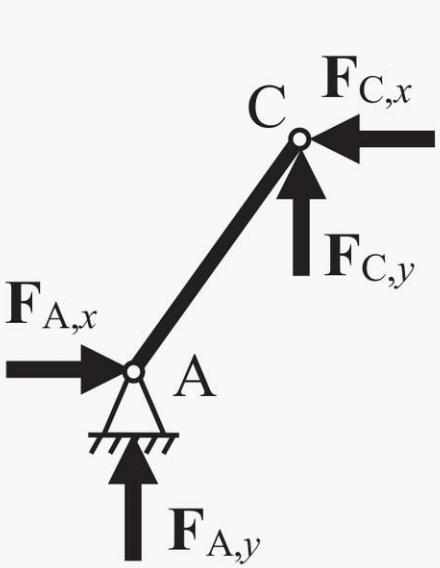
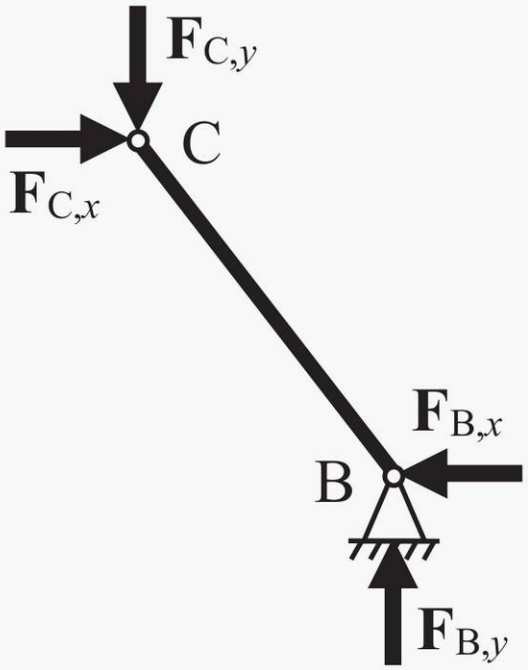
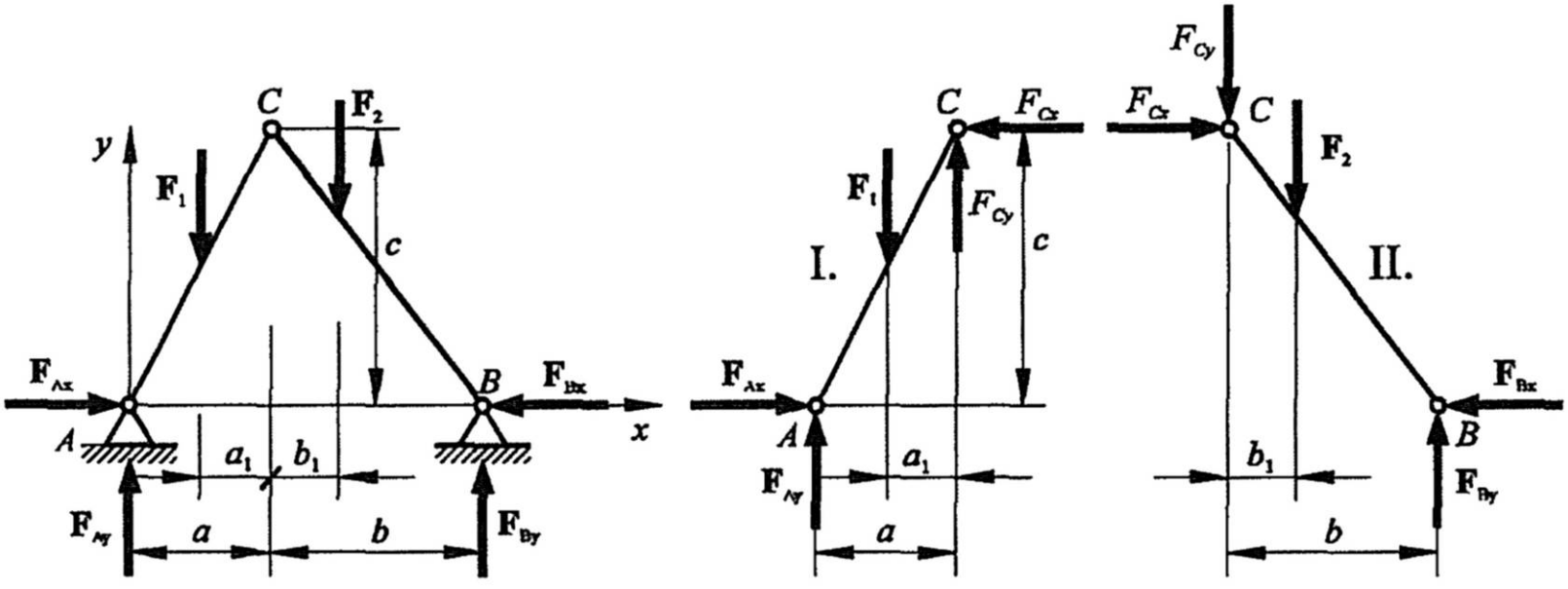
Háromcsuklós tartó







Háromcsuklós tartó



Teljes tartó vizsgálata:

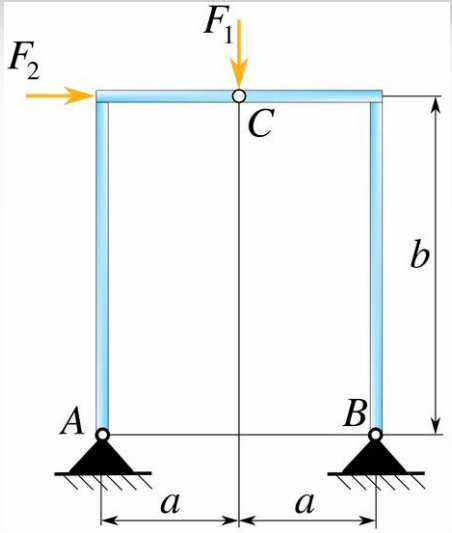
$$\sum M_A = 0 = F_1 \cdot (a - a_1) + F_2 \cdot (a + b_1) - F_{By} \cdot (a + b) \quad \rightarrow \quad F_{By}$$
$$\sum F_y = 0 = -F_{Ay} + F_1 + F_2 - F_{By} \quad \rightarrow \quad F_{Ay}$$
$$\sum F_x = 0 = F_{Ax} - F_{Bx} \quad \rightarrow \quad F_{Ax} = F_{Bx}$$

A – C tartószakasz vizsgálata:

$$\sum M_C = 0 = F_1 \cdot a_1 + F_{Ax} \cdot c - F_{Ay} \cdot a \quad \rightarrow \quad F_{Ax}$$

Háromcsuklós tartó támaszerői

$a = 2{,}0\,m$
$b = 5{,}0\,m$
$F_1 = 100\,kN$
$F_2 = 50\,kN$



Teljes tartó vizsgálata:

$$\sum M_A = 0 = F_1 \cdot a + F_2 \cdot b - F_{By} \cdot 2 \cdot a = 100{,}0\,kN \cdot 2{,}0\,m + 50{,}0\,kN \cdot 5{,}0\,m - F_{By} \cdot 4{,}0\,m = 200{,}0\,kNm + 250{,}0\,kNm - F_{By} \cdot 4{,}0\,m$$

$$F_{By} = \frac{450{,}0\,kNm}{4{,}0\,m} = 112{,}5\,kN\ (\uparrow)$$

$$\sum F_y = 0 = -F_{Ay} + F_1 - F_{By} = -F_{Ay} + 100{,}0\,kN - 112{,}5\,kN$$

$$F_{Ay} = = 12{,}5\,kN\ (\downarrow)$$

$$\sum F_x = 0 = F_{Ax} + F_2 - F_{Bx}$$

B – C tartószakasz vizsgálata:

$$\sum M_C = 0 = -F_{By} \cdot a + F_{Bx} \cdot b = -112{,}5\,kN \cdot 2{,}0\,m + F_{Bx} \cdot 5{,}0\,m = -225{,}0\,kNm + F_{Bx} \cdot 5{,}0\,m$$

$$F_{Bx} = \frac{225{,}0\,kNm}{5{,}0\,m} = 45{,}0\,kN\ (\leftarrow)$$

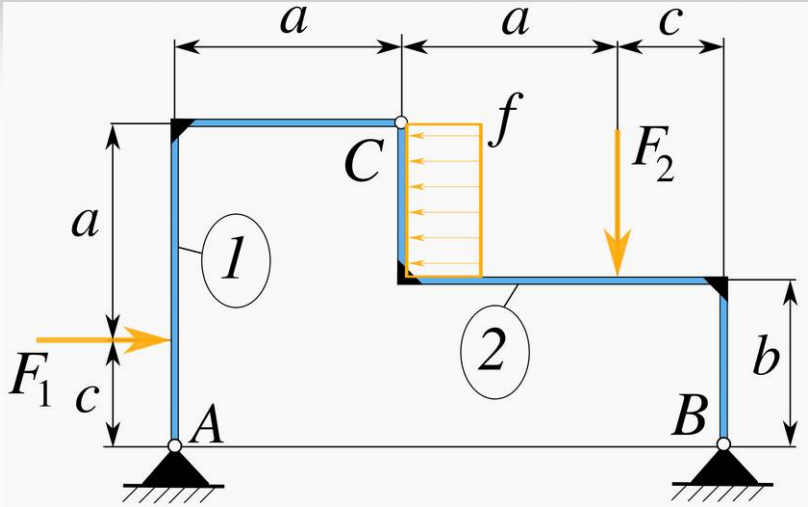
$$\sum F_x = 0 = F_{Ax} + F_2 - F_{Bx} = F_{Ax} + 50{,}0\,kN - 45{,}0\,kN = F_{Ax} + 5{,}0\,kN$$

$$F_{Ax} = = 5{,}0\,kN\ (\leftarrow)$$

Háromcsuklós tartó támaszerői

$a = 2{,}0\,m$
$b = 1{,}5\,m$
$c = 1{,}0\,m$
$F_1 = 3\,kN$
$F_2 = 2\,kN$
$f = 4\,kN/m$

$d = a + c - b = 2{,}0 + 1{,}0 - 1{,}5 = 1{,}5\,m$



Teljes tartó vizsgálata:

$$\sum M_A = 0 = F_1 \cdot c + F_2 \cdot 2{\cdot}a - f \cdot d \cdot \left(b + \frac{d}{2}\right) - F_{By} \cdot (2{\cdot}a + c)$$
$$= 3{,}0\,kN \cdot 1{,}0\,m + 2{,}0\,kN \cdot 4{,}0\,m - 4{,}0\,kN/m \cdot 1{,}5\,m \cdot 2{,}25\,m - F_{By} \cdot 5{,}0\,m$$
$$= 3{,}0\,kNm + 8{,}0\,kNm - 13{,}5\,kNm - F_{By} \cdot 5{,}0\,m = -2{,}5\,kNm - F_{By} \cdot 5{,}0\,m$$

$$F_{By} = \frac{2{,}5{,}0\,kNm}{5{,}0\,m} = 0{,}5\,kN\ (\downarrow)$$

$$\sum F_y = 0 = -F_{Ay} + F_2 + F_{By} = -F_{Ay} + 2{,}0\,kN + 0{,}5\,kN$$

$$F_{Ay} = = 2{,}5\,kN\ (\uparrow)$$

$$\sum F_x = 0 = F_{Ax} + f \cdot d - F_{Bx}$$

A – C tartószakasz vizsgálata:

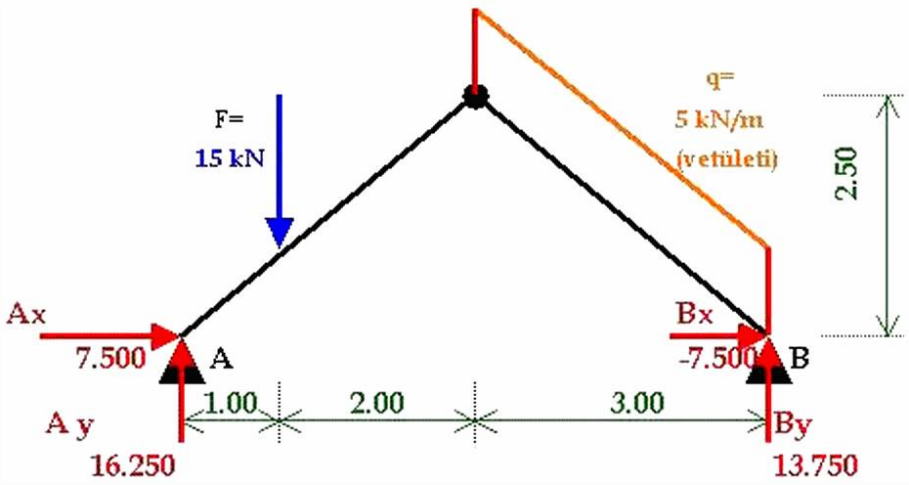
$$\sum M_C = 0 = -F_1 \cdot a + F_{Ay} \cdot a + F_{Ax} \cdot (a + c) = -3{,}0\,kN \cdot 2{,}0\,m + 2{,}5\,kN \cdot 2{,}0\,m + F_{Ax} \cdot 3{,}0\,m$$
$$= -6{,}0\,kNm + 5{,}0\,kNm + F_{Ax} \cdot 3{,}0\,m = -1{,}0\,kNm + F_{Ax} \cdot 3{,}0\,m$$

$$F_{Ax} = \frac{1{,}0\,kNm}{3{,}0\,m} = 0{,}33\,kN\ (\leftarrow)$$

$$\sum F_x = 0 = F_{Ax} + F_1 - f \cdot d - F_{Bx} = -0{,}33\,kN + 3{,}0\,kN - 4{,}0\,kN/m \cdot 1{,}5\,m + F_{Bx} = -3{,}33\,kN + F_{Bx}$$

$$F_{Bx} = = 3{,}33\,kN\ (\rightarrow)$$

Háromcsuklós tartó támaszerői



Teljes tartónál nyomatéki egyenlet "A", majd "B" pontra:

$\sum M(A)=0=15 \cdot 1 + 5 \cdot 3 \cdot 4.5 - By \cdot 6$ $By=(15 \cdot 1 + 5 \cdot 3 \cdot 4.5)/6=$ **13.750 kN(↑)**

$\sum M(B)=0=Ay \cdot 6 - 15 \cdot 5 - 5 \cdot 3 \cdot 1.5$ $Ay=(15 \cdot 5+5 \cdot 3 \cdot 1.5)/6=$ **16.250 kN(↑)**

Ellenőrzés y irányú erőkre:

$\sum Fy= -16.25 + 15 + 5 \cdot 3 - 13.75=$ **0.000**

Csuklónál szétbontás, csak a bal oldal vizsgálata. Nyomatéki egyenlet a "C" pontra:

$\sum M(C)=0=16.25 \cdot 3 - Ax \cdot 2.5 - 15 \cdot 2$ $Ax=(16.25 \cdot 3 - 15 \cdot 2)/2.5=$ **7.500 kN(→)**

Nyomatéki egyenlet a "C" pontra a jobb oldali részből:

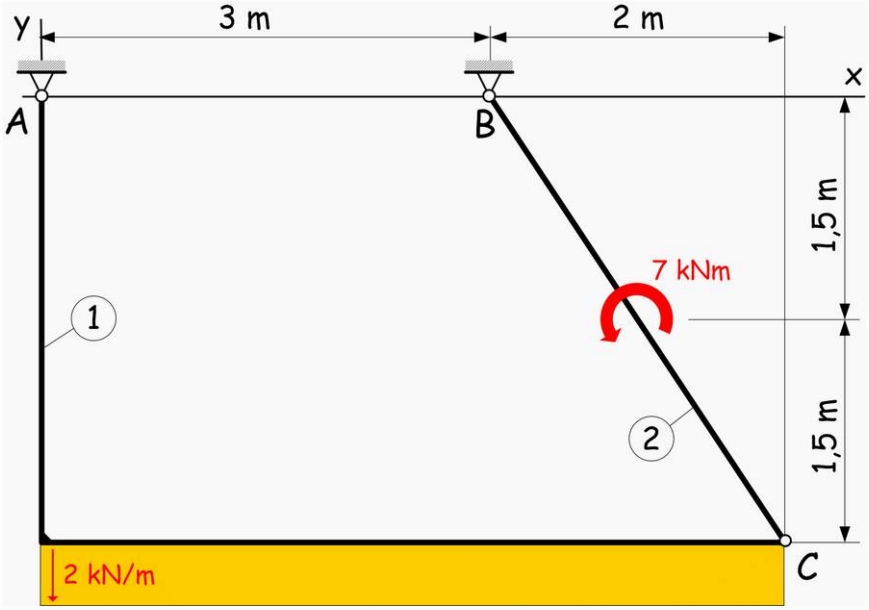
$\sum M(C)=0=5 \cdot 3 \cdot 1.5 - 13.75 \cdot 3 - Bx \cdot 2.5$

$Bx=(5 \cdot 3 \cdot 1.5 - 13.75 \cdot 3)/2.5=$ **-7.500 kN(←)**

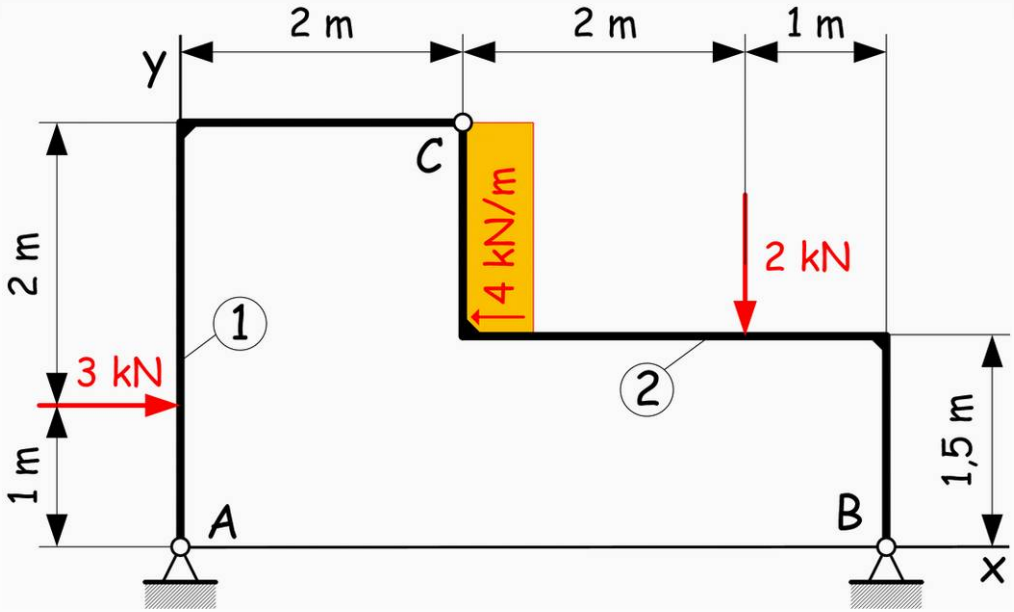
Ellenőrzés x irányú erőkre:

$\sum Fx=Ax + Bx=7.5 -7.5 =$ **0.000**

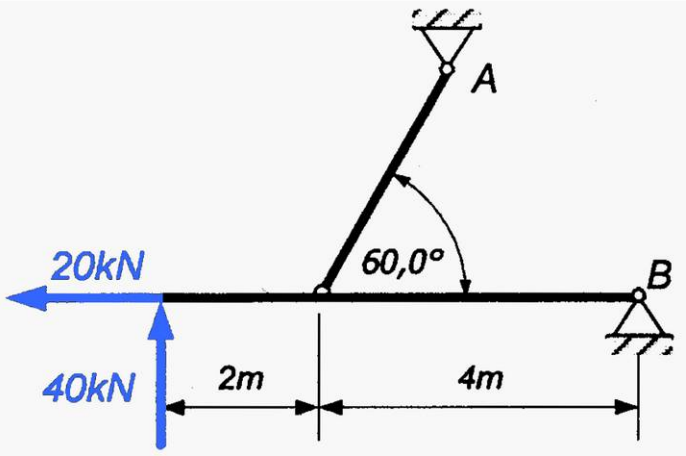
Háromcsuklós tartó támaszerői



Háromcsuklós tartó támaszerői

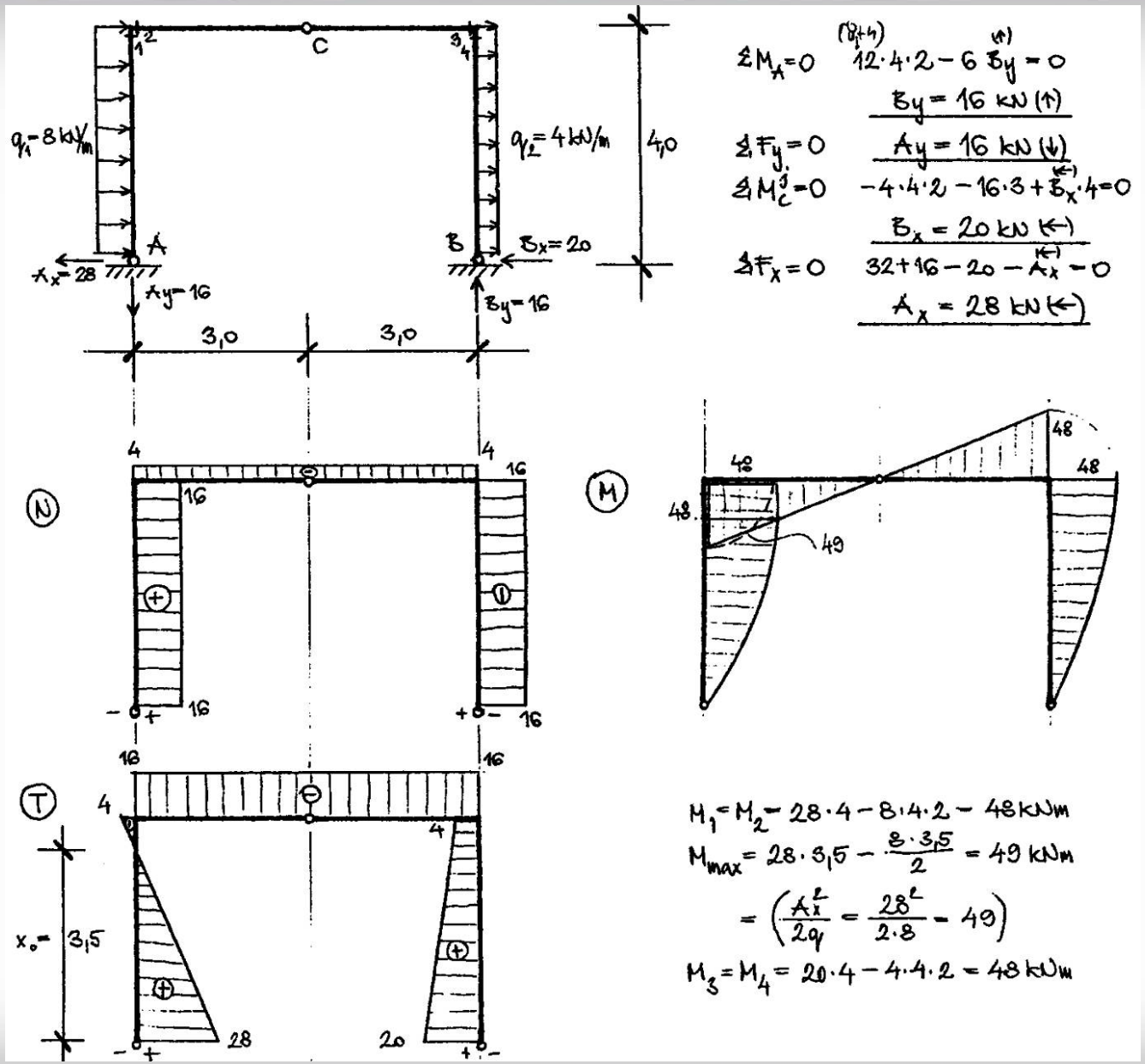


Háromcsuklós tartó támaszerői



$$\vec{F}_A = \left(-60\vec{e}_y - 34,6\vec{e}_z\right) kN \ , \ \vec{F}_B = \left(20\vec{e}_y + 54,6\vec{e}_z\right) kN$$

Háromcsuklós tartó belsőerő ábrái



$\Sigma M_A = 0 \quad 12 \cdot 4 \cdot 2 - 6\, B_y = 0$

$B_y = 16$ kN (↑)

$\Sigma F_y = 0 \quad A_y = 16$ kN (↓)

$\Sigma M_C^j = 0 \quad -4 \cdot 4 \cdot 2 - 16 \cdot 3 + B_x \cdot 4 = 0$

$B_x = 20$ kN (←)

$\Sigma F_x = 0 \quad 32 + 16 - 20 - A_x = 0$

$A_x = 28$ kN (←)

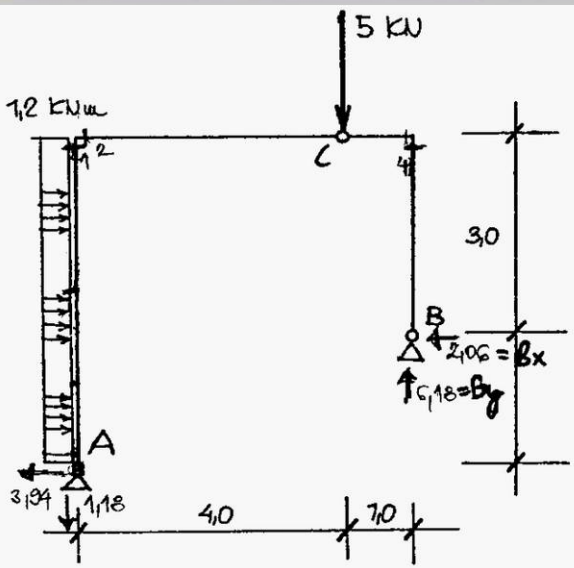
$M_1 = M_2 = 28 \cdot 4 - 8 \cdot 4 \cdot 2 = 48$ kNm

$M_{max} = 28 \cdot 3{,}5 - \frac{8 \cdot 3{,}5}{2} = 49$ kNm

$= \left(\frac{A_x^2}{2q} = \frac{28^2}{2 \cdot 8} = 49 \right)$

$M_3 = M_4 = 20 \cdot 4 - 4 \cdot 4 \cdot 2 = 48$ kNm

Háromcsuklós tartó belsőerő ábrái



$$\Sigma M_A = 0 = 1{,}2 \cdot 5 \cdot 2{,}5 + 5 \cdot 4 - B_y \cdot 5 - B_x \cdot 2$$

$$\Sigma M_{C_j} = 0 = B_x \cdot 3 - B_y \cdot 1 = 0$$

$$B_x = \frac{B_y}{3}$$

$$\Sigma M_1;\ B_y = \frac{5 \cdot 1{,}2 \cdot \frac{5}{2} + 5 \cdot 4 - B_x 2}{5}$$

$$B_y = 7 - 0{,}4\, B_x$$

$$B_x = \frac{1 \cdot B_y}{3} - \frac{B_y}{3}$$

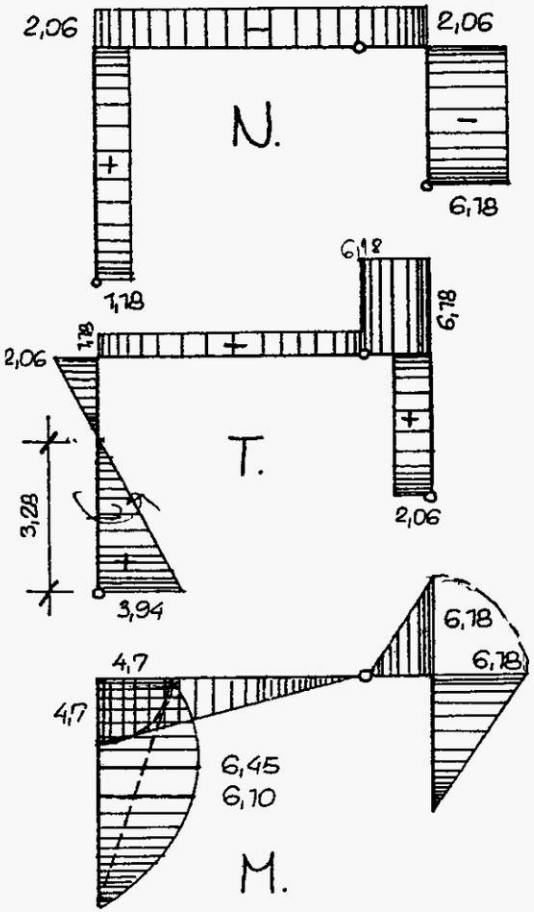
$$B_y = 7 - \frac{0{,}4 \cdot B_y}{3}$$

$$B_y = \frac{21}{3{,}4} = 6{,}18\ \text{kN} \uparrow$$

$$B_x = 2{,}06\ \text{kN} \leftarrow$$

$$\Delta_y = 5 - 6{,}18 = -1{,}18\ \text{kN} \downarrow$$

$$\Delta_x = 6 - 2{,}06 = 3{,}94\ \text{kN} \leftarrow$$



$$x_0 = \frac{3{,}94}{1{,}2} = 3{,}28\ \text{m}$$

$$M_{bf}^{1;2} = 5 \cdot 3{,}94 - 5 \cdot 1{,}2 \cdot 2{,}5 = 4{,}7$$

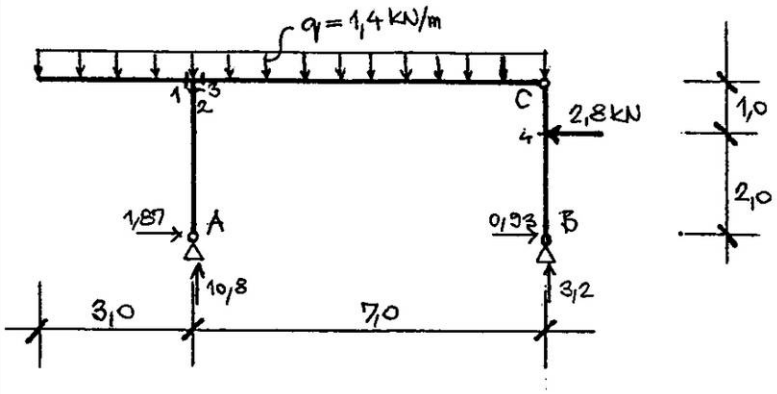
$$M_{max}^{(+)} = \frac{4{,}7}{2} + \frac{1{,}2 \cdot 5^2}{8} = 6{,}70\ \text{kNm}$$

$$M_{max}^{(+)} = 3{,}94 \cdot 3{,}28 - \frac{1{,}2 \cdot 3{,}28^2}{2} =$$

$$M_{max}^{(+)} = 6{,}45\ \text{kNm}$$

$$M_{3;4} = -2{,}06 \cdot 3 = -6{,}18\ \text{kNm}$$

Háromcsuklós tartó belsőerő ábrái



$\sum M^A = 0 = 2 \cdot 10 \cdot 1{,}4 - 2 \cdot 2{,}8 - 7\,B_y$

$B_y = \frac{22{,}4}{7} = 3{,}2\,kN\,(\uparrow)$

$\sum F_y = 0 = 10 \cdot 1{,}4 - 3{,}2 + A_y$

$A_y = 10{,}8\,kN\,(\uparrow)$

$\sum M^c_{jobb} = +1 \cdot 2{,}8 - 3B_x = 0$

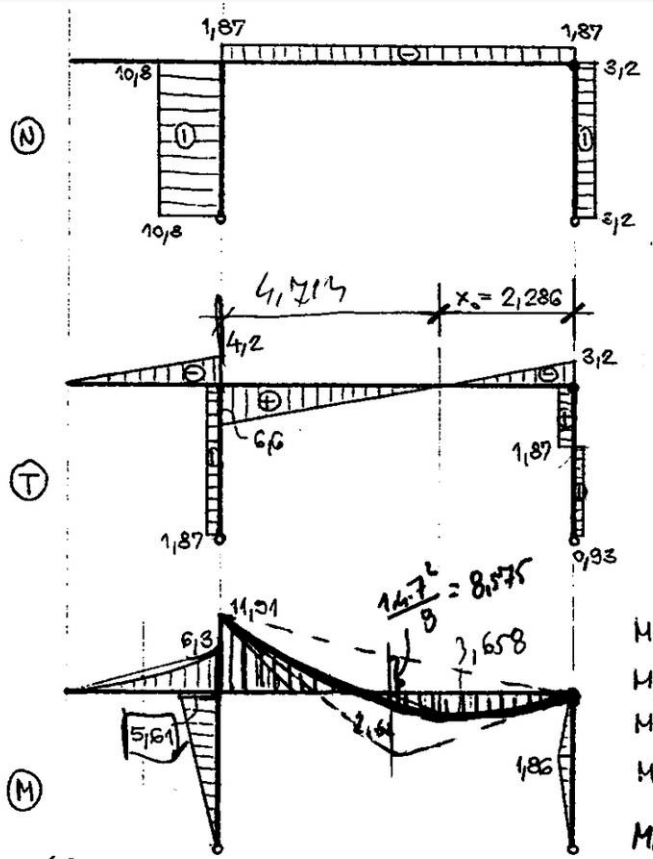
$B_x = \frac{2{,}8}{3} = 0{,}93\,kN\,(\rightarrow)$

$\sum P_x = 0 = -2{,}8 + 0{,}93 + A_x$

$A_x = 1{,}87\,kN\,(\rightarrow)$

Ellenőrzés:

$\sum M^c_{bal} = -\frac{10^2}{2} \cdot 1{,}4 - 3 \cdot 1{,}87 + 7 \cdot 10{,}8 = 0$



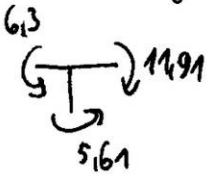
$M_1 = \frac{3^2}{2} \cdot 1{,}4 = 6{,}3\,kNm$

$M_2 = 3 \cdot 1{,}87 = 5{,}61\,kNm$ ✓

$M_3 = 6{,}3 + 5{,}61 = 11{,}91\,kNm$

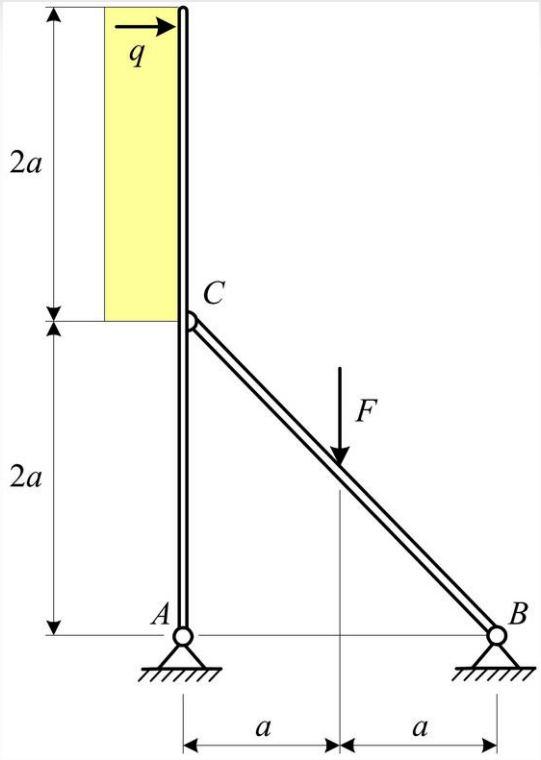
$M_4 = 2 \cdot 0{,}93 = 1{,}86\,kNm$

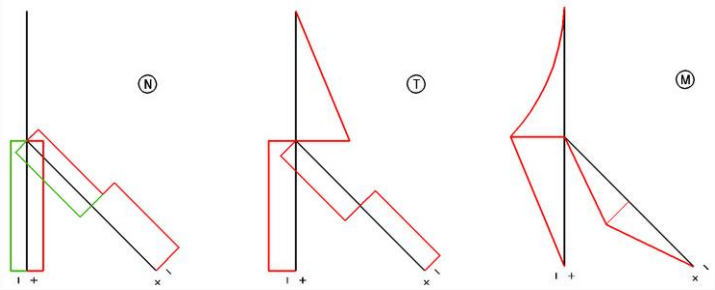
$M_{max} = 3{,}2 \cdot \frac{4{,}286}{2} = 3{,}658\,kNm$



$M_{max} = -1{,}4 \cdot \frac{7{,}714^2}{2} - 1{,}87 \cdot 3 + 10{,}8 \cdot 4{,}714 = 3{,}658\,kNm$

Háromcsuklós tartó – HF1





Teljes tartó vizsgálata:

$$\sum M_A = 0 = F \cdot a + q \cdot 2a \cdot 3a - B_y \cdot 2a = F \cdot a + 6 \cdot q \cdot a^2 - B_y \cdot 2a$$

$$B_y = \frac{F \cdot a + 6 \cdot q \cdot a^2}{2a} = \frac{F}{2} + 3 \cdot q \cdot a \ (\uparrow)$$

$$\sum F_y = 0 = F + A_y + B_y = F + A_y - \frac{F}{2} - 3 \cdot q \cdot a = A_y + \frac{F}{2} - 3 \cdot q \cdot a$$

$$A_y = \frac{F}{2} - 3 \cdot q \cdot a \ (\uparrow) \quad \left[\frac{F}{2} > 3 \cdot q \cdot a \right]$$

$$A_y = 3 \cdot q \cdot a - \frac{F}{2} \ (\downarrow) \quad \left[\frac{F}{2} < 3 \cdot q \cdot a \right]$$

$$\sum F_x = 0 = A_x + q \cdot 2a + B_x$$

B – C tartószakasz vizsgálata:

$$\sum M_C = 0 = F \cdot a - B_y \cdot 2a + B_x \cdot 2a = a \cdot \left(F - 2 \cdot B_y + 2 \cdot B_x \right)$$

$$B_x = B_y - \frac{F}{2} = 3 \cdot q \cdot a \ (\leftarrow)$$

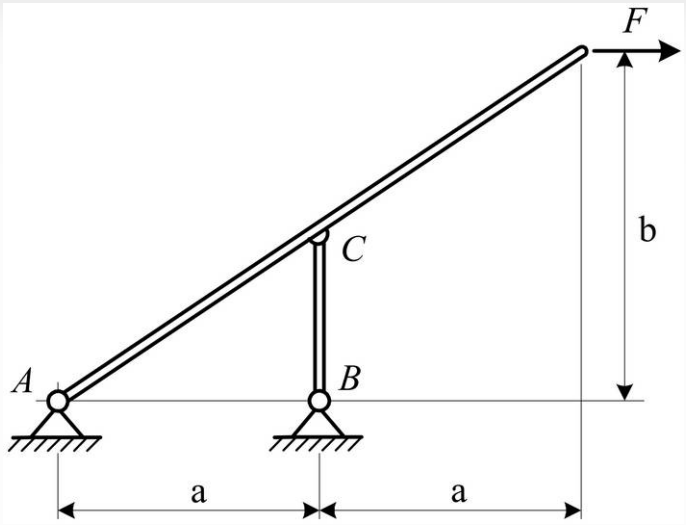
Teljes tartó vizsgálata:

$$\sum F_x = 0 = A_x + q \cdot 2a + B_x = A_x + q \cdot 2a - q \cdot 3a = A_x - q \cdot a$$

$$A_x = q \cdot a \ (\rightarrow)$$

Az igénybevételi ábrák alakhelyesek.
A terhelés függvényében az arányok és az előjelek változhatnak.

Háromcsuklós tartó – HF2



Teljes tartó vizsgálata :

$$\sum M_A = 0 = F \cdot b - B_y \cdot a$$

$$B_y = \frac{F \cdot b}{a} \; (\uparrow)$$

$$\sum F_y = 0 = A_y + B_y = A_y - \frac{F \cdot b}{a}$$

$$A_y = \frac{F \cdot b}{a} \; (\downarrow)$$

$$\sum F_x = 0 = A_x + B_x + F$$

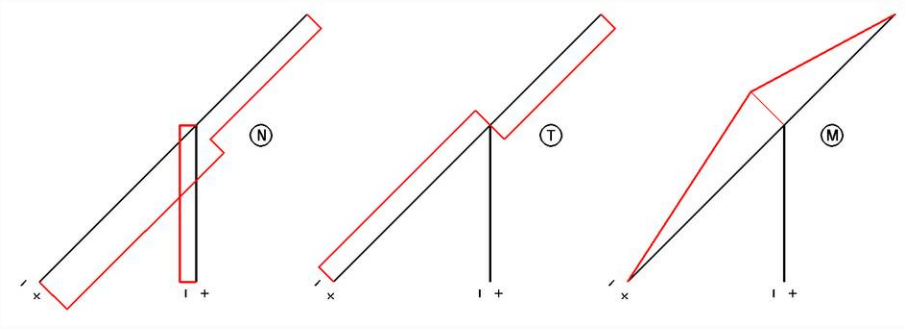
B – C tartószakasz vizsgálata :

$$B_x = 0$$

Teljes tartó vizsgálata :

$$\sum F_x = 0 = A_x + F + B_x = A_x + F$$

$$A_x = F \; (\leftarrow)$$



Az igénybevételi ábrák alakhelyesek .
A terhelés függvényében az arányok és az előjelek változhatnak .